Contextualist theories of the indicative conditional and

Stalnaker’s Thesis

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Abstract

Lewis (1976) argued that ‘there is no way to interpret a conditional connective so that, with sufficient generality, the probabilities of conditionals will equal the appropriate conditional probabilities’. However, as Lewis and others have subsequently recognized, Lewis’ triviality results go through only on the assumption that ‘if’ is not context-sensitive. This leaves a question that has not been adequately addressed: what are the prospects of a context-sensitive theory of ‘if’ that complies with Stalnaker’s Thesis? I offer one interesting constraint on any such theory. I argue that no context-sensitive theory satisfies Stalnaker’s Thesis if it satisfies three plausible assumptions: first, that the truth of an indicative is determined by the world of evaluation and by the set of worlds in the relevant epistemic context in which the antecedent is true; second, that one can learn an indicative conditional without learning that the antecedent and consequent are both true; third, that belief revision is conservative in the sense that it does not reduce the probabilities to zero unnecessarily. The result gives us a clearer picture of the real costs of a truth-conditional context-sensitive Stalnaker’s Thesis-compliant
semantics.

For philosophers who accepted Stalnaker’s Thesis (ST), the claim that the probability of an indicative conditional is equal to the conditional probability of the consequent on the antecedent, Lewis’ triviality results (Lewis, 1976) spelled trouble. The results were strengthened by Hájek (1993), Hájek and Hall (1994), Milne (2003), and others. The results keep coming: Hájek (2011) and Bradley (2007) are recent examples. The common wisdom is that triviality results force a choice: abandon ST, or adopt a non-truth-conditional semantics for indicatives. However, this conclusion is too hasty, as van Fraassen (1976), Lewis (1986) and Hájek (1993) have already recognized: Lewis-style results in fact only show triviality for the class of non-context-sensitive truth-conditional theories of the indicative (I spell this out in §1).

Since Lewis’ results only show that no truth-conditional context-in-sensitive theory of indicatives complies with ST, the job of the triviality theorist is only half-done, since the available results do not remove the hope of finding a truth-conditional context-sensitive theory that would satisfy ST. In the absence of parallel results for context-sensitive theories, the friends of ST will construe Lewis’ argument as an argument for context-sensitivity (in fact, there are some positive reasons to so construe it, see §1). So the following question is pressing: what are the prospects of an ST-compliant context-sensitive theory of indicatives? Surprisingly, little work has been done on this issue.

We have a partial answer to the question just raised. Bas van Fraassen (1976) early on provided an example of a context-sensitive theory that satisfies ST. Andrew Bacon (2014) recently improved on van Fraassen’s Stalnaker-Bernoulli models, removing some of their limitations.

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1 It is also well-known that one can avoid the triviality results by adopting a truth-conditional semantics with non-classical truth-values, as in Stalnaker and Jeffrey (1994). In this paper I restrict my attention to truth-conditional theories with classical truth-values.

2 A few results do apply to context-sensitive theories as well – see §2 for more details.

3 Kratzer (1986) has another response to Lewis’ argument, a response that suggests that the intuition behind ST really applies to probability conditionals, sentences of the form ‘if P, then probably Q’; these are true just in case the conditional probability is sufficiently high. But Kratzer’s response does not respect ST for ordinary indicative conditionals, and so is irrelevant here.
These results are extremely interesting, but the theories that we are offered are quite complicated, and van Fraassen’ approach has won few adherents. So while van Fraassen’s results answer the question of the tenability of ST in the affirmative, they leave the larger question of the prospects of a truth-conditional ST semantics unresolved. To answer that question, we need to find which features of these models are essential and which not, and to evaluate the plausibility of the essential features. In the end, one would like to know the real cost of a truth-conditional semantics that complies with ST.

This last question can be answered only by further work on triviality results for context-sensitive theories of the indicative. Such triviality results would show the essential limitations of context-sensitive models of ST, and thus show the real cost of accepting a truth-conditional ST-compliant semantics. We already have two such results: Stalnaker (1976) and Hájek and Hall (1994). Stalnaker’s result shows that no truth-conditional semantics that supports Stalnaker’s logic C2 can comply with ST. But Stalnaker’s proof makes essential use of the axiom of weakened transitivity, \((\phi \rightarrow \psi) \& (\psi \rightarrow \phi) \& (\phi \rightarrow \chi) \models (\phi \rightarrow \chi)\), and several authors, beginning with van Fraassen, have found it suspect. In addition, there is Hajek’s proof that the models of ST are necessarily infinite. The result is interesting, but it is not clear what principled reasons there are to insist that the models can be finite. So it is of interest to see what other triviality results are available for context-sensitive theories. In §2 I offer one such result.

The result I offer points to another essential feature of van Fraassen & Bacon models, and thus of all truth-conditional theories that comply with ST. I will argue that no such theory can satisfy all of three plausible principles. According to Moderate Context-Dependence (M), the truth of an indicative conditional \(A \rightarrow C\) at a world \(w\) in a context \(c\) is determined by the choice of \(w\) and by the set of \(A\)-worlds belonging to \(c\). Moderate Context-Dependence thus

\footnote{For criticism and further references see Bacon (2013) and Ahmed (2011).}
implies that the truth of an indicative depends neither on the probability distribution on \( c \) nor on which \( \neg A \)-worlds belong to \( c \). According to the Learnability Condition (L) it is possible to learn that \( A \rightarrow C \) is true without learning \( A \& C \). Finally, according to Conservativity, rationally revising one's beliefs in light of evidence \( E \) will not collapse the probability of any proposition \( A \) to zero if \( p(A \& E) \) was non-zero before the revision.

In view of these constraints, it seems that Lewis' original intuition proves largely correct: the best hope for the ST-theorist is to pursue a non-truth-conditional semantics, or else a semantics with non-classical truth-values. More importantly, however, the new result sets a clear task for the truth-conditionalist: to justify and motivate the rejection of M or L (since Conservativity seems hard to dispute). In §1 below, I will sketch Lewis' original proof, and explain just where the assumption that \( \rightarrow \) is not context-dependent comes in. In §2, I spell out my result.

1 Lewis' proof

Consider the simplest version of Lewis' result. Assume that \( A \rightarrow C \) expresses a proposition, and assume ST: \( p(A \rightarrow C) = p(C|A) \). By the probability calculus we have that

\[
(\text{i}) \quad p(A \rightarrow C) = p(A \rightarrow C|C)p(C) + p(A \rightarrow C|\neg C)p(\neg C)
\]

Assume further the Import-Export Lemma:

\[
(\text{2}) \quad p((A \& B) \rightarrow C) = p(A \rightarrow C|B)
\]

Now, pick \( A \) and \( C \) such that \( p(A) < 1 \), \( p(A \& C) > 0 \) and \( p(A \& \neg C) > 0 \). Combining (i) and (2), and assuming ST, we get
\( p(A \rightarrow C) = p(C | A) = p(A \rightarrow C | C) p(C) + p(A \rightarrow C | \neg C) p(\neg C) = p(C | A \& C) p(C) + p(C | A \& \neg C) p(\neg C) = p(C) \)

\( p(A \rightarrow C | C) = p((A \& C) \rightarrow C) = p(C | A \& C) = 1, \) by an application of (2), and then ST; likewise \( p(A \rightarrow C | \neg C) = p((A \& \neg C) \rightarrow C) = p(C | A \& \neg C) = 0). \)

But, that is absurd: we have proven that A and C are probabilistically independent just on the assumption that both \( p(A \& C) \) and \( p(A \& \neg C) \) are positive, and \( p(A) < 1. \) The real action is in the argument for the key lemma (2). First define \( p_B() \) as \( p( | B). \) Then defense of the lemma (2) goes as follows:

\[
\begin{align*}
(4) & \quad p(A \rightarrow C | B) = p_B(A \rightarrow C) & \text{(by definition of } p_B()) \\
(5) & \quad p_B(A \rightarrow C) = p_B(C | A) & \text{(by ST for } p_B()) \\
(6) & \quad p_B(C | A) = p_B(C \& A) / p_B(A) & \text{(by definition of } p( | )) \\
(7) & \quad p_B(C \& A) / p_B(A) = p(C \& A | B) / p(A | B) & \text{(by definition of } p_B()) \\
(8) & \quad p(C \& A | B) / p(A | B) = p(C \& A \& B) / p(A \& B) & \text{(by definition of } p( | )) \\
(9) & \quad p(C \& A \& B) / p(A \& B) = p((A \& B) \rightarrow C) & \text{(by ST for } p()) 
\end{align*}
\]

Equalities (4) and (7) follow from the definition of \( p_B. \) Equalities (5) and (9) from ST for \( p \) and \( p_B. \) Equalities (6) and (8) from the definition of conditional probability.

So what’s the catch? Why is the argument for (2) too hasty? Because the assumption that ST holds for \( p \) and \( p_B \) hides a significant further assumption, obscured in the proof leading up to (9): the assumption that \( \rightarrow \) is not context-sensitive. So, suppose that \( \rightarrow \) is context-sensitive. For example, it is a live possibility that \( \rightarrow \) is sensitive to the epistemic context: the set of worlds consistent with what is known, or taken for granted, in the context of utterance. Then let’s say that \( A \rightarrow_p C \) denotes the proposition expressed by ‘if A, then C’ in the epis-
temic context associated with $p()$, and $A \rightarrow_{pB} C$ the proposition expressed in the epistemic context associated with $p_{B}()$. The lemma as a whole, then, ought to give us

\[(10) \quad p(A \rightarrow_{p} C | B) = p((A \& B) \rightarrow_{p} C)\]

(it is the triviality of $p()$ that we are proving). And so, keeping track of context-sensitivity, (4) ought to be

\[(11) \quad p(A \rightarrow_{p} C | B) = p_{B}(A \rightarrow_{p} C)\]

But then, to pick up on the right-hand side of (11), (5) would have to be

\[(12) \quad p_{B}(A \rightarrow_{p} C) = p_{B}(C | A)\]

Note, however, that ST, applied to $p_{B}()$, does not give us (12), but only

\[(13) \quad p_{B}(A \rightarrow_{pB} C) = p_{B}(C | A)\]

And if $\rightarrow$ is context-sensitive, there is no reason to expect that (12) is true. The proof of the lemma goes through only on the assumption that $p_{B}(A \rightarrow_{pB} C) = p_{B}(A \rightarrow_{p} C)$ -- on the assumption that $\rightarrow$ is not context-sensitive. But this leaves open the possibility that the ST-theorist can escape triviality with a context-sensitive theory of the indicative.\(^5\)

How seriously ought we to take the possibility of a context-sensitive ST-compliant theory of the indicative? In his (1976), Lewis summarizes his conclusions as follows: ‘there is no way to interpret a conditional connective so that, with sufficient generality, the probabilities of conditionals will equal the appropriate conditional probabilities’ – rejecting, in effect, the context-sensitive option (Lewis 1976, p. 298). Lewis goes on: ‘But presumably our indicative

\[^{5}\] All other triviality results that I know of, besides the Stalnaker and Hajek ones mentioned above, are susceptible to the same kind of diagnosis: they depend on the assumption that the indicative is not context-sensitive.
conditional has a fixed interpretation, the same for speakers with different beliefs, and for
one speaker before and after a change in his beliefs. Else how are disagreements about a

But despite Lewis’ early misgivings context-sensitive theories of the indicative conditional
(and other modal constructions) – both in the tradition of Stalnaker (1975) and in the tradi-
tion of Kratzer (1986) – have been highly successful in the past thirty years. Certainly, a very
significant portion of the current work on the semantics of the conditional is on the context-
sensitive side. This alone is reason enough to consider the context-sensitive ST-compliant
option a live one. This is so despite the fact that few of the recent truth-conditional propos-
als are ST-compliant (although see Bacon (2013)). If context-sensitive theories of indicative
conditionals are to be taken seriously, then so are ST-compliant context-sensitive theories.

Could one have evidence that context-sensitivity is the right lesson to draw from Lewis-
style triviality results? To some extent, yes. If failure to acknowledge context-sensitivity,
rather than Stalnaker’s Thesis, is responsible for triviality, then we should expect other trivi-
ality results to be available, results that do not appeal to ST, but to other assumptions about
the semantics of the indicative conditional. And we do in fact have one such result: Bradley
(2007). Bradley shows that Cond-Cert, the claim that if \( p(A) > 0 \), then if \( p(X) = 1 \), then
\( p(A \rightarrow X) = 1 \) and if \( p(X) = 0 \), then \( p(A \rightarrow X) = 0 \), leads to triviality on the assumption
that \( \rightarrow \) is not context-sensitive. 7 So there is some positive reason to investigate the possibil-
ity of context-sensitive ST-compliant theories. This brings us back to the question I raised
at the outset: what are the prospects of a context-sensitive truth-conditional ST-compliant
semantics?

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6In contrast to his (1976), Lewis (1986) is much more cautious: ‘the proposal that \( \rightarrow \) has a non-uniform inter-
pretation, so that for each \( p() \) we have a \( \rightarrow_p \) that satisfies ST, but in general a different \( \rightarrow_{p'} \) for a different \( p() \), is
a rival hypothesis, an alternative way to explain Adams’ Thesis. It is unscathed by my arguments here and in the
previous paper.’ (1986, p. 581)

7see Bradley (2007) for Bradley’s own take on the significance of this result.
2 Triviality for context-sensitive theories

Let’s assume that $\rightarrow$ is context-sensitive in a particular way: sensitive to the epistemic context, $c$, – the set of worlds consistent with what is known, or taken for granted, in the context of utterance. Further, assume that $\rightarrow$ receives classical truth-values at every world in $c$. Let $p_c()$ be the epistemic probability distribution associated with $c$. These assumptions, together with ST, are clearly not enough to show that $\rightarrow$ is trivial, as van Fraassen has shown.

But three plausible assumptions give us what we need for a triviality result. Start with the broadly Ramseyan insight that evaluating an indicative is a matter of reasoning about some situation(s) in which the antecedent holds. It is a natural thought that since this is how we evaluate indicatives, their truth-value, if they have one, is a matter of what is the case in some relevant world(s) in which the antecedent holds. Of course the truth-value of an indicative at a world $w$ may also depend on the choice of $w$. If we accept this much, it is best to formulate the problem in terms of Stalnaker’s selection functions. So, for any context, $c$, we will have the indicative $A \rightarrow_c C$ true at $w$ in $c$ iff $f_c(A, w) \subseteq C$, where $f_c(A, w)$ selects some relevant (perhaps ‘closest’) $A$-world(s) in $c$, relative to $w$, and is in some way dependent on $c$. It is natural to assume that if $w \in A$, then $w \in f_c(A, w)$. With these assumptions, the question becomes: what constraints does satisfying ST impose on the selection function, $f_c$? In principle, $f_c$ could depend on $p_c()$, the probability distribution on $c$, and it could also depend on which $\neg A$ worlds there are in $c$. But the simplest and rather natural constraint is that $f_c(A, w)$ depends on nothing else than the choice of $w$ and the set of $A$-worlds in $c$. Thus, it is of interest to explore the following constraint:

*Moderate Context-Dependence (M)*

Given two contexts $c$ and $c'$, if $c$ and $c'$ include the same $A$-worlds, then for every world $w$ that is both in $c$ and $c'$, $f_c(A, w) = f_{c'}(A, w)$.
Moderate context-sensitivity is simple because its alternatives – dependence on the probability distribution, or on the worlds in which the antecedent is false – introduce more complicated patterns of dependence. It is natural because it is natural to think that the contextually available antecedent-worlds compete with each other for closeness to the world of evaluation, and so which antecedent-worlds make the cut depends on the available competition, and nothing else. If the indicative conditional is moderately context-dependent, for any pair of contexts \( c \) and \( c' \) that share their \( A \)-worlds, the truth-value of \( A \rightarrow C \) will be the same in every world shared by \( c \) and \( c' \).

To get some mileage out of moderate context-dependence, one needs to have some handle on how the different rationally admissible contexts can be related. Two assumptions are needed: some constraint on what propositions can serve as an agent’s total evidence, and some minimal principle of belief revision.

The central issue is whether one can pass from one context to another by learning a conditional. I will employ the slightly weaker assumption that for some context \( c \) and some conditional \( A \rightarrow C \) there is a proposition \( B \) that can be one’s total evidence such that \( B \) entails \( A \rightarrow C \) but does not entail \( A \& C \) – in other words, that in some contexts one can learn the conditional \( A \rightarrow C \) without learning that the antecedent and the consequent obtain:

**Learnability Condition (L)**

There is a context \( c \) and an indicative \( A \rightarrow C \) true in some worlds in \( c \), such that it is possible to learn that \( A \rightarrow C \) is true without learning that \( A \& C \).

It is not clear whether this is always possible for an arbitrary choice of \( c \) and \( A \rightarrow C \). But it seems very plausible that this is often possible: for example, if I learn that the wire is live,

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8. Moderate context-dependence is similar to what Bacon calls the Harper condition: if \( E \) is the agent’s total evidence in a context \( c \) (so \( E \) is the conjunction of all the propositions known in \( c \)), and \( f(A, w) \) the ur-selection function associated with the initial context of total ignorance, then the Harper condition states that \( f_c(A, w) = f(A \& E, w) \). Note, however, that moderate context-dependence is weaker than Harper’s condition: it allows, for example, that \( f(A \& E, w) \neq f_E(A, w) \).
I learn that I will get a shock if I touch it, although whether I will in fact touch it remains open.

With moderate context-dependence and learnability, we can construct the following simple proof. Let’s say that $c$ and $A \rightarrow C$ are the context and the conditional whose existence is asserted by L, and let B entail $A \rightarrow C$ without entailing $A \& C$, again in accordance with L. Now, let $c'$ be the context that results from learning $B \lor A$ (if $B$ can be one’s total evidence in $c$, then so can $B \lor A$).

Now, $c$ and $c'$ share all their $A$-worlds, and thus Moderate Context-Dependence applies: for every world $w$ shared by $c$ and $c'$, the truth-value of $A \rightarrow C$ at $w$ in $c$ and $c'$ is the same. We can express it thus: $A \rightarrow C$ is true at $w$ iff $A \rightarrow C$ is true at $w$, for all $w \in c \cap c'$. And that means that every $\neg A$-world in $c'$ is a $A \rightarrow C$-world: $A \rightarrow C$ is true throughout $\neg A$ in $c'$. Thus, $p_{c'}(A \rightarrow C | \neg A) = 1$.

On the other hand, by ST, $p_{c'}(A \rightarrow C) = p_{c'}(C | A)$. Now, we know by the probability calculus that $p_{c'}(A \rightarrow C) = p_{c'}(A \rightarrow C | A)p_{c'}(A) + p_{c'}(A \rightarrow C | \neg A)p_{c'}(\neg A)$. Since $(A \rightarrow C) \& A$ and $A \& C$ are equivalent, it follows that $p_{c'}(A \rightarrow C | \neg A) = p_{c'}(C | A)$. But we already determined that $p_{c'}(A \rightarrow C | \neg A) = 1$. So $p_{c'}(C | A) = 1$, and $p_{c'}(A \& \neg C) = 0$. This is an unpalatable result. It conflicts with the intuitively very minimal condition on belief revision:

**Conservativity in Belief Revision (C)**

If $p_c(A \& E) > 0$, and $p_{c'}()$ is the result of updating $c$ on $E$, then $p_{c'}(A) > 0$.\(^9\)

In sum, moderate context-dependence, learnability, conservativity and Stalnaker’s Thesis are inconsistent. So the present result shows the challenges that the truth-conditional ST-theorist must confront: justify abandoning either M or L (C seems above reproach). Aban-

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\(^9\)C is analogous to a conservativity principle that Bradley (2007) appeals to: (Pres-Cert) $P(A) > 0, P(X) = 1 \Rightarrow P_{A}(X) = 1$. Certainty is to be preserved, but so also is uncertainty.
donoing L seems difficult in view of the readily available examples like the live wire case above. But abandoning M can be interesting, if one can justify a more complex dependence of the selection function on contexts.

References


