Physics 4481-7681; CS 4812; AEP 4812-7681 ps7

Four problems, two pages, covers lectures 23-28, due Tue evening 10 Dec 2024

Problem 1: Clifford measurement

In ps5#1B, the result of measurement in the stabilizer formalism was described as:
i) "Suppose the first qubit is measured. If ±ZIII…I is already in the stabilizer group, then the qubit measures uniquely as 0 or 1, resp. If that is not the case, then it is a property of stabilizer states that 0 or 1 will be measured with equal probability."
ii) "According to whether 0 or 1 is measured, the associated stabilizer ±ZIII…I is added as a generator, *and* from the 2^n original stabilizer group elements choose $n-1$ that commute with ±ZIII…I to form a new set of n generators."

A. Show that i) above holds for any n-qubit Clifford state.

B. Show that ii) is always possible, i.e., that there is always a choice of $n-1$ stabilizer elements that commute with ±ZIII…I, and that the full Stabilizer group after measurement continues to contain 2^n elements.

Problem 2: Bell state expectation values

In ps5#2.a.ii showed that $\langle\psi_{11}|\hat{a}\cdot\vec{\sigma}\otimes\hat{b}\cdot\vec{\sigma}|\psi_{11}\rangle = -\hat{a}\cdot\hat{b}$, where $|\psi_{11}\rangle$ is the Bell state $|\psi_{11}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle$. For practice with the density matrix formalism, redo this calculation using $\langle\psi_{11}|\hat{a}\cdot\vec{\sigma}\otimes\hat{b}\cdot\vec{\sigma}|\psi_{11}\rangle = \mathrm{Tr}\rho_{11}\ \hat{a}\cdot\vec{\sigma}\otimes\hat{b}\cdot\vec{\sigma}$ as follows:

a) Recall that the sixteen Pauli operators $P_1\otimes P_0$ (where $P_i\in\{I,X,Y,Z\}$ form a basis for operators on the 2-qubit state space, so that ρ_{11} can be written as a linear combination of these basis operators. Expand ρ_{11} in that basis and calculate $\mathrm{Tr}\rho_{11}\ \hat{a}\cdot\vec{\sigma}\otimes\hat{b}\cdot\vec{\sigma}$ (using the usual $\mathrm{Tr}\sigma_i\sigma_j = 2\delta_{ij}$).

b) Similarly determine ρ_{ij} in the $P_1\otimes P_0$ basis for the other Bell states $|\psi_{00}\rangle = \frac{1}{\sqrt{2}}\big(|00\rangle + |11\rangle\big)$, $|\psi_{01}\rangle = \frac{1}{\sqrt{2}}\big(|00\rangle - |11\rangle\big)$, $|\psi_{10}\rangle = \frac{1}{\sqrt{2}}\big(|01\rangle + |10\rangle\big)$, and show that $\mathrm{Tr}\rho_{ij}\ \hat{a}\cdot\vec{\sigma}\otimes\hat{b}\cdot\vec{\sigma}$ for those others doesn't quite match $-\hat{a}\cdot\hat{b}$ (hence why $|\psi_{11}\rangle$ was chosen for that problem, to simplify the geometric picture given in the solutions).

For the next two problems consider the three wavefunctions:

A. $|\psi\rangle_n = \alpha|00\ldots0\rangle_n + \beta|11\ldots1\rangle_n$

(generalized GHZ, superposition of all 0s and all 1s)

B. $|\psi\rangle_n = \sum_{i=0}^{n-1} \gamma_i |w_i\rangle$

(generalized W state, where $|w_0\rangle = |10\ldots0\rangle$, $|w_1\rangle = |010\ldots0\rangle)$, …, $|w_{n-1}\rangle = |00\ldots01\rangle)$

C. $|\psi\rangle_n = \frac{1}{2^{n/4}} \sum_{x=0}^{2^{n/2}-1} |x\rangle_{n/2}|x\rangle_{n/2}$

(only for n even, a diagonal sum of the $n/2$-dimensional subspaces)

Problem 3: Entanglement entropies

For each of A,B,C above:

i) Calculate the reduced density matrix of the first $n-1$ qubits (by tracing out the rightmost qubit), then the first $n-2$ qubits (by tracing out the next rightmost qubit), and so on down to the reduced density matrix of only the first qubit. (It should be possible to write this in general form, depending on the number of qubits in the remaining subsystem.)

ii) Calculate the von Neumann entropy of the subsystems (from size $n-1$ qubits down to 1), which gives equivalently their bipartite entanglement entropies with their complementary subsystems.

Problem 4: CKW inequalities

For each of A,B,C above:

i) Calculate the successive tangles $\tau_{A_0A_k}$ for $k = 1, \ldots, n-1$ of the reduced density matrices for the first qubit with each of the rest.

ii) Calculate the tangle $\tau_{A_0(A_1\ldots A_{n-1})}$ (equivalently $4\det\rho_{A_0}$)

iii) Check whether the CKW inequality $\sum_{k=1}^{n-1}\tau_{A_0A_k} \leq \tau_{A_0(A_1\ldots A_{n-1})}$ is saturated by the results of i,ii)