"Phase Estimation"

First recall unitary U has eigenvalues

$$\lambda_k = e^{2\pi i \theta_k}$$

diagonal basis $U = \begin{pmatrix} \lambda_0 & & \\ & \ddots & \\ & & \lambda_{N-1} \end{pmatrix}$

$$U^\dagger U = \begin{pmatrix} |\lambda_0|^2 & & \\ & \ddots & \\ & & |\lambda_{N-1}|^2 \end{pmatrix}$$

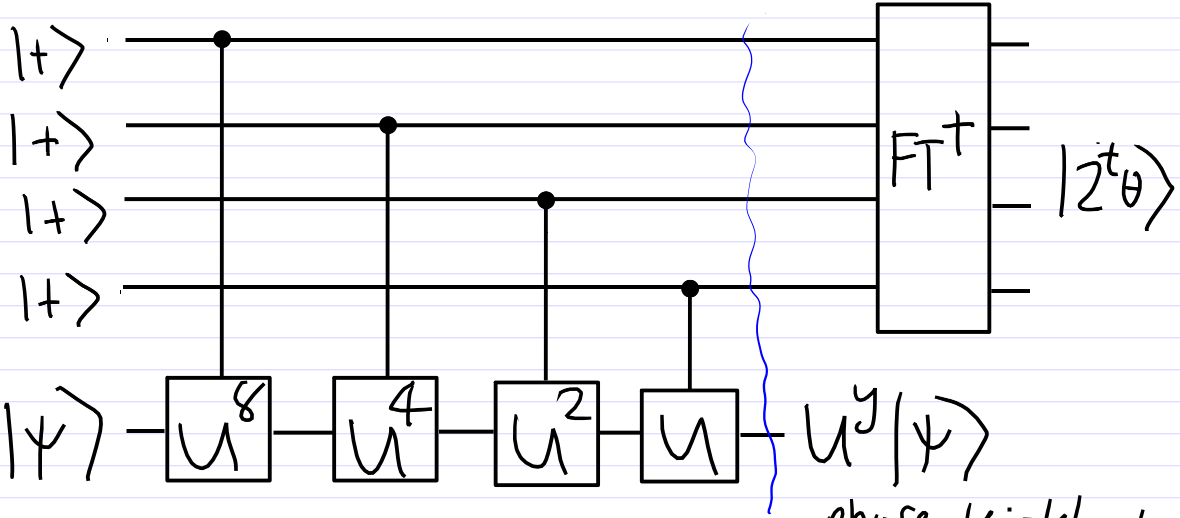
but $U^\dagger = U^{-1}$ so $|\lambda_k|^2 = 1$

and eigenvalues have unit norm.

Now find the period $U^x = 1$

acting on a specific eigenstate

$$U|\psi\rangle = e^{2\pi i \theta} |\psi\rangle$$



phase kickback

$$\frac{1}{2^{t/2}} \sum_{0 \le y < 2^t} e^{2\pi i y \theta} |y\rangle$$

Use boxed equation on next page with $\theta = x/2^t$ to see that final output of above circuit is $|x\rangle = |2^t \theta\rangle$

Recall we showed

$$U_{FT}|x\rangle = \frac{1}{2^{n/2}} \sum_y e^{2\pi i xy/2^n} |y\rangle$$

and

$$U_{FT}^{\dagger}|y\rangle = \frac{1}{2^{n/2}} \sum_x e^{-2\pi i xy/2^n} |x\rangle$$

implies that

$$\boxed{U_{FT}^{\dagger} \frac{1}{2^{n/2}} \sum_y e^{2\pi i xy/2^n} |y\rangle = |x\rangle}$$

(just $U_{FT}^{\dagger} U_{FT} = 1$)