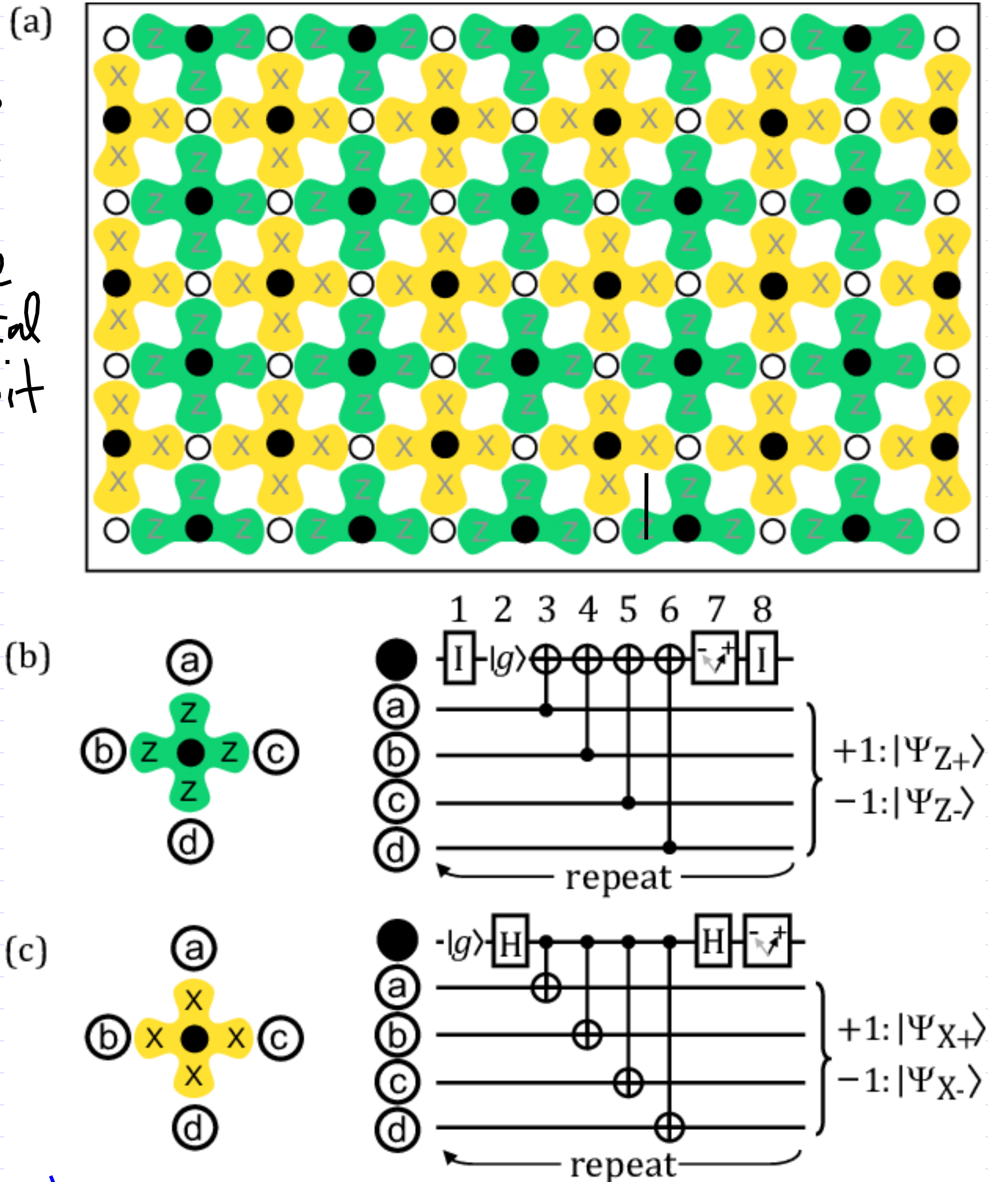
This will all be one logical qubit



arXiv: 1208.0928

each measurement reduces the dimension of the Hilbert space by a factor of 2.

Start with $4 \cdot 6 + 3 \cdot 5 = 39$ data qubits so 2^{39} dim space.

But $4 \cdot 5 + 3 \cdot 6 = 38$ measurement qubits so $2^{39}/2^{38} = 2$

$\Rightarrow$ 1 logical qubit

For example, 2 qubits

$|00\rangle \; |01\rangle \; |10\rangle \; |11\rangle$

measure $Z_0 Z_1 = +1 \Rightarrow |00\rangle, |11\rangle$

$= -1 \Rightarrow |01\rangle, |10\rangle$

Joint eigenstates of

$Z_0 Z_1$ $X_0 X_1$ (they commute)

$Z_0 Z_1$	$X_0 X_1$	
1	1	$\frac{1}{\sqrt{2}}(\|00\rangle + \|11\rangle)$
1	-1	$\frac{1}{\sqrt{2}}(\|00\rangle - \|11\rangle)$
-1	1	$\frac{1}{\sqrt{2}}(\|01\rangle + \|10\rangle)$
-1	-1	$\frac{1}{\sqrt{2}}(\|01\rangle - \|10\rangle)$

"Bell Basis"

For 4 qubits, there are eight $Z_a Z_b Z_c Z_d = +1$ eigenstates:

$|0000\rangle, |0011\rangle, \ldots, |1100\rangle, |1111\rangle$

(all with even # of 1's)

Similarly, eight $Z_a Z_b Z_c Z_d = -1$ eigenstates:

$|0001\rangle, |0010\rangle, \ldots, |1101\rangle, |1110\rangle$

(all with odd # of 1's)

Same for $X_a X_b X_c X_d$ in terms of $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$

$+1$ $|++++\rangle, |++--\rangle, \ldots, |--++\rangle, |----\rangle$

-1 $|+++-\rangle, |--+-\rangle, \ldots, |--+-\rangle, |---+\rangle$

Now consider error syndromes

$$Z_a Z_b Z_c Z_d \underbrace{X_a}_{\text{error on qubit } a} |\psi\rangle$$

$$= -X_a Z_a Z_b Z_c Z_d |\psi\rangle = -X_a |\psi\rangle$$

(if started in +1 eigenstate)

Similarly

error on qubit b

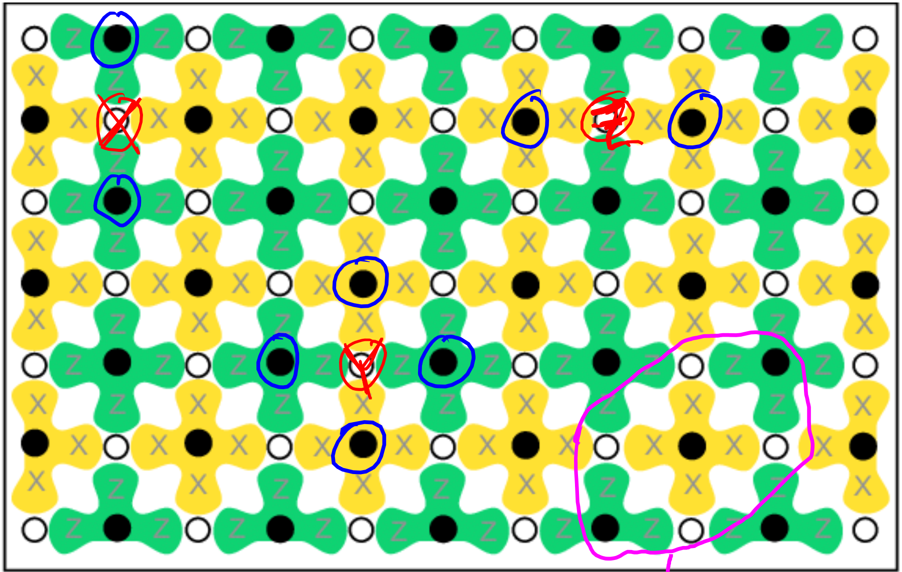
$$X_a X_b X_c X_d \, Z_b |\psi\rangle$$

$$= -Z_b X_a X_b X_c X_d |\psi\rangle = -Z_b |\psi\rangle$$

○ = flipped measurement value

○ = error on data qubit

a →



← b

c ↑

a = X error

b = Z error

c = Y error

39 ○ "data"

38 ● "meas"

2d or 1 qubit

1912.09410

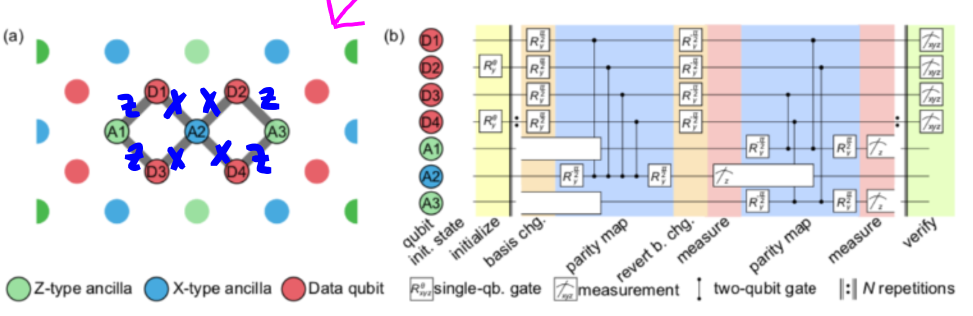
Repeated Quantum Error Detection in a Surface Code

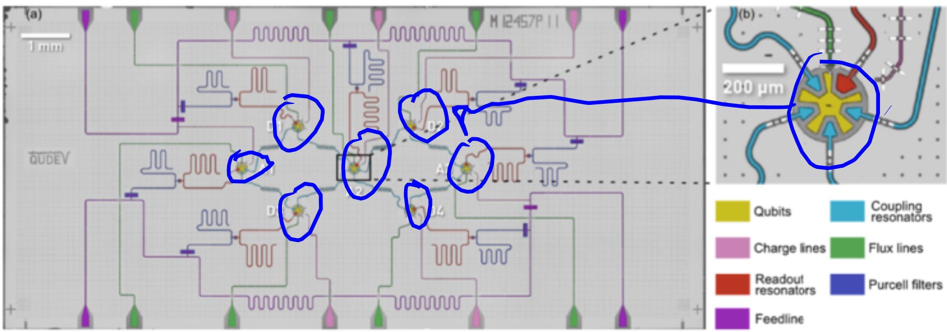
Christian Kraglund Andersen,[1,*] Ants Remm,[1] Stefania Lazar,[1] Sebastian Krinner,[1] Nathan Lacroix,[1] Graham J. Norris,[1] Mihai Gabureac,[1] Christopher Eichler,[1] and Andreas Wallraff[1]

[1] *Department of Physics, ETH Zurich, CH-8093 Zurich, Switzerland*

(Dated: December 20, 2019)

The realization of quantum error correction is an essential ingredient for reaching the full potential of fault-tolerant universal quantum computation. Using a range of different schemes, logical qubits can be redundantly encoded in a set of physical qubits. One such scalable approach is based on the surface code. Here we experimentally implement its smallest viable instance, capable of repeatedly detecting any single error using seven superconducting qubits, four data qubits and three ancilla qubits. Using high-fidelity ancilla-based stabilizer measurements we initialize the cardinal states of the encoded logical qubit with an average logical fidelity of 96.1%. We then repeatedly check for errors using the stabilizer readout and observe that the logical quantum state is preserved with a lifetime and coherence time longer than those of any of the constituent qubits when no errors are detected. Our demonstration of error detection with its resulting enhancement of the conditioned logical qubit coherence times in a 7-qubit surface code is an important step indicating a promising route towards the realization of quantum error correction in the surface code.





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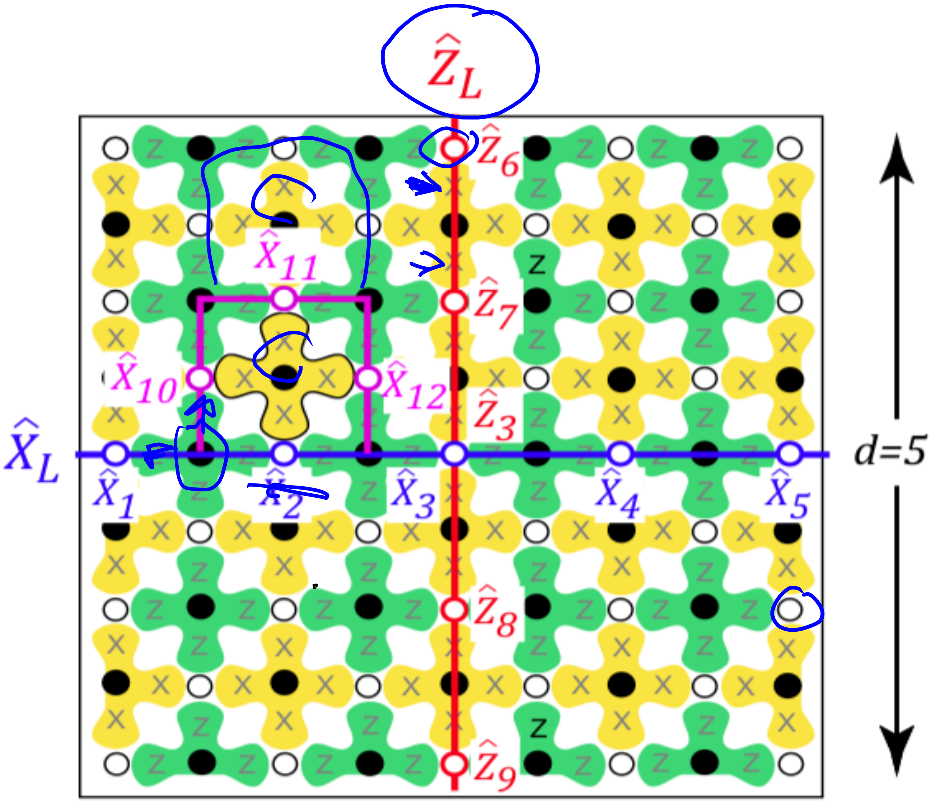
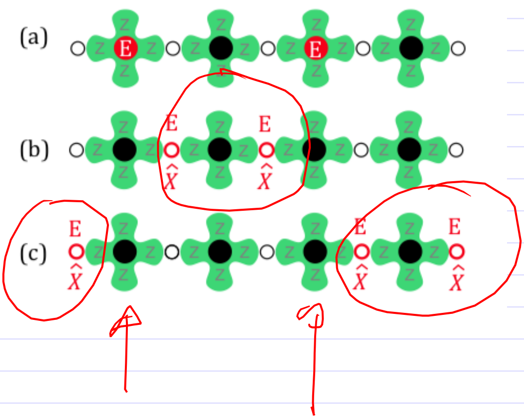
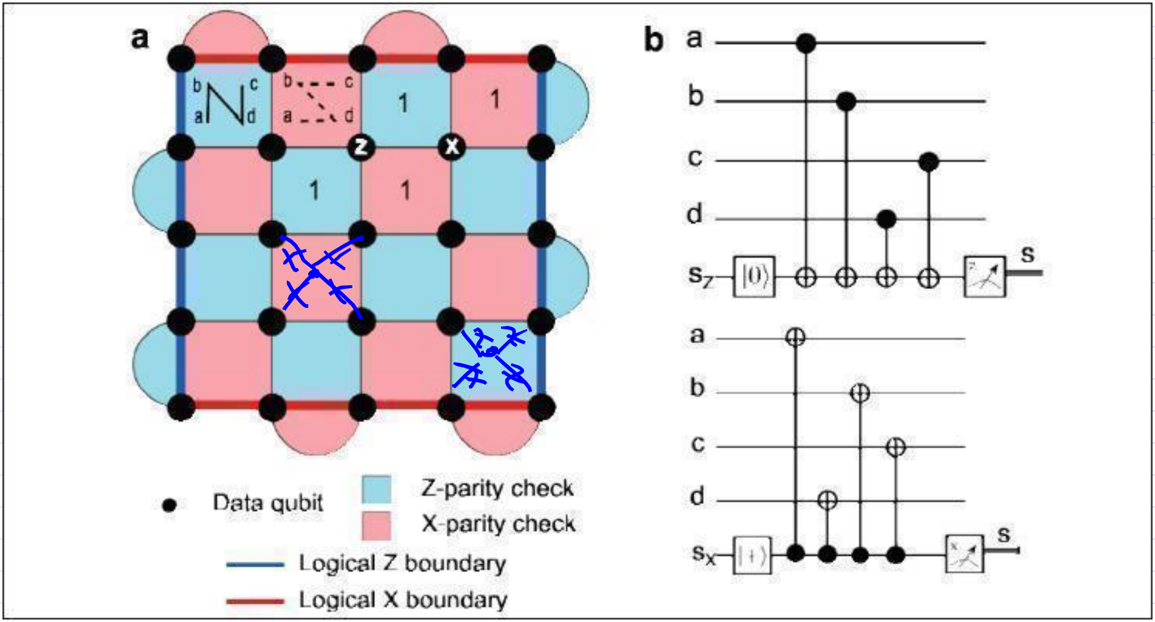


Still need logical operators $\overline{X}_L, \overline{Z}_L$ satisfying

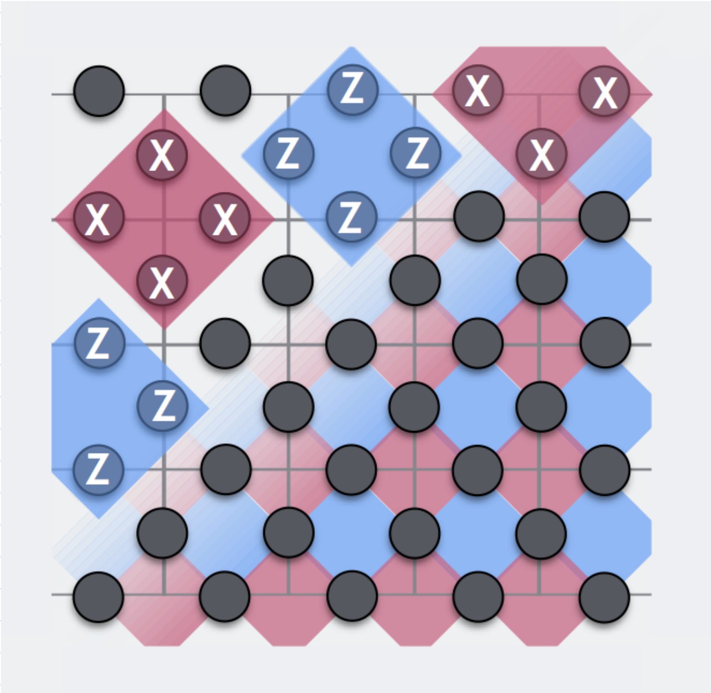
$\overline{X}_L^2 = \overline{Z}_L^2 = 1$, $\quad \overline{X}_L \overline{Z}_L = -\overline{Z}_L \overline{X}_L$



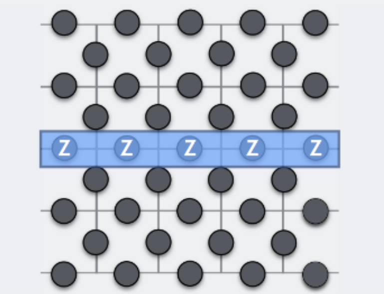
[12] Actually the surface code does not need to completely identify errors; it is sufficient that it identifies errors or chains of errors that are topologically equivalent to the actual errors, meaning any differences can be written as products of stabilizers.


a
b
c
a
d
b
c
a
d
1
1
Z
X
1
1
Data qubit
Z-parity check
X-parity check
Logical Z boundary
Logical X boundary
b
a
b
c
d
s_Z
$|0\rangle$
s
a
b
c
d
s_X
$|+\rangle$
s

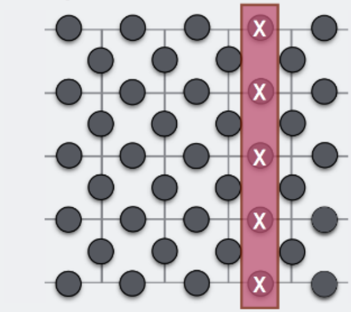
(IBM group)

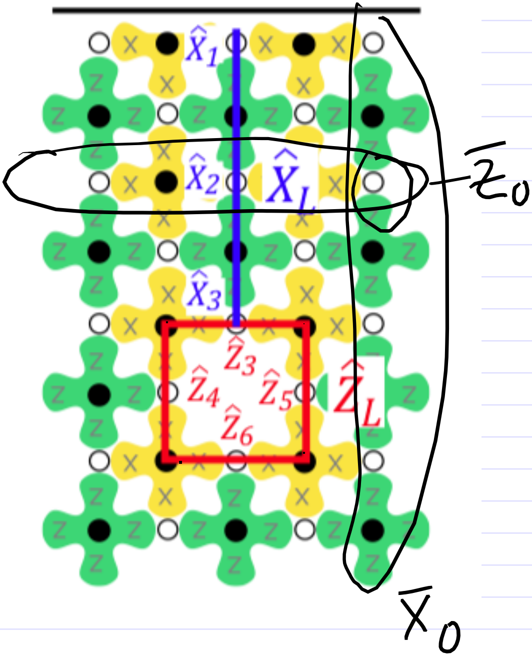


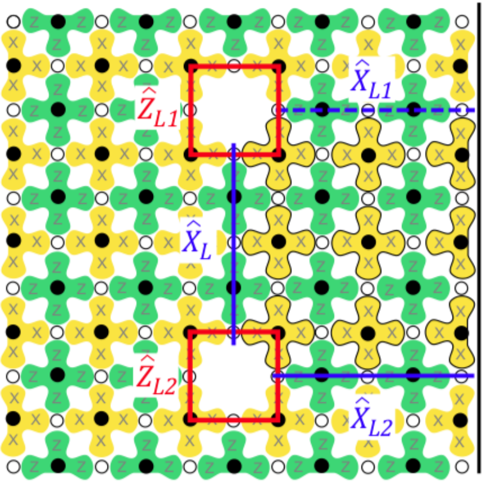
- logical **Z** operator:

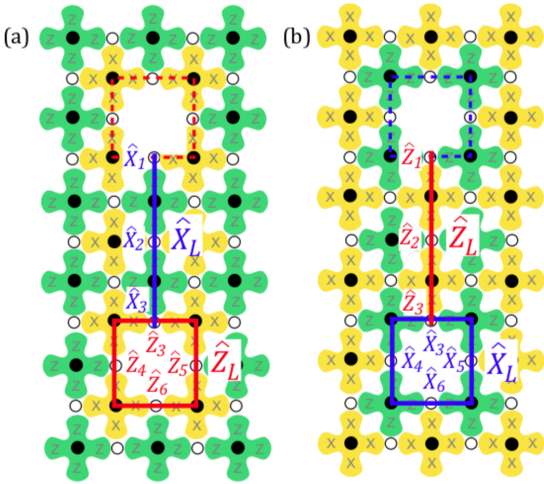


- logical **X** operator:




$\hat{X}_1$
$\hat{X}_2$
$\hat{X}_L$
$\hat{X}_3$
$\hat{Z}_3$
$\hat{Z}_4$
$\hat{Z}_5$
$\hat{Z}_6$
$\hat{Z}_L$
$\bar{Z}_0$
$\bar{X}_0$


$\hat{X}_{L1}$
$\hat{Z}_{L1}$
$\hat{X}_L$
$\hat{Z}_{L2}$
$\hat{X}_{L2}$


(a)
(b)
$\hat{X}_1$
$\hat{X}_2$
$\hat{X}_L$
$\hat{X}_3$
$\hat{Z}_3$
$\hat{Z}_4$
$\hat{Z}_5$
$\hat{Z}_6$
$\hat{Z}_L$
$\hat{Z}_1$
$\hat{Z}_2$
$\hat{Z}_L$
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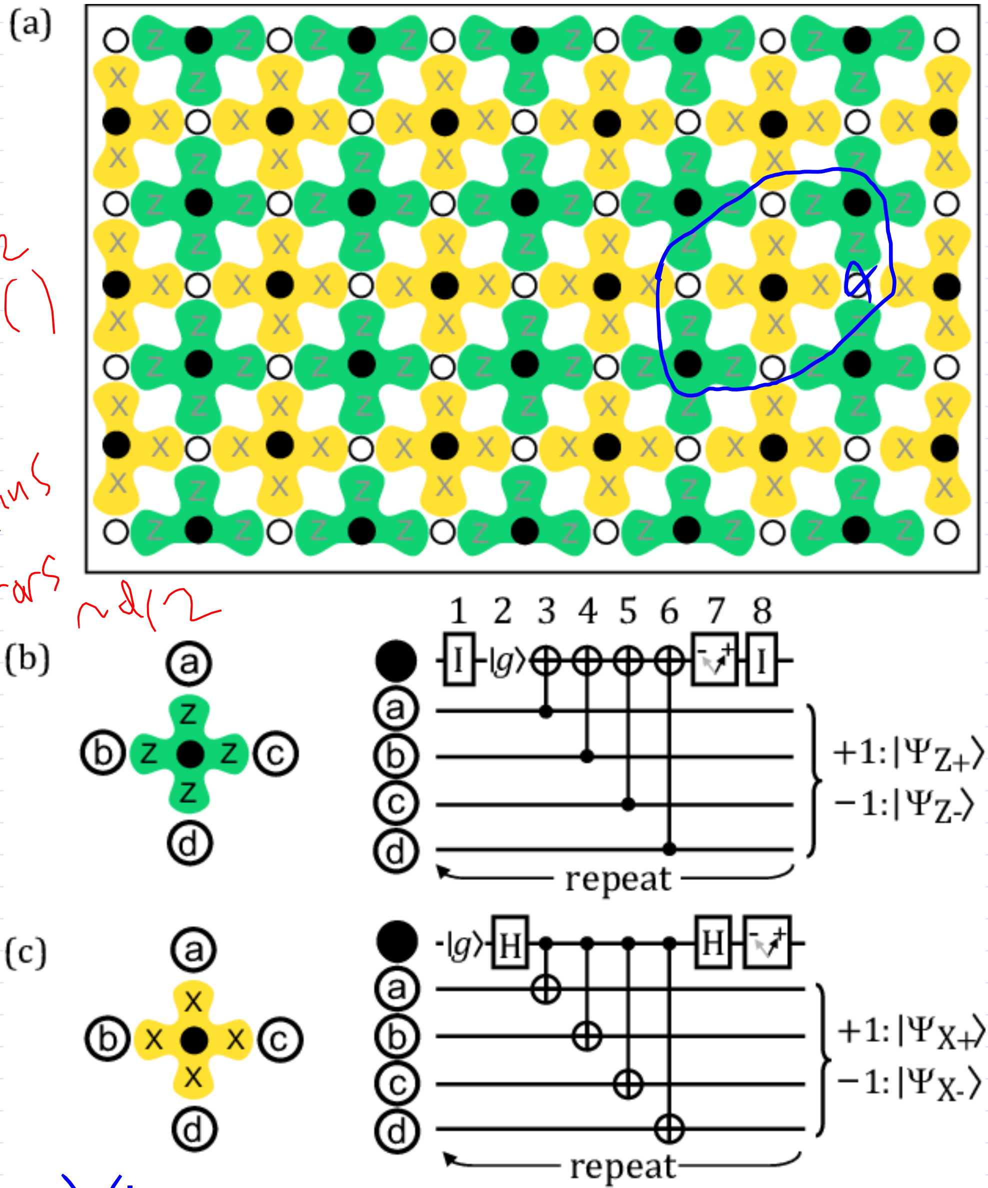
$\hat{X}_3$
$\hat{X}_4$
$\hat{X}_5$
$\hat{X}_6$
$\hat{X}_L$



p_e^d

$p_e^{d/2}$

$p_e(\ )$

$\sim d$

chains of errors

$\sim d/2$

arXiv:1208.0928

1912.09410

Repeated Quantum Error Detection in a Surface Code

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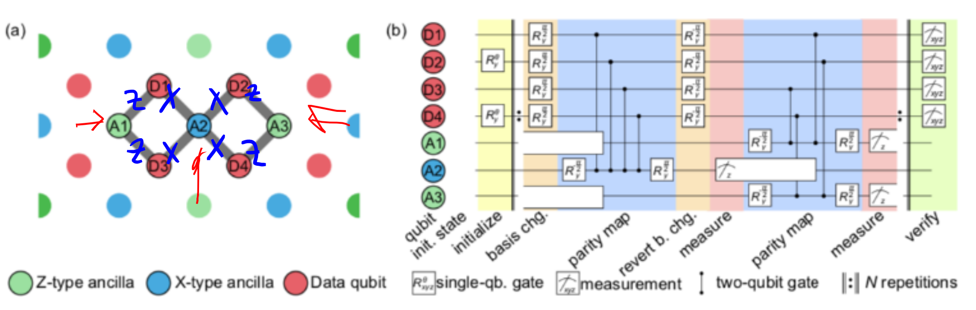
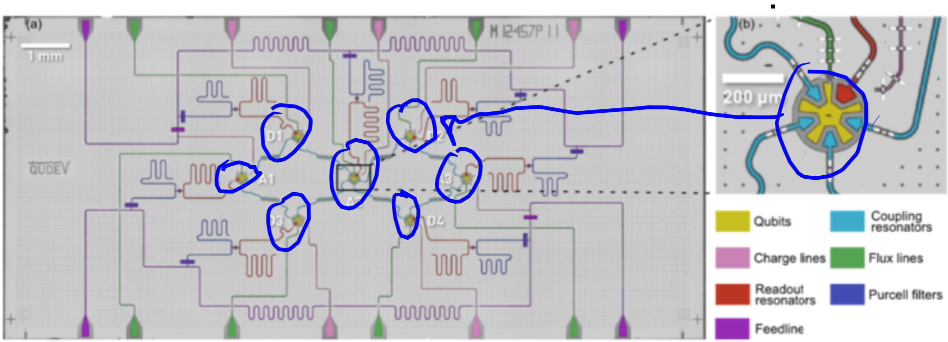
The realization of quantum error correction is an essential ingredient for reaching the full potential of fault-tolerant universal quantum computation. Using a range of different schemes, logical qubits can be redundantly encoded in a set of physical qubits. One such scalable approach is based on the surface code. Here we experimentally implement its smallest viable instance, capable of repeatedly detecting any single error using seven superconducting qubits, four data qubits and three ancilla qubits. Using high-fidelity ancilla-based stabilizer measurements we initialize the cardinal states of the encoded logical qubit with an average logical fidelity of 96.1%. We then repeatedly check for errors using the stabilizer readout and observe that the logical quantum state is preserved with a lifetime and coherence time longer than those of any of the constituent qubits when no errors are detected. Our demonstration of error detection with its resulting enhancement of the conditioned logical qubit coherence times in a 7-qubit surface code is an important step indicating a promising route towards the realization of quantum error correction in the surface code.





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In the surface code, as in any stabilizer code, errors are detected by observing changes in the stabilizer measurement outcomes. Such syndromes are typically measured by entangling the stabilizer operators with the state of ancilla qubits, which are then projectively measured to yield the stabilizer outcomes. The surface code consists of a $d \times d$ grid of data qubits with d^2-1 ancilla qubits, each connected to up to four data qubits [28]. The code can detect $d-1$ errors and correct up to $\lfloor (d-1)/2 \rfloor$ errors per cycle of stabilizer measurements. In particular, the stabilizers of the $d=2$ surface code, see Fig. 1, are given by

$$\overset{A_2}{X_{D1}X_{D2}X_{D3}X_{D4}}, \qquad \overset{A_1}{Z_{D1}Z_{D3}}, \qquad \overset{A_3}{Z_{D2}Z_{D4}}. \tag{1}$$

For the code-distance $d=2$, it is only possible to detect a single error per round of stabilizer measurements and once an error is detected, the error can not be unambiguously identified, e.g. one would obtain the same syndrome outcome for an X-error on D1 and on D3.

Here, we use the following logical qubit operators

$$Z_L = Z_{D1}Z_{D2}, \quad \text{or} \quad Z_L = Z_{D3}Z_{D4}, \tag{2}$$
$$X_L = X_{D1}X_{D3}, \quad \text{or} \quad X_L = X_{D2}X_{D4}, \tag{3}$$

such that the code space in terms of the physical qubit states is spanned by the logical qubit states

4 data

$$|0\rangle_L = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle), \tag{4}$$
$$|1\rangle_L = \frac{1}{\sqrt{2}}(|0101\rangle + |1010\rangle). \tag{5}$$

$= X_L |0\rangle_L$

To encode quantum information in the logical subspace, we initialize the data qubits in a separable state, chosen such that after a single cycle of stabilizer measurements and conditioned on ancilla measurement outcomes being $|0\rangle$, the data qubits are encoded into the target logical qubit state. In this work, we demonstrate this probabilistic preparation scheme for the logical states $|0\rangle_L$, $|1\rangle_L$, $|+\rangle_L = (|0\rangle_L + |1\rangle_L)/\sqrt{2}$ and $|-\rangle_L = (|0\rangle_L - |1\rangle_L)/\sqrt{2}$ and we perform repeated error detection on these states.

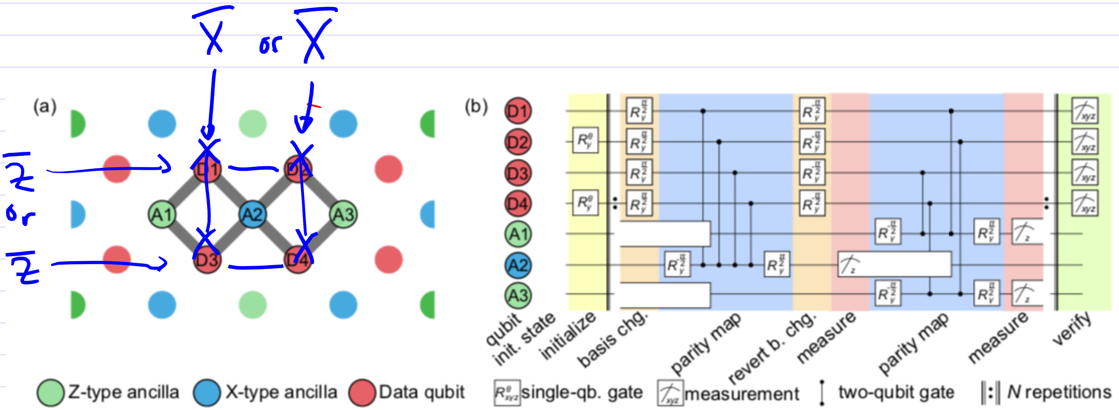


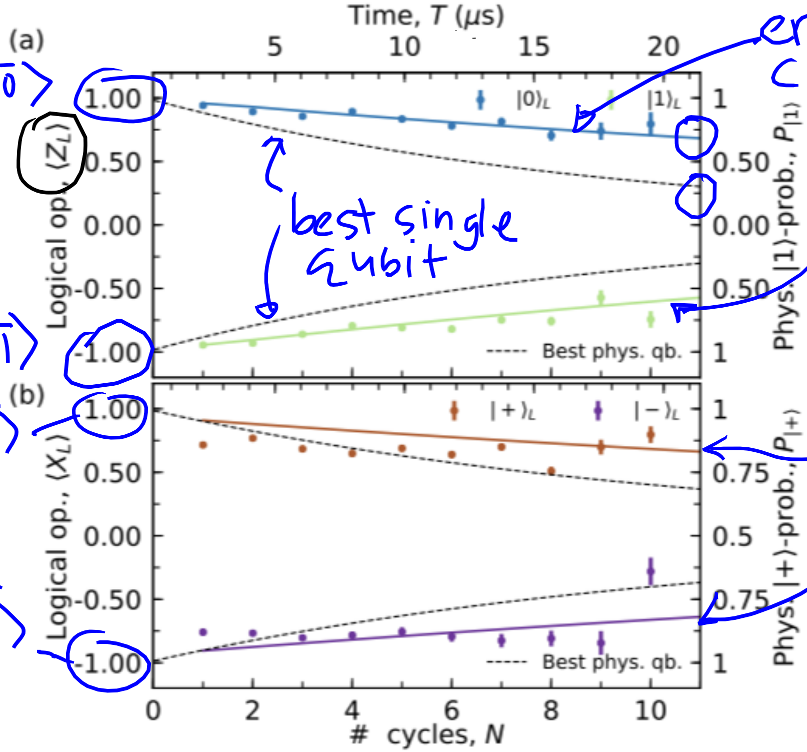
FIG. 1. Seven qubit surface code. (a) The surface code consists of a two-dimensional array of qubits. Here the data qubits are shown in red an the ancilla qubits for measuring X-type (Z-type) stabilizers in blue (green). The smallest surface code consists of seven qubits indicated by the data qubits D1-D4 and the ancilla qubits A1-A3. (b) Gate sequence for quantum error detection using the seven qubit surface code. Details of the gate sequence are discussed in the main text.

$\langle \bar{0}|\bar{Z}_L|\bar{0}\rangle$

$\langle \bar{1}|\bar{Z}_L|\bar{1}\rangle$

$\langle \bar{+}|\bar{X}_L|\bar{+}\rangle$

$\langle \bar{-}|\bar{X}_L|\bar{-}\rangle$



best single qubit

error corrected by surface code does better!

here too

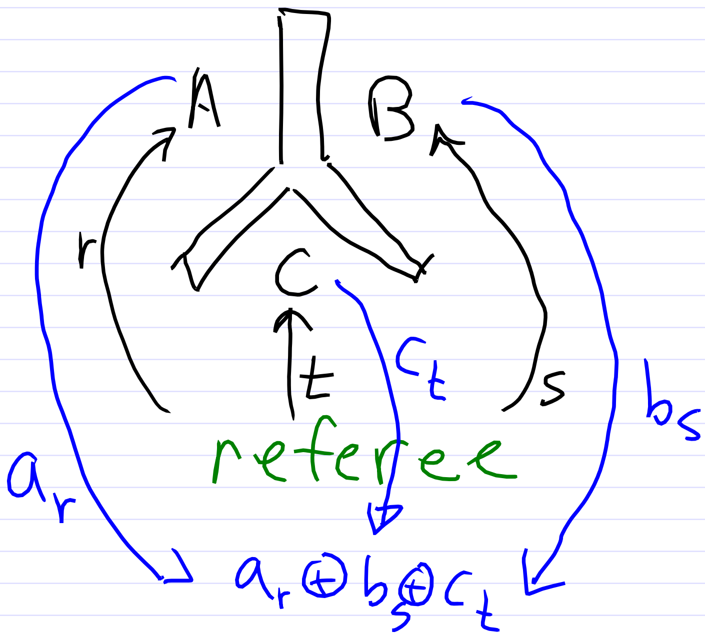
FIG. 5. Repeated quantum error detection. The expectation values of (a) the logical Z_L operator and (b) the logical X_L operator as a function of N, the number of stabilizer measurement cycles. The expectations values are shown for the prepared $|0\rangle_L$ (blue), $|1\rangle_L$ (green), $|+\rangle_L$ (brown) and $|-\rangle_L$ (purple) states. The solid lines indicate the corresponding values obtained from master equation simulations. Also shown (dashed lines, right axis) are the (a) qubit decay of the $|1\rangle$-state with the best measured T_1 value and (b) the physical qubit decay of the $|+\rangle$-state with the best measured T_2 value. (c) Total success probability p_s for detecting no errors during N cycles of stabilizer measurements for the $|0\rangle_L$ data shown in (a) and the corresponding values from numerical simulations. (d) Probability of observing k ancilla qubits in the $|1\rangle$ state for each measurement cycle and conditioned on having detected no error in any of the previous $N-1$ cycles. The data corresponds to the initial $|0\rangle_L$ state presented in (a).

Physics		"CS"
experiment (to understand nature)	$\Leftrightarrow$	games: optimize performance in various settings
locality	$\Leftrightarrow$	no communication
hidden variable can't explain statistics	$\Leftrightarrow$	classical strategy can't do as well as quantum

GHZ game

A, B, C are given r, s, t
from {000, 011, 110, 101}

goal: a, b, c $\quad a \oplus b \oplus c = r \vee s \vee t$



if A receives $r=0 \Rightarrow a_0$

$r=1 \quad a_1$

" B " $s=0,1 \quad b_0, b_1$

" C " $t=0,1 \quad c_0, c_1$

r, s, t	$a \oplus b \oplus c$	$r \vee s \vee t$
0 0 0	$a_0 \oplus b_0 \oplus c_0 =$	0
0 1 1	$a_0 \oplus b_1 \oplus c_1 =$	1
1 1 0	$a_1 \oplus b_1 \oplus c_0 =$	1
1 0 1	$a_1 \oplus b_0 \oplus c_1 =$	1

$$0 \oplus 0 \oplus 0 = 1$$

inconsistent! can only solve 3 of 4, so best is 75%

QM A, B, C share

$$|\Psi_{GHZ}\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)$$

if receive $r,s,t = 0$, measure in X basis

" " " 1 " " " Y "

measure in X basis $\Leftrightarrow$ apply H

" " Y " $\Leftrightarrow$ " $HS^\dagger$

$$SH|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + i|1\rangle\right) = |+i\rangle$$

$$S = \begin{pmatrix} 1 & \\ & +i \end{pmatrix} \qquad SH|1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - i|1\rangle\right) = |-i\rangle$$

$|\pm i\rangle$ are eigenstates of $Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

Define

$$\left.\begin{aligned} |\varphi^{\pm}\rangle &= \frac{1}{\sqrt{2}}\left(|00\rangle \pm |11\rangle\right) \\ |\psi^{\pm}\rangle &= \frac{1}{\sqrt{2}}\left(|01\rangle \pm |10\rangle\right) \end{aligned}\right\} \text{“Bell Basis”}$$

Note

$$H \otimes H\, |\varphi^{+}\rangle = |\varphi^{+}\rangle$$

$$H \otimes H\, |\varphi^{-}\rangle = |\psi^{+}\rangle$$

and

$$(HS^{\dagger}) \otimes (HS^{\dagger})\, |\varphi^{+}\rangle = |\psi^{+}\rangle$$

$$(HS^{\dagger}) \otimes (HS^{\dagger})\, |\varphi^{-}\rangle = |\varphi^{+}\rangle$$

$$H \otimes H \otimes H\, |\psi_{GHZ}\rangle = \frac{1}{2}\left(|000\rangle + |011\rangle + |101\rangle + |110\rangle\right)$$

even # 1's

$r,s,t = 0,0,0$

Always win!

$$H \otimes HS^{\dagger} \otimes HS^{\dagger}\, |\psi_{GHZ}\rangle = \frac{1}{2}\left(|001\rangle + |010\rangle + |100\rangle + |111\rangle\right)$$

odd # 1's

$r,s,t \in \{0,1,1;\ 1,0,1;\ 1,1,0\}$

$$H\otimes 1\otimes 1\,|\psi_{GHZ}\rangle$$

$$= \frac{1}{2}\Big((|0\rangle+|1\rangle)|0\rangle|0\rangle + (|0\rangle-|1\rangle)|1\rangle|1\rangle\Big)$$

$$= \frac{1}{\sqrt{2}}\Big(|0\rangle\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)+|1\rangle\frac{1}{\sqrt{2}}(|00\rangle-|11\rangle)\Big)$$

$$= \frac{1}{\sqrt{2}}\Big(|0\rangle|\varphi^+\rangle + |1\rangle|\varphi^-\rangle\Big)$$

$$H\otimes H|\varphi^+\rangle = \frac{1}{2\sqrt{2}}\Big[(|0\rangle+|1\rangle)(|0\rangle+|1\rangle)\pm(|0\rangle-|1\rangle)(|0\rangle-|1\rangle)\Big]$$

$$= |\varphi^+\rangle$$

$$H\otimes H|\varphi^-\rangle = |\psi^+\rangle$$

$$(HS^\dagger\otimes HS^\dagger)|\varphi^+\rangle = |\psi^+\rangle$$

$$\frac{1}{2\sqrt{2}}\Big[(|0\rangle+|1\rangle)(|0\rangle+|1\rangle)\pm(-i)(|0\rangle-|1\rangle)(-i)(|0\rangle-|1\rangle)\Big]$$

$$(HS^\dagger\otimes HS^\dagger)|\varphi^-\rangle = |\varphi^+\rangle$$

$$|\psi_{GHZ}\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|\varphi^+\rangle + |1\rangle|\varphi^-\rangle\right)$$

$$H\otimes H\otimes H|\psi_{GHZ}\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|\varphi^+\rangle + |1\rangle|\psi^+\rangle\right)$$

$$= \frac{1}{2}\left(|000\rangle + |011\rangle + |101\rangle + |110\rangle\right)$$

even # 1's

$$H\otimes(HS^\dagger)\otimes(HS^\dagger)|\psi_{GHZ}\rangle$$

$$= \frac{1}{\sqrt{2}}\left(|0\rangle|\psi^+\rangle + |1\rangle|\varphi^+\rangle\right)$$

$$= \frac{1}{2}\left(|001\rangle + |010\rangle + |100\rangle + |111\rangle\right)$$

odd # 1's !

Notice if we measure $H \otimes H \otimes H \, |\Psi\rangle$

x_2, x_1, x_0 satisfy $\boxed{x_0 \oplus x_1 \oplus x_2 = 0.}$

Therefore $x_0 = x_1 \oplus x_2$

But Einstein would expect

same x_0

regardless of H's on 1, 2:

(1) This was with H not $HS^\dagger$ on 1,2

(2) but measuring 1,2 doesn't affect 0

(3) So X_0 must be "predisposed" even before measuring

(4) + moreover even before deciding whether to apply H or $HS^\dagger$

The same argument applies to X_1, X_2.
And if X_i^S is the result of measuring after applying $HS^\dagger$ to qubit i, the X_i^S are also independent of what is (or is not) done to other qubits.
But those "predispositions" are not mutually consistent:

Experimentally, they must satisfy

$$\left\{\begin{aligned} X_0 \oplus X_1^s \oplus X_2^s &= 1 \\ X_0^s \oplus X_1 \oplus X_2^s &= 1 \\ X_0^s \oplus X_1^s \oplus X_2 &= 1 \end{aligned}\right.$$

with sum: $X_0 \oplus X_1 \oplus X_2 = 1$

so it is not possible to preassign values to X_0, X_1, X_2 consistent with all four measurement possibilities, since with all H's,

$$X_0 \oplus X_1 \oplus X_2 = 0.$$

and Einstein's "elements of reality" [hidden variable] argument says they're the <u>same</u> X_i, regardless of $HS^\dagger$ applied to other two.

Do the experiment:

w/ two $HS^\dagger$ always an odd # of 1s.

w/ three H always an even # of 1s.

So there are <u>no</u> consistent pre-assignments/predispositions/hidden variables/elements of reality

~~Einstein~~ × QM ✓