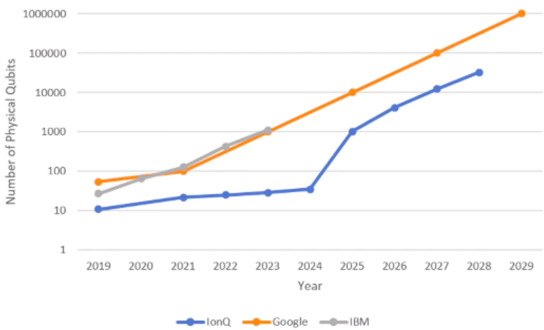
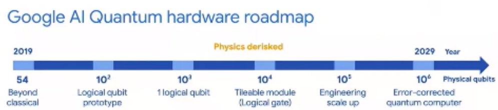
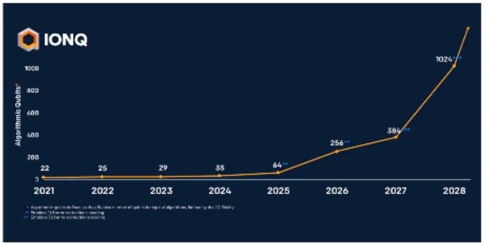
Physics 4481-7681 / CS 4812
Lec 15, 12 Oct 2023

Hardware comparison: **vendor roadmaps**

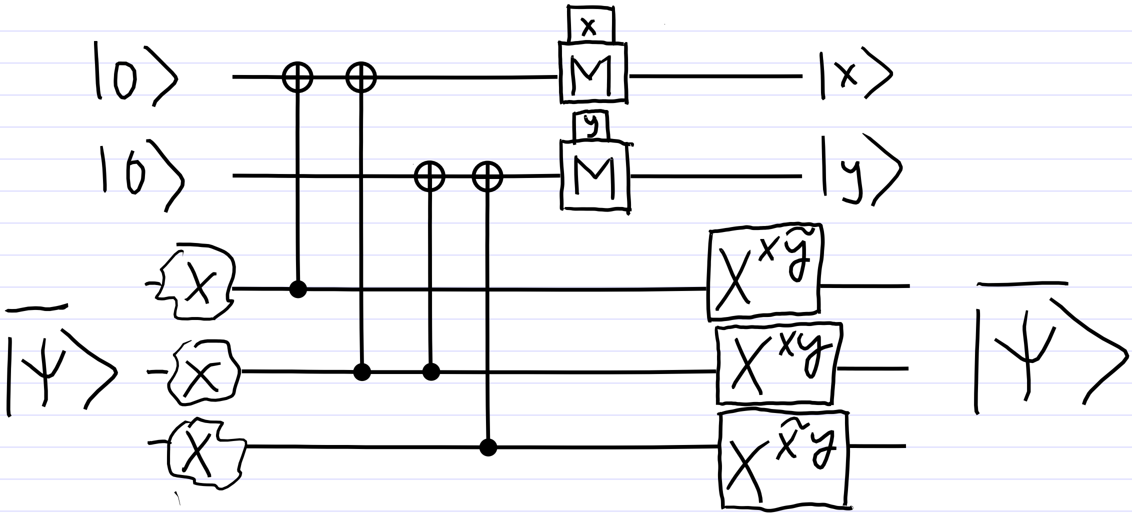


- IonQ, Google, and IBM are all predicting ~1000 physical qubits in either **2023** (Google/IBM) or **2025** (IonQ).
- IonQ is predicting ~32,000 physical qubits in 2028. Google is predicting 1,000,000 physical qubits in 2029, and IBM does not make a concrete prediction this far out.

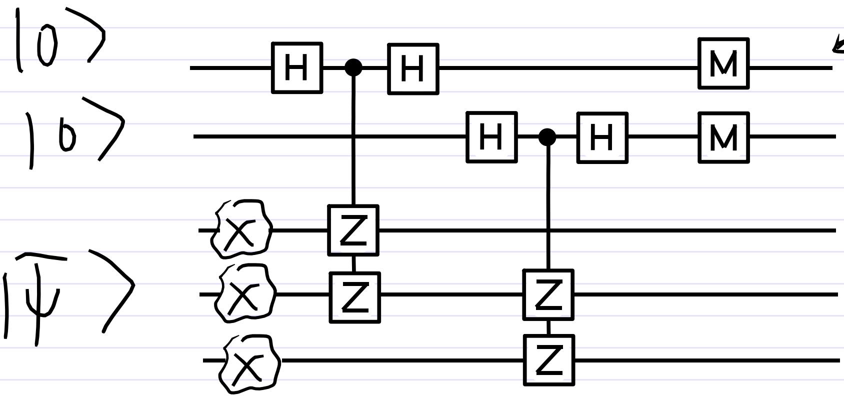




$$|\bar{\psi}\rangle = \alpha|\bar{0}\rangle + \beta|\bar{1}\rangle$$



Use $X = HZH$



"measures" $Z_1 Z_2$

measure $Z_0 Z_1$

$$V\left(\frac{1+(-1)^x V}{2}\right) = (-1)^x\left(\frac{1+(-1)^x V}{2}\right)$$

$$\rho_x^V$$

Specify 5 qubit code

$M_0 = ZXXZI$ $\qquad \underline{M_i^2 = 1}$

$M_1 = XXZ1Z$

$M_2 = XZ1ZX$

$M_3 = Z1ZXX$

$[X_i, Z_j] = 0 \quad i \neq j$

Commutator

$[A,B] = AB - BA$

$[M_i, M_j] = 0$

$M_4 = I\,Z\,X\,X\,Z$?

not independent $= M_0 M_1 M_2 M_3$

Code words

$$|\bar{0}\rangle = \frac{1}{4}(1+M_0)(1+M_1)(1+M_2)(1+M_3)|00000\rangle$$

$$|\bar{1}\rangle = \frac{1}{4}(1+M_0)(1+M_1)(1+M_2)(1+M_3)|11111\rangle$$

Normalized? $\quad (1+M_i)^2 = 2(1+M_i)$

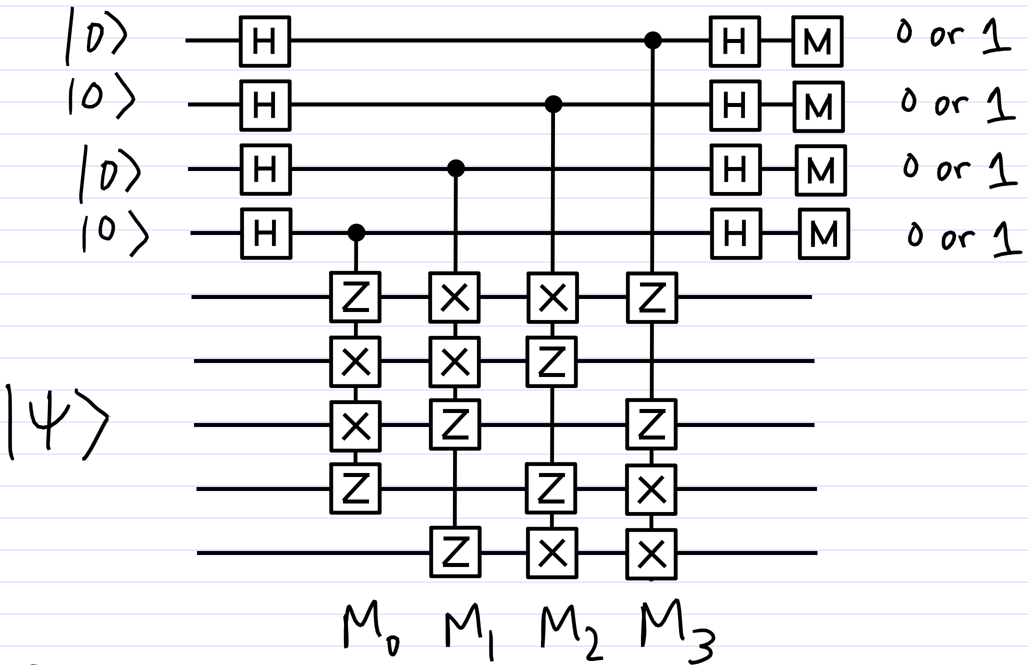
$$\langle\bar{0}|\bar{0}\rangle = \frac{1}{16}16\langle 0^5|\prod_i(1+M_i)|0^5\rangle = \underline{1}$$

$\langle\bar{1}|\bar{1}\rangle = \underline{1} \qquad \bar{X} = X_0 X_1 X_2 X_3 X_4$

$\langle\bar{1}|\bar{0}\rangle = \langle\bar{0}|\bar{1}\rangle = 0 \qquad \bar{Z} = Z_0 Z_1 Z_2 Z_3 Z_4$

$|\bar{1}\rangle = \bar{X}|\bar{0}\rangle \qquad \bar{Z}|\bar{0}\rangle = |\bar{0}\rangle \qquad [\bar{X}, M_i] = 0$

$|\bar{0}\rangle = \bar{X}|\bar{1}\rangle \qquad \bar{Z}|\bar{1}\rangle = -|\bar{1}\rangle \qquad [\bar{Z}, M_i] = 0$



5-qubit code, encoding circuit.

To initialize state to $|\bar{0}\rangle$:

measure M_i's, projects to

$\alpha|\bar{0}\rangle + \beta|\bar{1}\rangle$ space.

measure $\bar{Z}$, gives $|\bar{0}\rangle$ or $|\bar{1}\rangle$

if $|\bar{1}\rangle$, apply $\bar{X}|\bar{1}\rangle = |\bar{0}\rangle$

$M_0 = Z_1 X_2 X_3 Z_4$ $\qquad$ $M_2 = Z_3 X_4 X_0 Z_1$

$M_1 = Z_2 X_3 X_4 Z_0$ $\qquad$ $M_3 = Z_4 X_0 X_1 Z_2$

Now see that the M_i characterize the 16 spaces $\underbrace{\mathbb{1}}_{\text{uncorrupted}}$ $\underbrace{X_i\ Y_i\ Z_i}_{\text{15 corruptions}}$:

	X_0	Y_0	Z_0	X_1	Y_1	Z_1	X_2	Y_2	Z_2	X_3	Y_3	Z_3	X_4	Y_4	Z_4	$\mathbb{1}$
M_0	+	+	+	−	−	+	+	−	−	+	−	−	−	−	+	+
M_1	−	−	+	+	+	+	−	−	+	+	−	−	+	−	−	+
M_2	+	−	−	−	−	+	+	+	+	−	−	+	+	−	−	+
M_3	+	−	−	+	−	−	−	−	+	+	+	+	−	−	+	+

each column is a unique error signature. Just look at whether the given operator commutes or anti-commutes with M_i.

e.g., (start of 1st column)

$$M_0 X_0 |\psi\rangle = X_0 M_0 |\psi\rangle = +X_0|\psi\rangle$$

$$M_1 X_0 |\psi\rangle = -X_0 M_1 |\psi\rangle = -X_0|\psi\rangle$$

Recall $|\bar{0}\rangle = \frac{1}{4} \prod_i (1 + M_i) |0^5\rangle$

$$|\bar{1}\rangle = \frac{1}{4} \prod_i (1 + M_i) |1^5\rangle$$

have $M_0, M_1, M_2, M_3 = +1, +1, +1, +1$

Suppose: measure M_0, M_1, M_2, M_3
as $+1, -1, +1, -1$

How to correct error?

Well $+-+-$ is the X_2 column,
so the state has an X_2 error

$$X_2 |\psi\rangle$$

To correct, apply X_2

$$X_2 X_2 |\psi\rangle = |\psi\rangle$$

How to implement arbitrary U?

- Universal gate set

$CNOT, Z, H, T$ $\quad T^4 = Z$ ($\pi/8$ gate)

gives any U

difficulty with 5-qubit code:

"hard" to get H, $CNOT$

instead use:

7-qubit "Steane code"

7-qubit Steane Code $\Rightarrow$

$X\,X\,X\,X\,I\,I\,I = M_2$ | $N_2 = Z\,Z\,Z\,Z\,1\,1\,1$

$X\,\cancel{X}\,I\,I\,X\,X\,I = M_1$ | $N_1 = Z\,Z\,1\,1\,Z\,Z\,1$

$X\,I\,X\,I\,X\,I\,X = M_0$ | $N_0 = Z\,1\,Z\,1\,Z\,1\,Z$

Z_0 error $\quad M_i Z_0|\Psi\rangle = -Z_0 \underbrace{M_i|\Psi\rangle}$

$\Leftrightarrow M_0 = -1 \quad\quad = -Z_0|\Psi\rangle$

X_0 error $\Leftrightarrow N_0 = -1$

Y errors $\Rightarrow$ same error syndrom for both N_i, M_i

$Z_0 X_2$ $\quad M_0 = -1 \quad N_0 = -1 \quad N_1 = N_2 = M_1 = M_2 = +1$

$X_i Z_j \Leftrightarrow Y_0$ error

some 2 qubit errors $\quad 2(3n+1) \leq 2^n$

$4 \quad 8$

7-qubit code

$M_0 = X_0 X_4 X_5 X_6$ $\quad N_0 = Z_0 Z_4 Z_5 Z_6$

$M_1 = X_1 X_3 X_5 X_6$ $\quad N_1 = Z_1 Z_3 Z_5 Z_6$

$M_2 = X_2 X_3 X_4 X_6$ $\quad N_2 = Z_2 Z_3 Z_4 Z_6$

$M_i^2 = 1 = N_i^2$ $\quad [M_i, M_j] = [N_i, N_j] = 0$

$[M_i, N_j] = 0$

$$|\bar{0}\rangle = \frac{1}{2^{3/2}}(1+M_0)(1+M_1)(1+M_2)|0^7\rangle$$

$$|\bar{1}\rangle = \frac{1}{2^{3/2}}\prod_{i=0}^{2}(1+M_i)|1^7\rangle$$

$$\langle\bar{0}|\bar{0}\rangle = \frac{1}{2^3}2^3\langle 0^7|\prod_i(1+M_i)|0^7\rangle$$

$(1+M_i)^2 = 2(1+M_i)$

(only contribution)

same for: $\langle\bar{1}|\bar{1}\rangle = 1$

$\langle\bar{0}|\bar{1}\rangle = \langle\bar{1}|\bar{0}\rangle = 0$

(odd/even)

$$A|v_1\rangle = \lambda_1 |v_1\rangle$$

$$A|v_2\rangle = \lambda_2 |v_2\rangle$$

$$(\langle v_2| A)|v_1\rangle = \lambda_2 \langle v_2|v_1\rangle$$

$$= \langle v_2|(A|v_1\rangle) = \lambda_1 \langle v_2|v_1\rangle$$

$$(\lambda_2 - \lambda_1)\langle v_2|v_1\rangle = 0$$

$$\lambda_1 \neq \lambda_2 \Rightarrow \langle v_2|v_1\rangle = 0$$

$\bar{X} = X_0 X_1 \ldots X_6$

$\bar{Z} = Z_0 Z_1 \ldots Z_6$

$\bar{X}|\bar{0}\rangle = \bar{1} \quad \bar{X}|\bar{1}\rangle = |\bar{0}\rangle$

$\bar{Z}|\bar{0}\rangle = |\bar{0}\rangle \quad \bar{Z}|\bar{1}\rangle = -|\bar{1}\rangle$

$\bar{X}^2 = \bar{Z}^2 = 1 \quad \bar{X}\bar{Z} = -\bar{Z}\bar{X}$.

Now $\bar{H} = H_0 H_1 H_2 H_3 H_4 H_5 H_6 = H^{\otimes 7}$

need to show:

$\bar{H}|\bar{0}\rangle = \frac{1}{\sqrt{2}}(|\bar{0}\rangle + |\bar{1}\rangle) \quad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$\bar{H}|\bar{1}\rangle = \frac{1}{\sqrt{2}}(|\bar{0}\rangle - |\bar{1}\rangle)$

$\langle\bar{0}|\bar{H}|\bar{0}\rangle = \langle\bar{0}|\bar{H}|\bar{1}\rangle = \langle\bar{1}|\bar{H}|\bar{0}\rangle$

$= -\langle\bar{1}|\bar{H}|\bar{1}\rangle = 1/\sqrt{2}$

$$\langle \bar{x} | \bar{H} | \bar{y} \rangle = \qquad\qquad \bar{H} M_i = N_i \bar{H}$$

$$\frac{1}{2^3} \langle \bar{0} | \bar{X}^x \prod_i (1+M_i)\ \bar{H}\ \prod_i (1+M_i) \bar{X}^y | \bar{0} \rangle$$

$$= \frac{1}{2^3} \langle \bar{0} | \bar{X}^x \prod_i (1+N_i)\ \bar{H}\ \prod_i (1+N_i) \bar{X}^y | \bar{0} \rangle$$

$$= 2^3 \langle \bar{0} | \bar{X}^x \bar{H} \bar{X}^y | \bar{0} \rangle$$

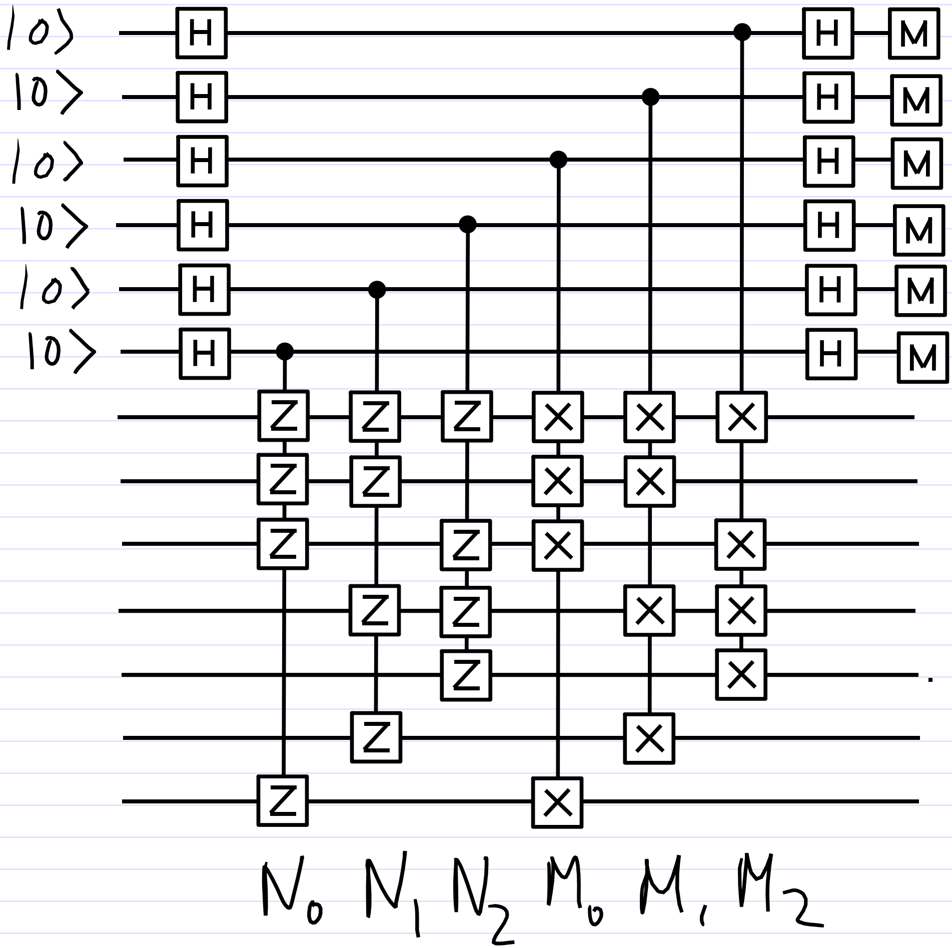
$$= 2^3 \left(\langle 0 | X^x H X^y | 0 \rangle \right)^7$$

$$= \cancel{2^3} \cancel{\left(\langle 0 | X^x H X^y | 0 \rangle \right)^6} \langle 0 | X^x H X^y | 0 \rangle$$

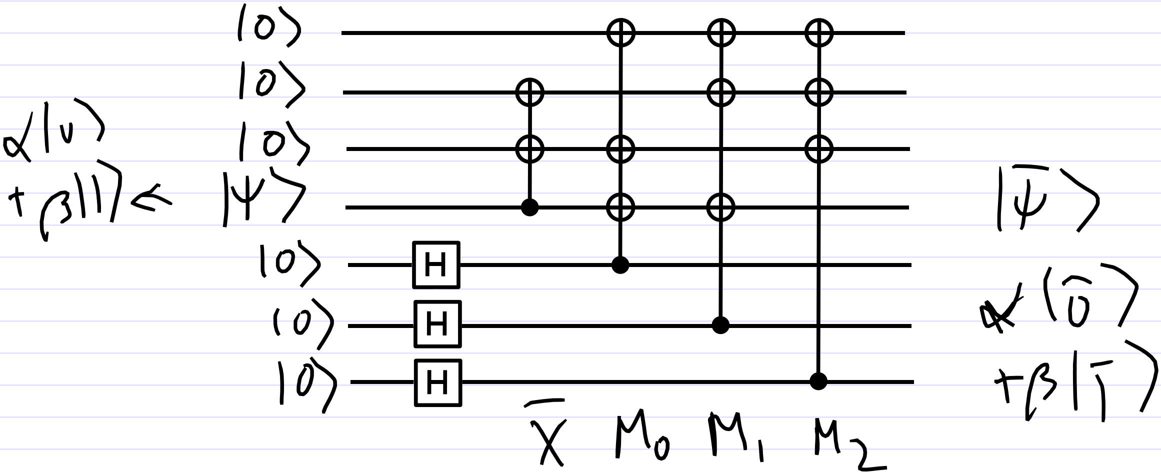
$$= \langle 0 | X^x H X^y | 0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

✓

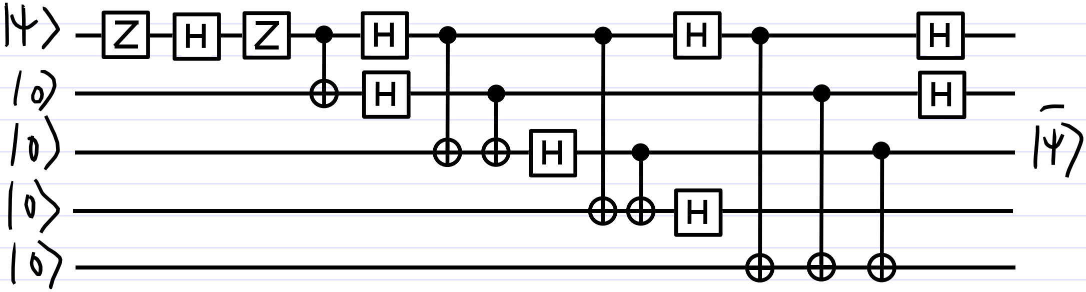
7-qubit measurement gates



7-qubit encoding circuit



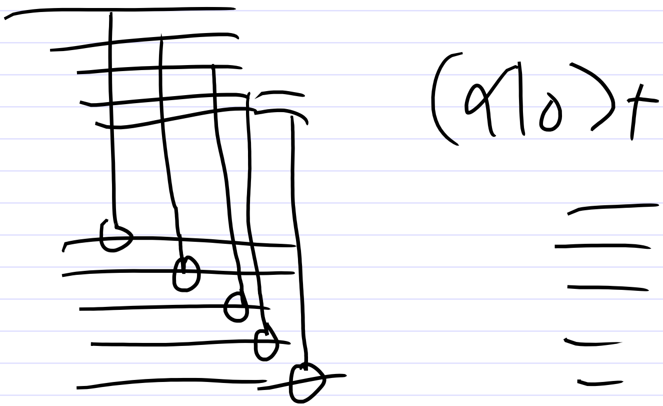
Compare to 5 qubit encoding circuit



5-qubit $\bar{H} \neq H^{\otimes 5}$

$\bar{S} \neq S^{\otimes 5}$

$\overline{CNOT} \neq (CNOT)^{\otimes 5}$



$(\alpha|0\rangle + \beta|1\rangle)|0000\rangle$

$\rightarrow \alpha|\bar{0}\rangle + \beta|\bar{1}\rangle$

@ $\bar{Z} = Z^{\otimes 7}$ $\bar{Z}|\bar{0}\rangle = |\bar{0}\rangle$

$\bar{Z}|\bar{1}\rangle = -|\bar{1}\rangle$

$\prod (1+M_i)|0^7\rangle$

each term $|0001111\rangle$

has even # of 1s

$\langle 1^7|$ odd # of 1s so

always 0.

encoded $|\psi\rangle = \alpha|\bar{0}\rangle + \beta|\bar{1}\rangle$

21 possible errors

$X_i \quad Y_i \quad Z_i$

+ 1 uncorrupted = 22

orthogonal 2d spaces

	1	X_0	X_1	X_2	X_3	X_4	X_5	X_6
M_0		•				•	•	•
M_1			•		•		•	•
M_2				•	•	•		•

	1	Z_0	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
N_0		•				•	•	•
N_1			•		•		•	•
N_2				•	•	•		•

error syndromes:

X_i error $\Rightarrow$ look at which N_i's flip sign

Z_i error $\Rightarrow$ look at which M_i's flip sign

Y_i error $\Rightarrow$ look at pattern of <u>both</u> M, N flips [have to be SAME]

$$M_j X_i |\psi\rangle = X_i |\psi\rangle$$

$N_0 = -1$ error?

X_0

$M_1 = M_2 = -1$ Z_3

(all 6) $M_0 = \dots = N_2 = -1$ Y_6

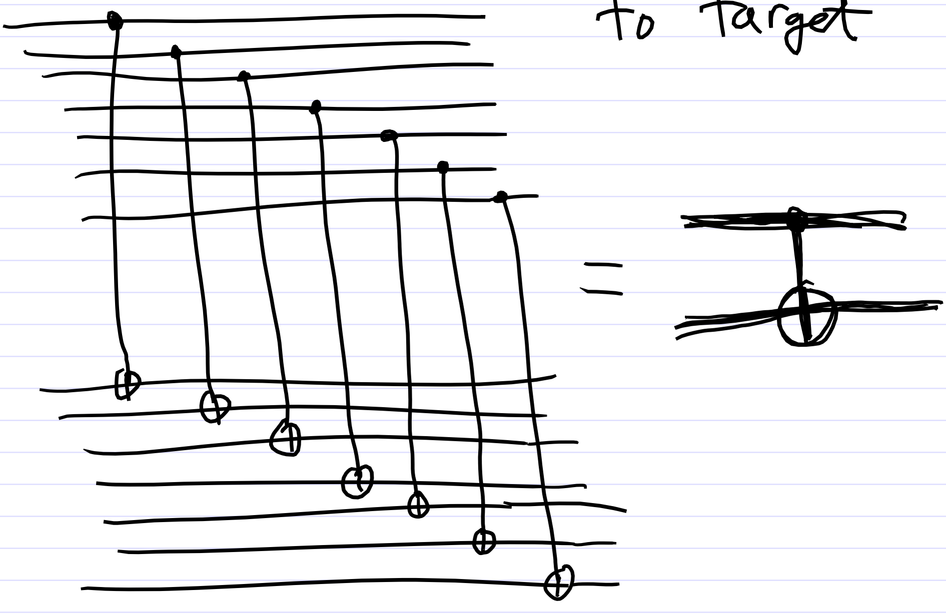
7-qubit $\overline{\text{CNOT}}$

also simple structure:

$$|\bar{0}\rangle = \prod_i (1+M_i)|0^7\rangle$$
$$|\bar{1}\rangle = \bar{X}|\bar{0}\rangle$$

If control is $|\bar{0}\rangle$, then pattern of M_i's applies $\prod_i (1+M_i)$ to target, no effect.

If control is $|\bar{1}\rangle$, then applies additional $\bar{X}$ to target



and $\overline{\text{CNOT}} = \text{CNOT}^{\otimes 7}$

$\overline{\text{CNOT}}$, $\bar{H}$, $\bar{X}$, $\bar{Z}$ all parallelize, 7-qubit code can be made fault tolerant,

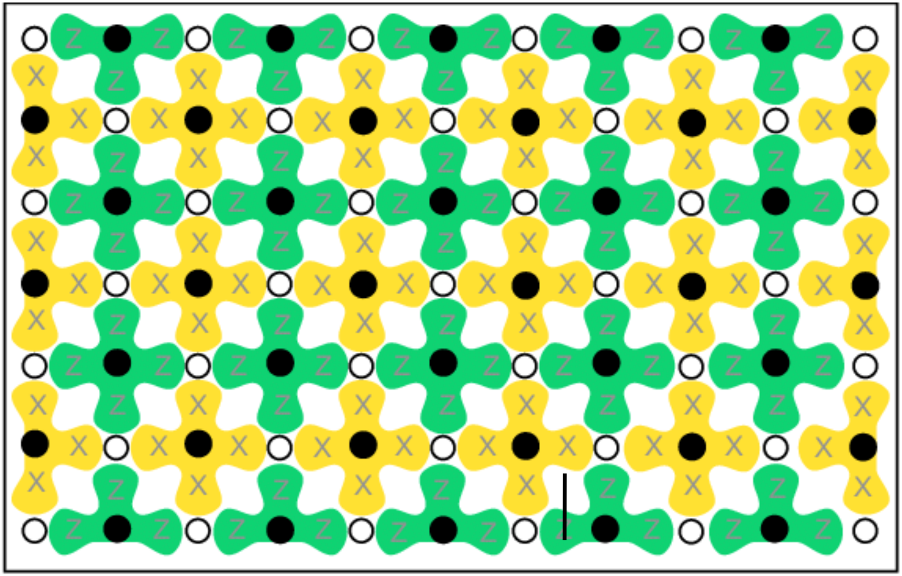
BUT current qubits still not stable enough –

Need surface code, thousands of physical qubits per logical qubit.

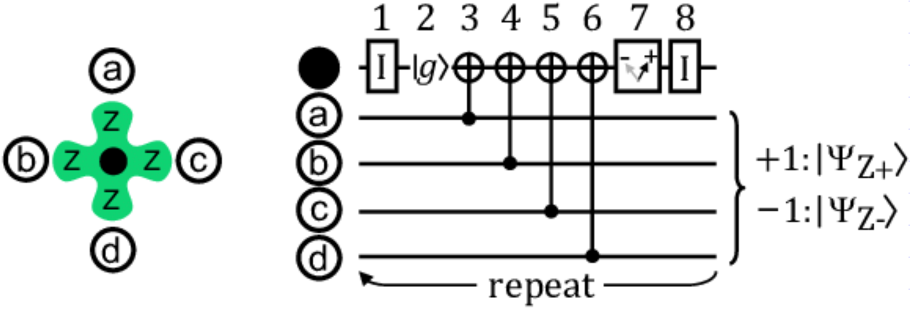
Then can preserve single qubit phase coherence for millions of years

This will all be one logical qubit

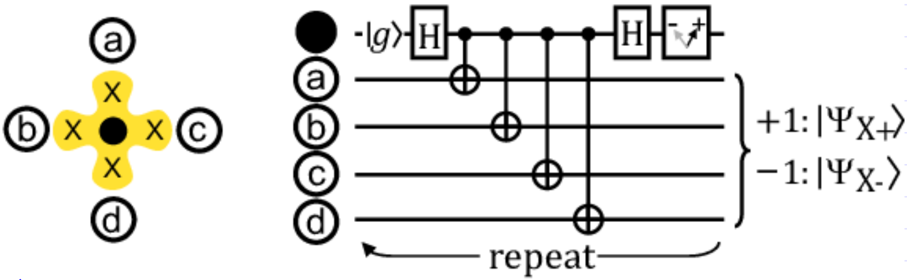
(a)



(b)



(c)

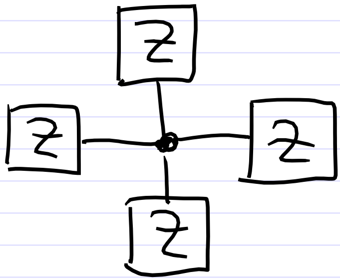


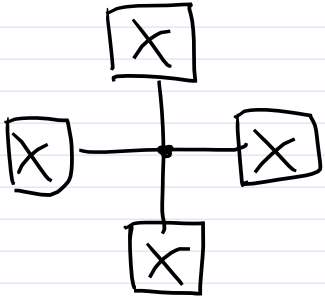
arXiv: 1208.0928

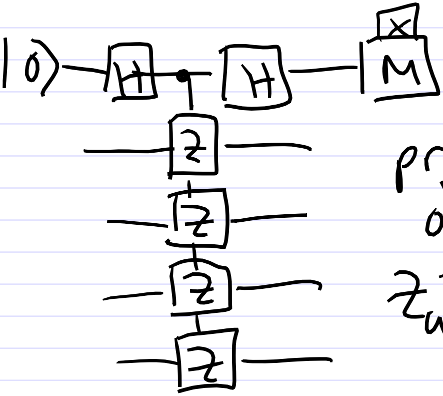
○ = "data" qubits $2^{39}/2^{38} = 2$

● = measurement qubits

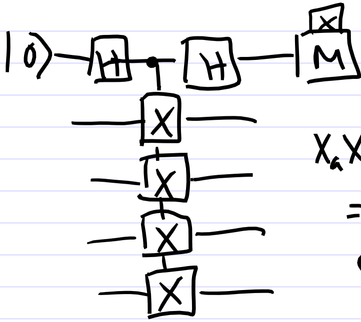
either measure $Z_a Z_b Z_c Z_d$ or $X_a X_b X_c X_d$ on neighboring qubits







projects onto $Z_a Z_b Z_c Z_d = (-1)^x$ eigenstate



$X_a X_b X_c X_d = (-1)^x$ eigenstate

5-qubit code M_0, M_1, M_2, M_3

± 1

7-qubit code $N_0, N_1, N_2, M_0, M_1, M_2$

"stabilizers"

~~Surface code~~

Arbitrarily many stabilizers of the form $Z_a Z_b Z_c Z_d$

$X_a X_b X_c X_d$

$|0000\rangle$ $|1111\rangle$ $|0011\rangle$

± 1 $|0101\rangle$

each measurement reduces the dimension of the Hilbert space by a factor of 2.

Start with $4 \cdot 6 + 3 \cdot 5 = 39$ data qubits so 2^{39} dim space.

But $4 \cdot 5 + 3 \cdot 6 = 38$ measurement qubits so $2^{39}/2^{38} = 2$

$\Rightarrow$ 1 logical qubit

For example, 2 qubits

$|00\rangle$ $|01\rangle$ $|10\rangle$ $|11\rangle$

measure $Z_0 Z_1 = +1 \Rightarrow |00\rangle, |11\rangle$

$= -1 \Rightarrow |01\rangle, |10\rangle$

Joint eigenstates of
$Z_0 Z_1$ $X_0 X_1$ (they commute)

$Z_0 Z_1$	$X_0 X_1$	
1	1	$\frac{1}{\sqrt{2}}(\lvert 00\rangle + \lvert 11\rangle)$
1	-1	$\frac{1}{\sqrt{2}}(\lvert 00\rangle - \lvert 11\rangle)$
-1	1	$\frac{1}{\sqrt{2}}(\lvert 01\rangle + \lvert 10\rangle)$
-1	-1	$\frac{1}{\sqrt{2}}(\lvert 01\rangle - \lvert 10\rangle)$

"Bell Basis"

For 4 qubits, there are eight $Z_a Z_b Z_c Z_d = +1$ eigenstates:

$|0000\rangle, |0011\rangle, \ldots, |1100\rangle, |1111\rangle$

(all with even # of 1's)

Similarly, eight $Z_a Z_b Z_c Z_d = -1$ eigenstates:

$|0001\rangle, |0010\rangle, \ldots, |1101\rangle, |1110\rangle$

(all with odd # of 1's)

Same for $X_a X_b X_c X_d$ in terms of $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$

$+1$ $|{+}{+}{+}{+}\rangle, |{+}{+}{-}{-}\rangle, \ldots, |{-}{-}{+}{+}\rangle, |{-}{-}{-}{-}\rangle$

-1 $|{+}{+}{+}{-}\rangle, |{-}{-}{+}{-}\rangle, \ldots, |{-}{-}{+}{-}\rangle, |{-}{-}{-}{+}\rangle$

Now consider error syndromes

$$Z_a Z_b Z_c Z_d \underbrace{X_a}_{\substack{\text{error} \\ \text{on qubit } a}} |\psi\rangle$$

$$= -X_a Z_a Z_b Z_c Z_d |\psi\rangle = -X_a |\psi\rangle$$

(if started in +1 eigenstate)

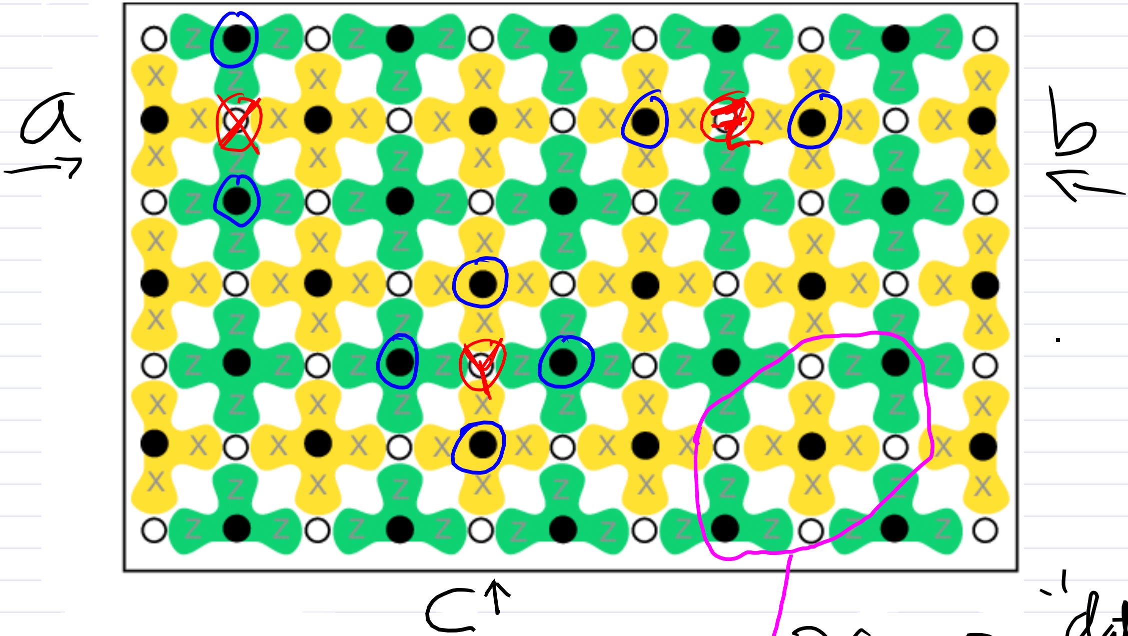
Similarly

$$X_a X_b X_c X_d \overset{\text{error on qubit } b}{Z_b} |\psi\rangle$$

$$= -Z_b X_a X_b X_c X_d |\psi\rangle = -Z_b |\psi\rangle$$

○ = flipped measurement value

○ = error on data qubit



39 ○ "data"

38 ● "meas"

2d or 1 qubit

a = X error

b = Z error

c = Y error

1912.09410

Repeated Quantum Error Detection in a Surface Code

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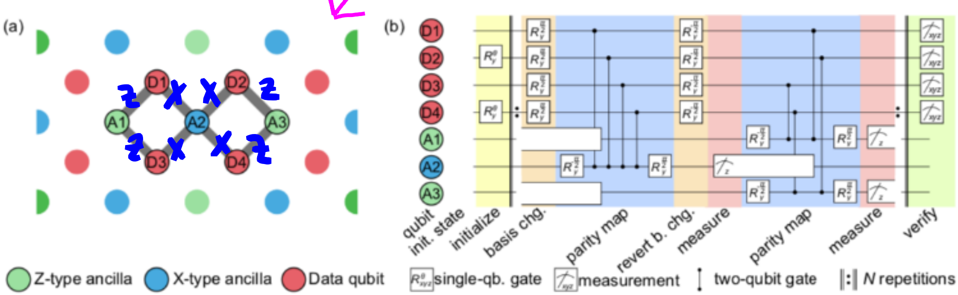
(Dated: December 20, 2019)

The realization of quantum error correction is an essential ingredient for reaching the full potential of fault-tolerant universal quantum computation. Using a range of different schemes, logical qubits can be redundantly encoded in a set of physical qubits. One such scalable approach is based on the surface code. Here we experimentally implement its smallest viable instance, capable of repeatedly detecting any single error using seven superconducting qubits, four data qubits and three ancilla qubits. Using high-fidelity ancilla-based stabilizer measurements we initialize the cardinal states of the encoded logical qubit with an average logical fidelity of 96.1%. We then repeatedly check for errors using the stabilizer readout and observe that the logical quantum state is preserved with a lifetime and coherence time longer than those of any of the constituent qubits when no errors are detected. Our demonstration of error detection with its resulting enhancement of the conditioned logical qubit coherence times in a 7-qubit surface code is an important step indicating a promising route towards the realization of quantum error correction in the surface code.



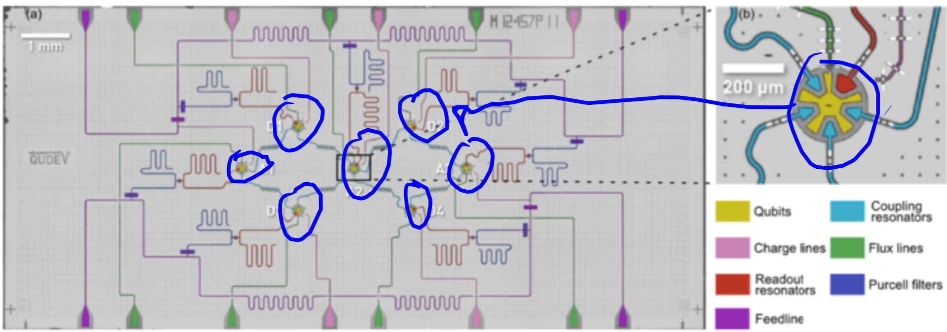


FIG. 2. Seven-qubit device. (a) False colored micrograph of the seven-qubit device used in this work. Transmon qubits are shown in yellow, coupling resonators in cyan, flux lines for single-qubit tuning and two-qubit gates in green, charge lines for single-qubit drive in pink, the two feedlines for readout in purple, transmission line resonators for readout in red and Purcell filters for each qubit in blue. (b) Enlarged view of the center qubit (A2) which connects to four neighboring qubits.

[Submitted on 13 Jul 2022 (v1), last revised 20 Jul 2022 (this version, v2)]

Suppressing quantum errors by scaling a surface code logical qubit

Practical quantum computing will require error rates that are well below what is achievable with physical qubits. Quantum error correction offers a path to algorithmically-relevant error rates by encoding logical qubits within many physical qubits, where increasing the number of physical qubits enhances protection against physical errors. However, introducing more qubits also increases the number of error sources, so the density of errors must be sufficiently low in order for logical performance to improve with increasing code size. Here, we report the measurement of logical qubit performance scaling across multiple code sizes, and demonstrate that our system of superconducting qubits has sufficient performance to overcome the additional errors from increasing qubit number. We find our distance-5 surface code logical qubit modestly outperforms an ensemble of distance-3 logical qubits on average, both in terms of logical error probability over 25 cycles and logical error per cycle ($2.914\% \pm 0.016\%$ compared to $3.028\% \pm 0.023\%$). To investigate damaging, low-probability error sources, we run a distance-25 repetition code and observe a 1.7×10^{-6} logical error per round floor set by a single high-energy event (1.6×10^{-7} when excluding this event). We are able to accurately model our experiment, and from this model we can extract error budgets that highlight the biggest challenges for future systems. These results mark the first experimental demonstration where quantum error correction begins to improve performance with increasing qubit number, illuminating the path to reaching the logical error rates required for computation.

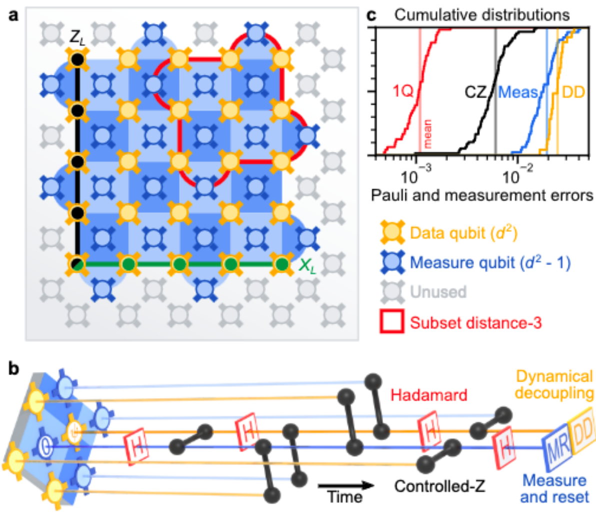


FIG. 1. **Implementing surface code logical qubits. a,** Schematic of a 72-qubit Sycamore device with a distance-5 surface code embedded, consisting of 25 data qubits (gold) and 24 measure qubits (blue). Each measure qubit is associated with a stabiliser (blue colored tile, dark: X, light: Z). Representative logical operators Z_L (black) and X_L (green) traverse the array, intersecting at the lower-left data qubit. The upper-right quadrant (red outline) is one of four subset distance-3 codes (the four quadrants) we compare to distance-5. **b,** Illustration of a stabiliser measurement, focusing on one data qubit (gold) and one measure qubit (blue), in perspective view with time progressing to the right. Each qubit participates in four controlled-Z (CZ) gates with its four nearest neighbours, interspersed with Hadamard gates (H), and finally, the measure qubit is measured and reset to $|0\rangle$. Data qubits perform dynamical decoupling (DD) while waiting for the measurement and reset. All stabilisers are measured in this manner concurrently. Cycle duration is 921 ns, including 500 ns measurement and 160 ns reset. **c,** Cumulative distributions of errors for single-qubit gates, CZ gates, measurement, and data qubit DD (idle during measurement and reset). Benchmarked in simultaneous operation using random circuit techniques, on the 49 qubits used in distance-5 and the four CZ layers from the stabiliser circuit [31, 32]. Vertical lines are means.