Quantum Error Correction

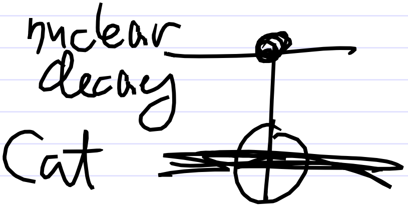
Coupled to environment.

Classically: bit flips

Qubits can have infinitesimal changes

But if measured, lose the state.

How can state be corrected without measuring it?



$$\frac{1}{\sqrt{2}}\left(|0\rangle|alive\rangle + |1\rangle|dead\rangle\right)$$

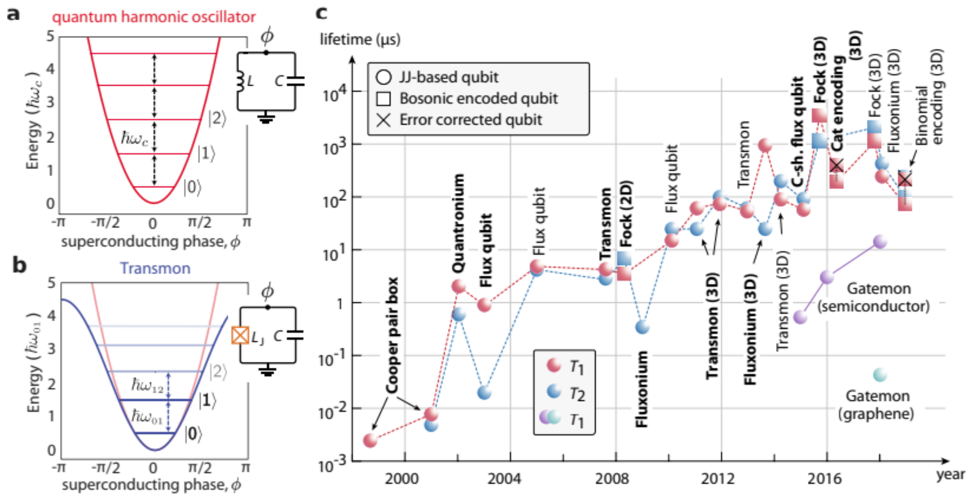


Figure 2

(a) The energy spectrum of a quantum harmonic oscillator (QHO). (b) The energy spectrum of the transmon qubit, showing how the introduction of the non-linear Josephson junction produces non-equidistant energy levels. (c) Evolution of lifetimes and coherence times in superconducting qubits. Bold font indicates the first demonstration of a given modality. 'JJ-based qubits' are qubits where the quantum information is encoded in the excitations of a superconducting circuit containing one or more Josephson junctions (see Sec. 2.1). 'Bosonic encoded qubits' are qubits where the quantum information is encoded in superpositions of multi-photon states in a QHO, and a Josephson junction circuit mediates qubit operation and readout (see Sec. 2.4). 'Error corrected qubits' represent qubit encodings in which a layer of active error-correction has been implemented to increase the encoded qubit lifetime. The charge qubit and transmon modalities are described in Sec. 2.1.1, flux qubit and the capacitively shunted flux qubit ('C-sh. flux qubit') are described in Sec. 2.1.2, and fluxonium and gatemon modalities are described in Sec. 5. The codes underlying the 'cat encoding' and 'binomial encoding' are discussed in Sec. 4.3. '(3D)' indicates a qubit embedded in a three-dimensional cavity. For encoded qubits, the non-error-corrected T_1 and T_2 times used in this figure are for the encoded, but not error-corrected, version of the logical qubit (see Refs. (11) and (12) for details). The references for the JJ-based qubits are (in chronological order) (34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48); the semiconductor-JJ-based transmons (gatemons) are Refs. (49, 50, 51); and the graphene-JJ-based transmon is Ref. (52). The bosonic encoded qubits in chronological order are Refs. (53, 54, 11, 55, 12).

(from arXiv:1905.13641)

single qubit coherence

Table 1 State of the art high-fidelity two-qubit gates in superconducting qubits

Acronym[a]	Layout[b]	First demonstration [Year]	Highest fidelity [Year]	Gate time
CZ (ad.)	T–T	DiCarlo et al. (72) [2009]	99.4%† Barends et al. (3) [2014]	40 ns
			99.7%† Kjaergaard et al. (73) [2020]	60 ns
$\sqrt{\text{iSWAP}}$	T–T	Neeley et al. (81)$^\circ$ [2010]	90%* Dewes et al. (74) [2014]	31 ns
CR	F–F	Chow et al. (75) [2011]	99.1%† Sheldon et al. (5) [2016]	160 ns
$\sqrt{\text{bSWAP}}$	F–F	Poletto et al. (76) [2012]	86%* ibid.	800 ns
MAP	F–F	Chow et al. (77) [2013]	87.2%* ibid.	510 ns
CZ (ad.)	T–(T)–T	Chen et al. (56) [2014]	99.0%† ibid.	30 ns
RIP	3D F	Paik et al. (78) [2016]	98.5%† ibid.	413 ns
$\sqrt{\text{iSWAP}}$	F–(T)–F	McKay et al. (79) [2016]	98.2%† ibid.	183 ns
CZ (ad.)	T–F	Caldwell et al. (80) [2018]	99.2%† Hong et al. (6) [2019]	176 ns
CNOT_L	BEQ-BEQ	Rosenblum et al. (13) [2018]	~99%$^\square$ ibid.	190 ns
CNOT_{T-L}	BEQ-BEQ	Chou et al. (82) [2018]	79%* ibid.	4.6 µs

Gates ordered by year of first demonstration. Gate time is for the highest fidelity gate.

[a]Full names: CZ (ad.): Adiabatic controlled phase, $\sqrt{\text{iSWAP}}$: square-root of the iSWAP, CR: Cross-resonance, $\sqrt{\text{bSWAP}}$: Square-root of the Bell-Rabi SWAP, MAP: Microwave activated phase, RIP: Resonator induced phase gate, CNOT_L: Logical CNOT, CNOT_{T-L}: Teleported logical CNOT.

[b]F is short 'fixed frequency', T is short for 'tunable'. For all non-bosonic encoded qubit gates, the qubits were of the transmon variety (except for the first demonstration of $\sqrt{\text{iSWAP}}$, using phase qubits, and first demonstration of CR which used capacitively shunted flux qubits). Terms in parenthesis is a coupling element. '3D F' is short for a fixed frequency transmon qubit in a three-dimensional cavity. 'BEQ' is short for bosonic encoded qubit (see Sec. 2.4).

$^\circ$Implemented with phase qubits.

†Determined by interleaved randomized Clifford benchmarking (70).

$^\square$Determined by repeated application of the gate to various input states and observing state fidelity decay as function of applied gates. See (13) for details.

*Determined by quantum process tomography.

Gates implemented on flux-tunable qubits.

All-microwave gates.

Combination of tunable and fixed frequency components.

Gates on bosonic encoded qubits.

(from arXiv:1905.13641)

gate times
and fidelities

Classical Error Correction

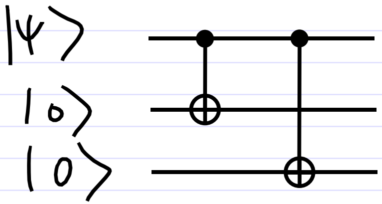
$$|\bar{0}\rangle = |0\rangle|0\rangle|0\rangle = |000\rangle$$

$$|\bar{1}\rangle = |1\rangle|1\rangle|1\rangle = |111\rangle$$

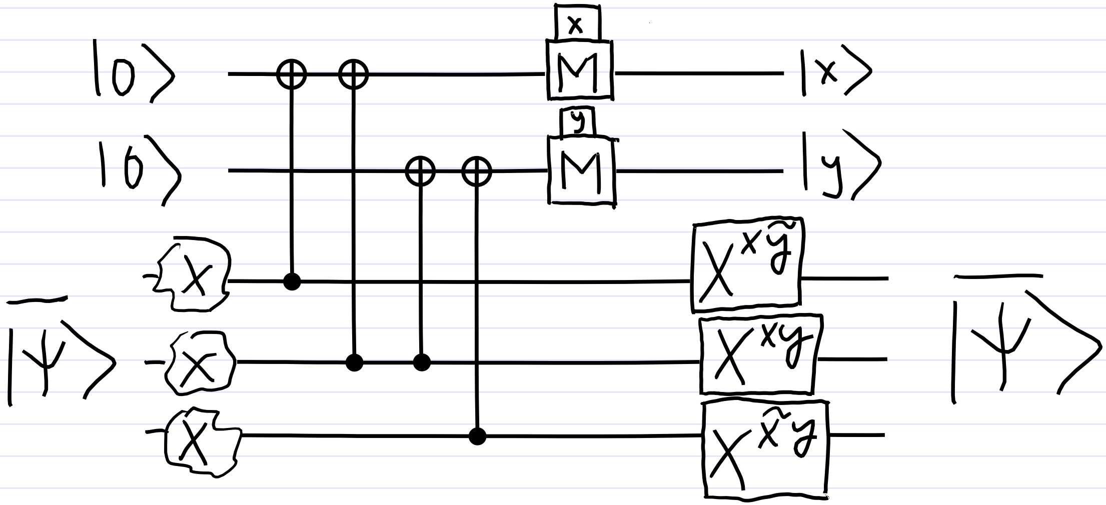
$|0\rangle|1\rangle|0\rangle$ $\begin{matrix} |\bar{0}\rangle \\ |\bar{1}\rangle \end{matrix}$? majority rule

Quantum mechanical:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



$$\alpha|000\rangle + \beta|111\rangle = |\bar{\psi}\rangle$$



restores state $|\bar{\psi}\rangle$
Learn nothing about α, β,
only relations within codewords

xy	
00	1
10	X_2
01	X_0
11	X_1

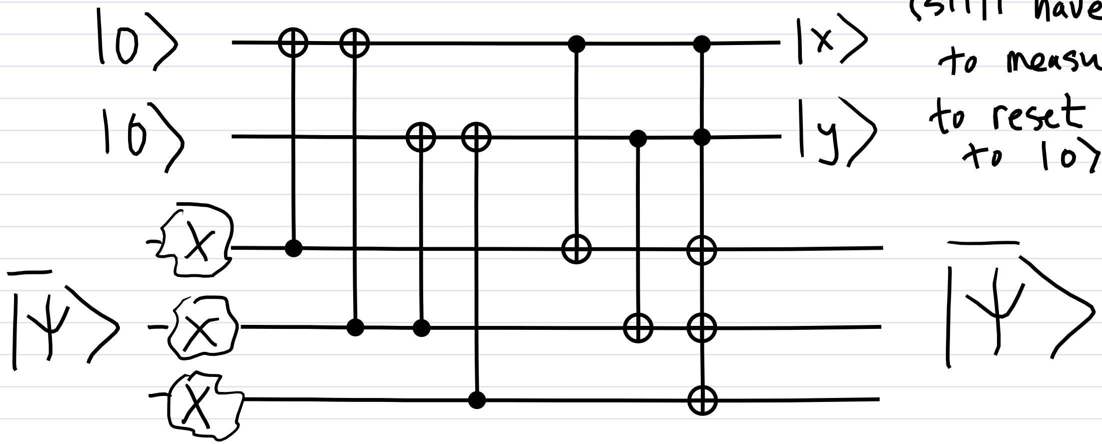
#dims #possible corruptions

$$2(3+1) = 2^3$$

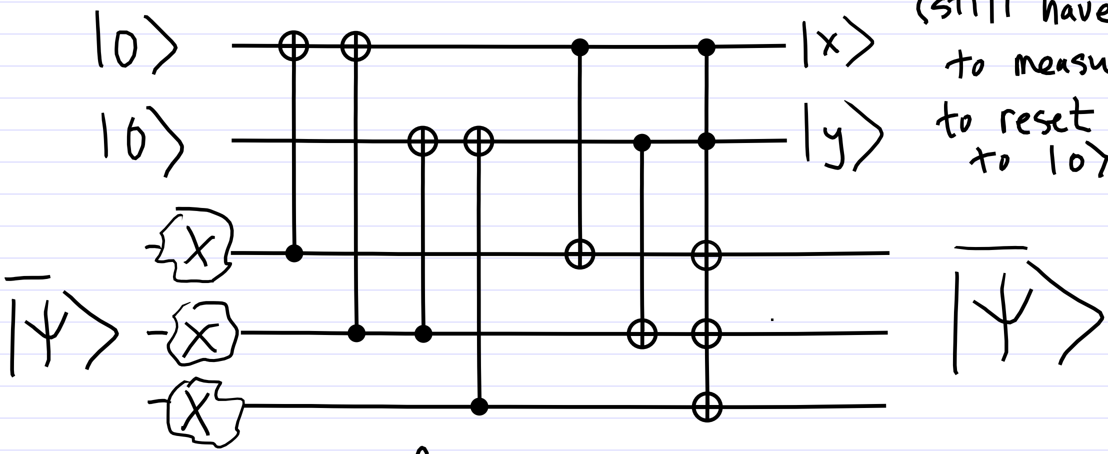
#qubits

$$2(n+1) \le 2^n \qquad n \ge 3$$

3 qubit codeword is minimum for correcting single bitflip error



(still have to measure to reset to |0>)



(still have to measure to reset to $|0\rangle$)

For generalization, consider eigenvalues of $Z_1 Z_2$, $Z_0 Z_1$

Error	$Z_1 Z_2$	$Z_0 Z_1$
$\underline{1}$	1	1
X_2	-1	1
X_1	-1	-1
X_0	1	-1
	$(-1)^x$	$(-1)^y$

e.g. $(Z_0 Z_1) X_0 \overline{|\psi\rangle} = -X_0 (Z_0 Z_1) \overline{|\psi\rangle} = -X_0 \overline{|\psi\rangle}$

$$\boxed{(Z_1 Z_2)\, X_2 |\bar{\psi}\rangle \overset{?}{=} \pm X_2 |\bar{\psi}\rangle}$$

$$|\bar{\psi}\rangle = \alpha|000\rangle + \beta|111\rangle$$

$$Z_0 Z_1 |\bar{\psi}\rangle = |\bar{\psi}\rangle$$

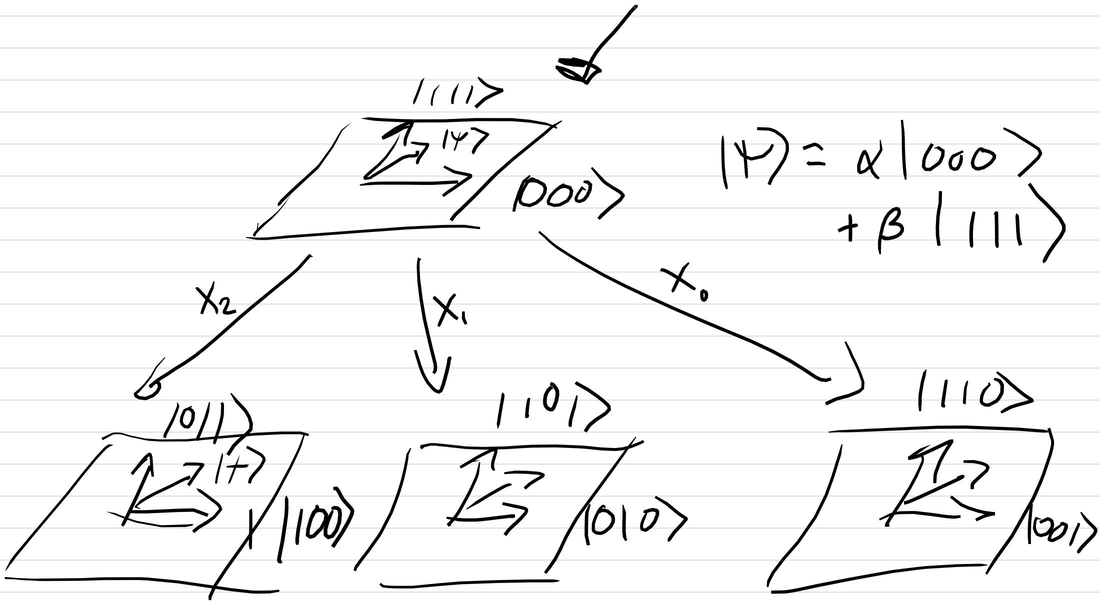
$$Z_1 Z_2 |\bar{\psi}\rangle = |\bar{\psi}\rangle$$

$$(Z_1 Z_2 \quad X_2) = -X_2 (Z_1 Z_2)$$

$$(Z \otimes Z \otimes \mathbb{1} \quad X \otimes 1 \otimes 1$$

$$\underline{(Z_0 Z_1)}\, X_2 |\bar{\psi}\rangle \overset{?}{=} \pm X_2 |\bar{\psi}\rangle$$

$(Z_0 Z_1)^2 = 1$ so has eigenvalues ± 1



"stabilizer formalism"

"Measure" an operator

$A^2 = 1$, A hermitian

$\Rightarrow$ eigenvalues $= \pm 1$

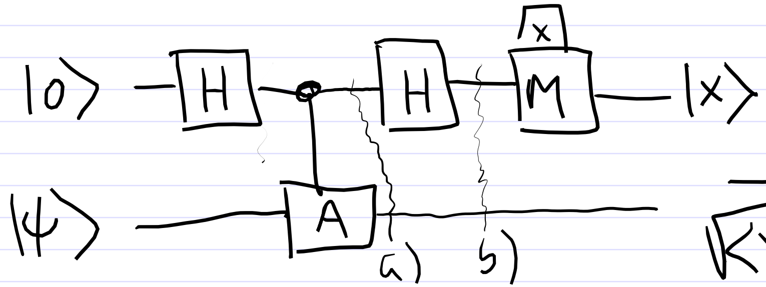
$$A = \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & -1 & & \\ & & & -1 & \\ & & & & \ddots \end{pmatrix}$$

$$P_0^A = \frac{1+A}{2} = \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 0 & & \\ & & & 0 & \\ & & & & \ddots \end{pmatrix}$$

$$P_1^A = \frac{1-A}{2} = \begin{pmatrix} 0 & & & & \\ & 0 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & \ddots \end{pmatrix}$$

Projectors onto ± 1 eigenspaces.

eigenvalue of P_x^A is $(-1)^x$



$$\frac{P_x^A|\psi\rangle}{\sqrt{\langle\psi|P_x^A|\psi\rangle}}$$

a) $C^A \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|\psi\rangle + |1\rangle A|\psi\rangle\right)$

b) $\frac{1}{2}\left((|0\rangle + |1\rangle)|\psi\rangle + (|0\rangle - |1\rangle)A|\psi\rangle\right)$

$$= \frac{1}{2}|0\rangle(1+A)|\psi\rangle + \frac{1}{2}|1\rangle(1-A)|\psi\rangle$$

$$= |0\rangle P_0^A|\psi\rangle + |1\rangle P_1^A|\psi\rangle$$

Note: if $A = Z$ for single qubit, coincides with usual notion of measurement

Now consider more than just bitflips:

$$|\Psi_0\rangle = \alpha|0\rangle|v\rangle_n + \beta|1\rangle|w\rangle_n$$

$$X|\Psi_0\rangle = |\Psi_1\rangle = \alpha|1\rangle|v\rangle_n + \beta|0\rangle|w\rangle_n$$

$$Z|\Psi_0\rangle = |\Psi_2\rangle = \alpha|0\rangle|v\rangle_n - \beta|1\rangle|w\rangle_n$$

$$XZ|\Psi_0\rangle = |\Psi_3\rangle = \alpha|1\rangle|v\rangle_n - \beta|0\rangle|w\rangle_n$$

These form a basis for the full 2d (complex) space of all corruptions.

$\Longrightarrow$ Any corruption can be expanded in terms of X, Z, XZ (bit flip, phase error, joint bitflip/phase error $\sim Y$)

If can correct these, can correct all!