\[ H \otimes H S^+ \otimes H S^+ |\psi_{G12}\rangle = \frac{1}{2} \left( [1001] + [1010] + [1000] + [1111] \right) \]

\[ = H S^+ \otimes H S^+ \otimes H S^+ |\psi_{G\text{12}}\rangle \]

\[ = H S^+ \otimes H S^+ \otimes H |\psi_{G\text{12}}\rangle \]

\[ |\psi_{G\text{12}}\rangle = \frac{1}{\sqrt{2}} \left( |1000\rangle + |1111\rangle \right) \]

\[ = \frac{1}{\sqrt{2}} \left( |10\rangle |\Psi^+\rangle + |11\rangle |\Psi^-\rangle \right) \]
Generation and verification of 27-qubit Greenberger-Horne-Zeilinger states in a superconducting quantum computer

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Generating and detecting genuine multipartite entanglement (GME) of sizeable quantum states prepared on physical devices is an important benchmark for highlighting the progress of near-term quantum computers. A common approach to certify GME is to prepare a Greenberger-Horne-Zeilinger (GHZ) state and measure a GHZ fidelity of at least 0.5. We measure the fidelities using multiple quantum coherences of GHZ states on 11 to 27 qubits prepared on the IBM Quantum ibmq_montreal device. Combinations of quantum readout error mitigation (QREM) and parity verification error detection are applied to the states. A fidelity of 0.546 ± 0.017 was recorded for a 27-qubit GHZ state when QREM was used, demonstrating GME across the full device with a confidence level of 98.6%. We benchmarked the effect of parity verification on GHZ fidelity for two GHZ state preparation embeddings on the heavy-hexagon architecture. The results show that the effect of parity verification, while relatively modest, led to a detectable improvement of GHZ fidelity.

RESULTS

Detecting genuine multipartite entanglement in GHZ states

GHZ states are highly entangled states. They can be prepared in gate-based quantum devices by initialising a single primary qubit to the $|+\rangle$ state and the other qubits to the $|0\rangle$ state, then CNOT gates are iteratively applied from the primary qubit (or any other qubit that has already had a CNOT applied in this manner) to each other qubit involved in the state, as shown in Figure 4. A GHZ state can be expressed as

$$|\text{GHZ}_N\rangle = \frac{|0\rangle \otimes N + |1\rangle \otimes N}{\sqrt{2}},$$

FIG. 2. Example preparation circuits for 7-qubit GHZ states. Whenever a CNOT is applied within these circuits, the GHZ state grows in size by 1 qubit. (a) An inefficient embedding where all CNOTs are applied from the primary qubit, resulting in a CNOT depth of 6. (b) An optimal embedding where CNOTs are applied in parallel from qubits that are already included in the growing GHZ state, resulting in a CNOT depth of 3. When constructing GHZ states, we aim to prepare them using a similar embedding to this optimal one modulo the qubit layout of the quantum device.
FIG. 1. History of experimentally prepared quantum states exhibiting $N$-qubit GME, where $N \geq 3$, with at least 95% confidence in gate-based quantum systems. The year is the date of first publication, to the best of our knowledge. The plot includes superconducting [13, 19], ion trap [8, 20, 23], photonic (polarisation) [24, 31], photonic (multiple degrees of freedom (DoF)) [22, 38], nitrogen-vacancy (NV) centres in diamond [39], neutral atom [35, 36], and quantum dot [37] systems. The circled marker for $N = 27$ in 2021 refers to the results of this work.

(a) GHZ state embedding 1 with a single parity checker qubit

(b) GHZ state embedding 2 with two parity checker qubits

FIG. 3. Diagrams showing how the GHZ states are constructed on the *ibmq_montreal* device. Blue vertices represent qubits of the GHZ state, green vertices represent parity checking qubits, and lighter beige vertices represent qubits that are not directly involved in the experiment. Arrows represent the direction of CNOTs (control (c) → target (t), denoted $\text{CNOT}_c^t$). Dark blue arrows indicate CNOTs used for constructing the GHZ state for all state sizes, light blue arrows indicate CNOTs growing the GHZ state, orange arrows indicate CNOTs used for parity verification, dotted beige lines indicate that no two-qubit gates are applied. (a) Shows the construction of the state sizes 11 to 19 beginning at qubit 2 and including the single parity checking qubit 13. The CNOT$_1^2$ gate is performed before the CNOT$_3^2$ gate. The states are incrementally grown from the initial 11 qubits by including qubits in the order: 15, 16, 18, 19, 21, 22, 23, and 25. (b) Shows the construction of the state sizes 19 to 25 beginning at qubit 13 and including the two parity checking qubits 2 and 24. The initial CNOTs are performed in the order CNOT$_{12}^{13}$, CNOT$_{14}^{13}$, CNOT$_{10}^{12}$, CNOT$_{15}^{12}$, CNOT$_{11}^{14}$, and CNOT$_{16}^{14}$. The states are incrementally grown from the initial 19 qubits by including qubits in the order: 9, 6, 20, 17, 0 and 26. The 26 and 27-qubit states do not use qubits 2 and 24 as parity checking qubits, instead the 26-qubit GHZ state includes qubit 2 with a CNOT$_3^2$ in its construction while the 27-qubit state additionally includes qubit 24 with a CNOT$_{23}^{24}$ gate.
\[ S_{ij} = C_{ij} C_{ji} C_{ij} \]
\[ Z = H \times H \]

\[ H S^+ |\psi\rangle \rightarrow |\psi\rangle \rightarrow S^+ |H\rangle \rightarrow H \times S^+ |\psi\rangle \]

\[ (\text{interchanges target and control}) \]

\[ |00\rangle \rightarrow |00\rangle \]

\[ |10\rangle \rightarrow |10\rangle \]

\[ |110\rangle \rightarrow |110\rangle \]

\[ |111\rangle \rightarrow |-111\rangle \]
\[ |111111\rangle \rightarrow -|111111\rangle \]
\[ |10\rangle \rightarrow |11011\rangle \rightarrow -|11011\rangle \]
\[ |110\rangle \rightarrow -|110\rangle \]
\[ |110011\rangle \rightarrow -|110011\rangle \]
Classically can't create from 1/2 qubit gates since $(110) \leftrightarrow (111)$ is an odd permutation
$n=5, m=1$

$$|x\rangle \rightarrow U_f |x\rangle$$

$$|y\rangle \rightarrow |y \oplus f(x)\rangle$$

$$g = (110001)$$

$$f(x) = a \cdot x = x_0 \oplus x_3 \oplus x_4$$

Particular case of $f$

$$f(x) = a \cdot x = a_0 x_0 \oplus a_1 x_1 \oplus \cdots \oplus a_{n-1} x_{n-1}$$

$$= \sum_{i=0}^{n-1} a_i x_i$$
'93 Bernstein-Vazirani

- bit 1-bit m=1
artificial? but unambiguous speed-up
choose some \( a \leq 2^n \)

\[ f(x) = a \cdot x = \oplus a_i x_i \] 

bitwise XOR

How many invocations of \( f \) to determine \( a \)?

Classically takes \( N = 2^n \) \( m \)

Choose \( x = 2^m \) \( x = (0, \ldots, 0, 1, 0, \ldots, 0) \)
then \( x \cdot a = a_m \)

\[ m = 0, \ldots, n-1 \] so \( n \) times to
determine each bit of \( a \)
QM!
can determine a with
single invocation of Uf!

factor of n speedup