Is $\text{BQP} \subset \text{NP}$? would mean for any problem we can solve with Q.C., must admit a short classical proof. (So far no counterexamples)

But if $\text{BQP} \subset \text{P}$ + $\text{BQP} \subset \text{NP}$

then where does it sit?

[Bernstein/Vazirani] $\text{BQP} \subset \text{PSPACE} \subset \text{EXP}$
\[ U_{m-1} \cdots U_1 U_0 |\psi\rangle = |\psi\rangle \]

\[ U_j \in SU(2^n) \]

\[ 2^n \times 2^n \text{ matrices} \]

\[ \sum (U)_{ab} (V)_{bc} (W)_{cd} \psi_d \]

\[ b, c, d \]

\[ \psi = \sum_{a} \]

\[ b = 10 \]

\[ c = 2 \]

\[ d = 4 \]

\[ U_{a/10} \cdot V_{10/25} \cdot W_{2/9} \cdot \psi_4 \]

(Feynman path integral)

Each "path" polynomial in space, but have to sum exponential number of them.
And to prove $P \neq BQP$

have to prove, e.g., Factoring $\notin P$

so as little hope as proving $P \neq NP$.

Another issue:

$P \subseteq BQP \subseteq PSPACE$

so proving $P \neq BQP$

$\Rightarrow P \neq PSPACE$

But that's also unsolved.

What else can we say?
NP-hard

GI, factoring

Solve any one of these, solves all...

Hamiltonian path
TSP
Knapsack packing

Comb. opt.

Graph coloring
3-SAT

\[ x_0 v x_1 v x_2 \]
\[ x_5 v x_8 v x_9 \]

Bounded Quantum Polynomial

"(e.g. Levenstein distance)"

Graph connectivity
Primality testing
Matrix determinant

Linear programming
Dijkstra for shortest path
Marriage Problem, ...
BQP provides at most "only" exponential speed-up. Because we can always classically simulate using \(2^n\times 2^n\) unitary matrices.

Recall Lec2 (Church-Turing + Extended CT): √CT \(\rightarrow\) can simulate physical processes to arbitrary accuracy with Turing machine equiv.

\(\times\) ECT \(\rightarrow\) using only polynomial (classical) resources

\(\downarrow\) (Shor/Simon algorithms)

modify to use polynomial quantum resources
Claims that BQP can solve general instances of NP-complete problems all violate the $\sqrt{\text{Grover speed-up}}$ bound. But can BQP or some other form of quantum resources help with some instances of these problems?

Instead consider "Adiabatic Quantum Computing" to tackle combinatoric problems directly.
\[ H = "Hamiltonian", \text{ a hermitian } (H = H^\dagger) \text{ operator whose eigenvalues } \lambda_i \text{ give energies of eigenstates} \]

\[ H | \psi_i \rangle = \lambda_i | \psi_i \rangle \]

\[ | \psi \rangle = \sum_i \alpha_i | \psi_i \rangle \]

\[ \langle \psi | H | \psi \rangle = \sum_i \alpha_i^2 \lambda_i \]

\[ = \sum_i p_i \lambda_i = E \langle \lambda \rangle \]
Adiabatic Algorithm

1) start with an $H_{\text{init}}$ that has a simple ground state, e.g.,

$$H_{\text{init}} := \left( \begin{array}{cc} 1 & -1 \\ -1 & 1 \end{array} \right) \rightarrow 0 \text{ ground state}$$

$$|1\rangle \rightarrow 2$$

$$H_{\text{init}} = \sum_{i=0}^{n-1} H_i,$$ 

$$|1\rangle_{\text{init}} = |1\rangle$$

$$= |11\ldots1\rangle$$

2) “perturb” $H$ so that it becomes $H_{\text{final}}$, where the lowest energy (ground) state of $H_{\text{final}}$ is the solution to a problem of interest.
e.g. 3-SAT can be encoded with 3-qubit interactions

$$H_{\text{init}}^{(3)} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

wants all three qubits to be 1

$$x_i \land x_j \land x_k$$

(or with the 0 somewhere else, any other)

$$\overline{x_i} \land \overline{x_j} \land x_k$$

$$H_{\text{final}} = \sum_i H_i^{(3)}$$

3) Do it slowly

$$H_t = (1-t) H_{\text{init}} + t H_{\text{final}}$$

t \in [0, 1]$$
Adiabatic theorem:
Then the eigenstate continuously transitions to eigenstate of $H_{\text{final}}$ (if transition is slow enough) (slow compared to what?)

$\Delta E$, the "mass gap", sets the scale.

But seems that it's exponentially small in combinatorial problems of interest.

$\Rightarrow$ so they'll take exponential time
The company D-Wave has been building many qubit systems for years (though lower quality qubits), 1000-2000 a few years ago, with 5000 promised in near-term. Not excluded that it will work better on some problems.

But also not proven that it is more powerful than classical algorithms, including 'simulated annealing'. But intuition is to be able to tunnel through barrier, rather than have to climb out of false minimum. Quantum wave function might "see" global view of energy landscape.