7-qubit code

$M_0 = X_0 X_4 X_5 X_6 \quad N_0 = \mathbb{Z}_2 \mathbb{Z}_4 \mathbb{Z}_5 \mathbb{Z}_6$

$M_1 = X_1 X_3 X_5 X_6 \quad N_1 = \mathbb{Z}_1 \mathbb{Z}_3 \mathbb{Z}_5 \mathbb{Z}_6$

$M_2 = X_2 X_3 X_4 X_6 \quad N_2 = \mathbb{Z}_2 \mathbb{Z}_3 \mathbb{Z}_4 \mathbb{Z}_6$

$M_i^2 = 1 = N_i^2 \quad [M_i, M_j] = [N_i, N_j] = 0$

$[M_i, N_j] = 0$

$|0\rangle = \frac{1}{2^{3/2}} (1 + M_0)(1 + M_1)(1 + M_2)|0^7\rangle$

$|1\rangle = \frac{1}{2^{3/2}} \prod_{i=0}^{2} (1 + M_i) |1^7\rangle$
\[ \hat{X} = H_{0} H_{1} (H_{2} H_{3} H_{4} H_{5} H_{6}) = H^{07} \]

Now need to show:

\[ \hat{H}/0 = \pm \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \quad \hat{H}/1 = \pm \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \]

\[ \langle 0 | \hat{H} | 0 \rangle = \langle 0 | \hat{H} | 1 \rangle = \langle T | \hat{H} | 0 \rangle = -\langle T | \hat{H} | 1 \rangle = 1/\sqrt{2} \]
\[ \langle \bar{x}^1 \bar{H} \bar{y} \rangle = N_i \bar{H} \]
\[ \frac{1}{2^3} \langle 0^7 | \bar{x}^i \bar{H}^{I+M_i} \bar{x}^i \bar{H}^{I+M_i} \bar{x}^y \bar{1} \bar{0}^7 \rangle \]
\[ = \frac{1}{2^3} \langle 0^7 | \bar{x}^i \bar{H}^{I+N_i} \bar{x}^i \bar{H}^{I+N_i} \bar{x}^y \bar{1} \bar{0}^7 \rangle \]
\[ = 2^3 \langle 0^7 | \bar{x}^x \bar{H}^{x} \bar{x}^y \bar{1} \bar{0}^7 \rangle \]
\[ = 2^3 \left( \langle 0^1 | x^x H x^y 10 \rangle \right)^6 \langle 0^1 | x^x H x^y 10 \rangle \]
\[ = \langle 0^1 x^y H x^y 10 \rangle = \frac{1}{\sqrt{2}} (1, -1) \]
7-qubit encoding circuit

\[ |\psi\rangle = (\alpha |0\rangle + \beta |1\rangle) \]

Compare to 5 qubit encoding circuit
7-qubit CNOT also simple structure:

If control is $|0\rangle$, then pattern of $M_i$'s applies $\prod_i (1+M_i)$ to target, no effect.

If control is $|1\rangle$, then applies additional $\overline{X}$ to target.

$$|0\rangle = \prod_i (1+M_i) |0\rangle$$
$$|1\rangle = \overline{X} |0\rangle$$

$$\overline{\text{CNOT}} = \overline{\text{CNOT}} \otimes \overline{\text{H, X, Z}} \text{ all parallelize,}$$
so 7-qubit code can be made fault tolerant, BUT current qubits still not stable enough. Need surface code, thousands of physical qubits per logical qubit.

Then can preserve single qubit phase coherence for millions of years.
This will all be one logical qubit

\[ \text{arXiv: 1206.0928} \]
5-qubit code $M_0, M_1, M_2, M_3$

$+1$

7-qubit code $N_0, N_1, N_2, M_0, M_1, M_2$

"Stabilizers"

Surface code

Arbitrarily many stabilizers of the form $2 a \overline{Z}_0 2 \overline{Z}_d$

$X_a X_b X_c X_d$

$|0000\rangle$ $|1111\rangle$ $|0011\rangle$

$+1$

$|0101\rangle$
$0 = \text{"data" qubits} \quad \frac{2^{39}}{2^{38}} = 2 \quad \Phi$

$\bigcirc = \text{measurement qubits}$

either measure $Z_a Z_b Z_c Z_d$ or $X_a X_b X_c X_d$ on neighboring qubits

$|0\rangle \rightarrow H \rightarrow H \rightarrow X \rightarrow M$

projects onto $Z_a Z_b Z_c Z_d = (-1)^x$ eigenstate

$X_a X_b X_c X_d = (-1)^x$ eigenstate
Each measurement reduces the dimension of the Hilbert space by a factor of 2.

Start with $4 \cdot 6 + 3 \cdot 5 = 39$ data qubits so $2^{39}$ dim space.

But $4 \cdot 5 + 3 \cdot 6 = 38$ measurement qubits so $2^{39}/2^{38} = 2$

$\Rightarrow 1$ logical qubit

For example, 2 qubits

$|00\rangle, |01\rangle, |10\rangle, |11\rangle$

measure $Z_0 Z_1 = +1 \Rightarrow |100\rangle, |111\rangle$

$= -1 \Rightarrow |101\rangle, |110\rangle$
Joint eigenstates of $Z_0 Z_1$, $X_0 X_1$ (they commute)

\[
\begin{array}{cccc}
1 & 1 & \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) \\
1 & -1 & \frac{1}{\sqrt{2}} (|00\rangle - |11\rangle) \\
-1 & 1 & \frac{1}{\sqrt{2}} (|10\rangle + |11\rangle) \\
-1 & -1 & \frac{1}{\sqrt{2}} (|10\rangle - |11\rangle)
\end{array}
\]

"Bell Basis"
0 = flipped measurement value
0 = error on data qubit

a = X error
b = Z error
c = Y error

c^\uparrow

39 0  "data"
38  "meas"
2d or 1 qubit
Repeated Quantum Error Detection in a Surface Code

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The realization of quantum error correction is an essential ingredient for reaching the full potential of fault-tolerant universal quantum computation. Using a range of different schemes, logical qubits can be redundantly encoded in a set of physical qubits. One such scalable approach is based on the surface code. Here we experimentally implement its smallest viable instance, capable of repeatedly detecting any single error using seven superconducting qubits, four data qubits and three ancilla qubits. Using high-fidelity ancilla-based stabilizer measurements we initialize the cardinal states of the encoded logical qubit with an average logical fidelity of 96.1%. We then repeatedly check for errors using the stabilizer readout and observe that the logical quantum state is preserved with a lifetime and coherence time longer than those of any of the constituent qubits when no errors are detected. Our demonstration of error detection with its resulting enhancement of the conditioned logical qubit coherence times in a 7-qubit surface code is an important step indicating a promising route towards the realization of quantum error correction in the surface code.

FIG. 2. Seven-qubit device. (a) False colored micrograph of the seven-qubit device used in this work. Transmon qubits are shown in yellow, coupling resonators in cyan, flux lines for single-qubit tuning and two-qubit gates in green, charge lines for single-qubit drive in pink, the two feedlines for readout in purple, transmission line resonators for readout in red and Purcell filters for each qubit in blue. (b) Enlarged view of the center qubit (A2) which connects to four neighboring qubits.