Alice and Bob share a Bell state
\[ \Phi_{ab} = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) \]

Alice also has some (unknown) \( |\psi\rangle \)

\[ |\psi\rangle = \alpha |0\rangle + \beta |1\rangle \]

(See next page)

To recover \( |\psi\rangle \), Bob needs to apply:

If Alice measures:
\[ x, y = 0, 0 \]
\[ 0, 1 \]
\[ 1, 0 \]
\[ 1, 1 \]
\[ |\psi_{ab}\rangle = \left(\alpha|10\rangle + \beta|11\rangle\right) \text{ where } \alpha^2 + \beta^2 = 1 \]

Apply CNOT \(\text{CNOT} \Rightarrow\)

\[ |0\rangle \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) + |1\rangle \frac{1}{\sqrt{2}} (|10\rangle + |01\rangle) \]

Apply \(H\) \(\text{H}\) gives: \(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\)

To correct Bob needs to apply \(X\) \(\text{X}\)

\[ \frac{1}{2} |0\rangle_a |0\rangle_a (\alpha|10\rangle + \beta|11\rangle)_b \]

\[ + \frac{1}{2} |0\rangle |1\rangle (\alpha|11\rangle + \beta|0\rangle)_b \]

\[ + \frac{1}{2} |1\rangle |1\rangle (\alpha|10\rangle - \beta|11\rangle)_b \]

\[ + \frac{1}{2} |1\rangle |1\rangle (\alpha|11\rangle - \beta|0\rangle)_b \]

\[ z \times \]

\[ |\psi\rangle_b \]
For teleportation, we'll need

General swap circuit reduces for $|\psi\rangle|0\rangle$ to:

$|\psi\rangle$  

Also recall: implies the controlled circuit identity

$=$
\[ |\psi\rangle = H |0\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \]

\[ X |\psi\rangle = |\psi\rangle \]

\[ |\psi\rangle \]

\[ X \]

\[ Z \]

\[ H \]

\[ H \]

\[ X \]

\[ Z \]

\[ H \]

\[ X \]

\[ Z \]

\[ (measure, \ then \ classical \ control) \]

\[ H \]

\[ X \]

\[ X' \]

\[ Z' \]

\[ |\psi\rangle \]
Now consider Alice's qubit is in some entangled state $\sum_i |\psi_i\rangle |\Phi_i\rangle_{n-1}$. 

(Same argument teleports entanglement)
Figure 10.3: Entanglement swapping procedure depicted. Initially Alice and Bob share a Bell pair, as do Charlie and Diane and Alice and Charlie. Following a pair of teleportations from Alice to Bob and from Charlie to Diane we find that Bob and Diane now share a Bell pair (even though they never interacted directly).
Quantum Error Correction

Coupled to environment. Classically: bit flips. Qubits can have infinitesimal changes. But if measured, lose the state. How can the state be corrected without measuring it?

\[
\text{Cat} \quad \frac{1}{\sqrt{2}} (|0\rangle_{\text{alive}} + |1\rangle_{\text{dead}})
\]

nuclear decay
Figure 2
(a) The energy spectrum of a quantum harmonic oscillator (QHO). (b) The energy spectrum of the transmon qubit, showing how the introduction of the non-linear Josephson junction produces non-equidistant energy levels. (c) Evolution of lifetimes and coherence times in superconducting qubits. Bold font indicates the first demonstration of a given modality. ‘JJ-based qubits’ are qubits where the quantum information is encoded in the excitations of a superconducting circuit containing one or more Josephson junctions (see Sec. 2.1). ‘Bosonic encoded qubits’ are qubits where the quantum information is encoded in superpositions of multi-photon states in a QHO, and a Josephson junction circuit mediates qubit operation and readout (see Sec. 2.4). ‘Error corrected qubits’ represent qubit encodings in which a layer of active error-correction has been implemented to increase the encoded qubit lifetime. The charge qubit and transmon modalities are described in Sec. 2.1.1, flux qubit and the capacitively shunted flux qubit (‘C-sh. flux qubit’) are described in Sec. 2.1.2, and fluxonium and gatemon modalities are described in Sec. 5. The codes underlying the ‘cat encoding’ and ‘binomial encoding’ are discussed in Sec. 4.3. ‘(3D)’ indicates a qubit embedded in a three-dimensional cavity. For encoded qubits, the non-error-corrected $T_1$ and $T_2$ times used in this figure are for the encoded, but not error-corrected, version of the logical qubit (see Refs. (11) and (12) for details). The references for the JJ-based qubits are (in chronological order) (34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48); the semiconductor-JJ-based transmons (gatemons) are Refs. (49, 50, 51); and the graphene-JJ-based transmon is Ref. (52). The bosonic encoded qubits in chronological order are Refs. (53, 54, 11, 55, 12).

(from arXiv:1905.13641)

Single qubit coherence
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Layout</th>
<th>First demonstration [Year]</th>
<th>Highest fidelity [Year]</th>
<th>Gate time</th>
</tr>
</thead>
<tbody>
<tr>
<td>CZ (ad.)</td>
<td>T–T</td>
<td>DiCarlo et al. (72) [2009]</td>
<td>99.4%† Dewends et al. (3) [2014]</td>
<td>40 ns</td>
</tr>
<tr>
<td>√iSWAP</td>
<td>T–T</td>
<td>Neeley et al. (81) ⁹ [2010]</td>
<td>99.7%† Kjaergaard et al. (73) [2020]</td>
<td>60 ns</td>
</tr>
<tr>
<td>CR</td>
<td>F–F</td>
<td>Chow et al. (75) [2011]</td>
<td>99.1%† Sheldon et al. (5) [2016]</td>
<td>160 ns</td>
</tr>
<tr>
<td>√bSWAP</td>
<td>F–F</td>
<td>Poletto et al. (76) [2012]</td>
<td>86%* ibid.</td>
<td>800 ns</td>
</tr>
<tr>
<td>MAP</td>
<td>F–F</td>
<td>Chow et al. (77) [2013]</td>
<td>87.2%* ibid.</td>
<td>510 ns</td>
</tr>
<tr>
<td>CZ (ad.)</td>
<td>T–(T)–T</td>
<td>Chen et al. (56) [2014]</td>
<td>99.0%† ibid.</td>
<td>30 ns</td>
</tr>
<tr>
<td>RIP</td>
<td>3D F</td>
<td>Paik et al. (78) [2016]</td>
<td>98.5%† ibid.</td>
<td>413 ns</td>
</tr>
<tr>
<td>√iSWAP</td>
<td>F–(T)–F</td>
<td>McKay et al. (79) [2016]</td>
<td>98.2%† ibid.</td>
<td>183 ns</td>
</tr>
<tr>
<td>CZ (ad.)</td>
<td>T–F</td>
<td>Caldwell et al. (80) [2018]</td>
<td>99.2%† Hong et al. (6) [2019]</td>
<td>176 ns</td>
</tr>
<tr>
<td>CNOT&lt;sub&gt;L&lt;/sub&gt;</td>
<td>BEQ-BEQ</td>
<td>Rosenblum et al. (13) [2018]</td>
<td>≈99% MediaPlayer™ ibid.</td>
<td>190 ns</td>
</tr>
<tr>
<td>CNOT&lt;sub&gt;T–L&lt;/sub&gt;</td>
<td>BEQ-BEQ</td>
<td>Chou et al. (82) [2018]</td>
<td>79%* ibid.</td>
<td>4.6 μs</td>
</tr>
</tbody>
</table>

Gates ordered by year of first demonstration. Gate time is for the highest fidelity gate.

⁹ Implemented with phase qubits.

† Determined by interleaved randomized Clifford benchmarking (70).

⁻⁻ Determined by repeated application of the gate to various input states and observing state fidelity decay as function of applied gates. See (13) for details.

* Determined by quantum process tomography.

- Gates implemented on flux-tunable qubits.
- All-microwave gates.
- Combination of tunable and fixed frequency components.
- Gates on bosonic encoded qubits.

(from arXiv:1905.13641)
Classical Error Correction

\[ |\overline{0} \rangle = |0 \rangle |0 \rangle |0 \rangle = |000 \rangle \]

\[ |\overline{1} \rangle = |1 \rangle |1 \rangle |1 \rangle = |111 \rangle \]

\[ |0 \rangle |1 \rangle |1 \rangle \quad ? \quad \text{majority rule} \]

Quantum mechanical:

\[ |\psi \rangle = \alpha |0 \rangle + \beta |1 \rangle \]

\[ |\psi \rangle = \sqrt{\alpha^2 + \beta^2} |0 \rangle + \beta |1 \rangle \]

\[ \Rightarrow \quad |\psi \rangle = \sqrt{|\psi|} \]

\[ \Rightarrow \quad |\psi \rangle = \sqrt{\psi} \]
restores state $|1\psi\rangle$

Learn nothing about $\alpha, \beta$, only relations within codewords

$$
\begin{array}{cccc}
00 & 1 & X_2 & 11 \\
01 & X_0 & & X_1 \\
10 & & X_{xy} & \\
11 & X_{xy} & X_{xy} & \\
\end{array}
$$
\[ 2(3+1) = 2^3 \]

\[ 2(n+1) \leq 2^n \quad n \geq 3 \]

3 qubit codeword is minimum for correcting single bitflip error

(still have to measure to reset to \( |0\rangle \))