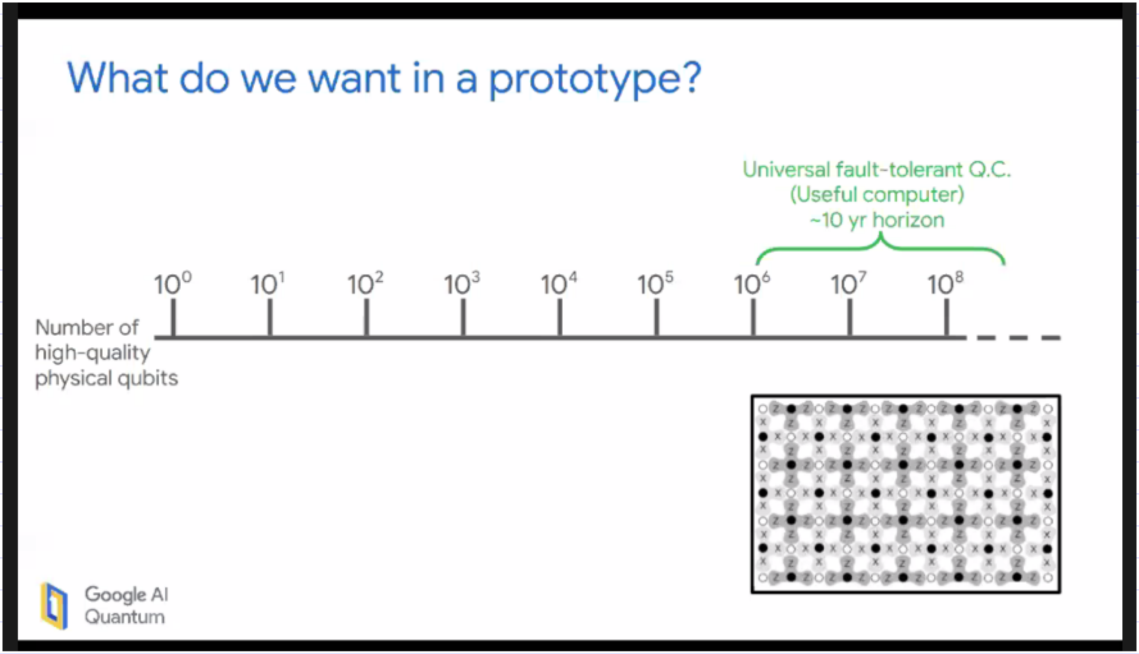
Lec 4 15 Sep 2020

Optical Society of America Conf
on-line 14 - 17 Sep 2020

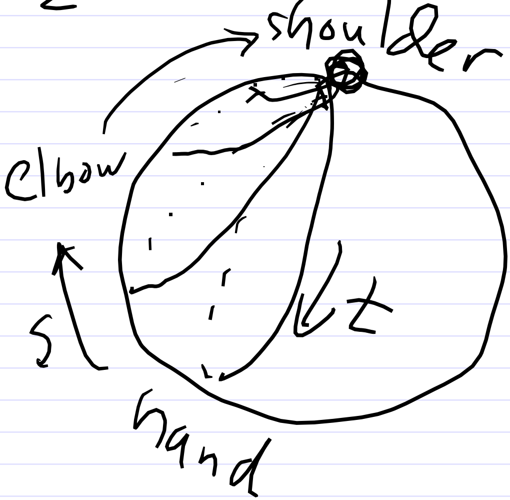
https://www.osa.org/en-us/meetings/topical_meetings/quantum/



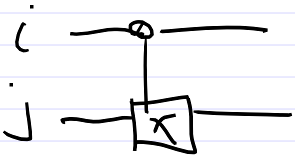
$$e^{i\frac{\theta}{2}\sigma_z}$$ θ runs from $0 \to 2\pi$

$$= \cos\frac{\theta}{2} + i\sigma_z \sin\frac{\theta}{2}$$

S^3

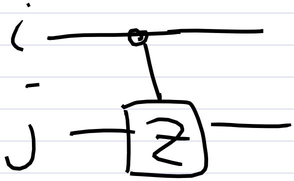


C_{ij}



$X = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$

C^Z_{ij}



$i=0$ do nothing

$i=1$ apply Z

$Z = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$

$Z|0\rangle = |0\rangle$

$Z|1\rangle = -|1\rangle$

$C^Z_{10}\overset{\text{ctrl}}{|0\rangle}|0\rangle = |0\rangle|0\rangle$

$C^Z_{10}|0\rangle|1\rangle = |0\rangle|1\rangle$

$C^Z_{10}|1\rangle|0\rangle = |1\rangle|0\rangle$

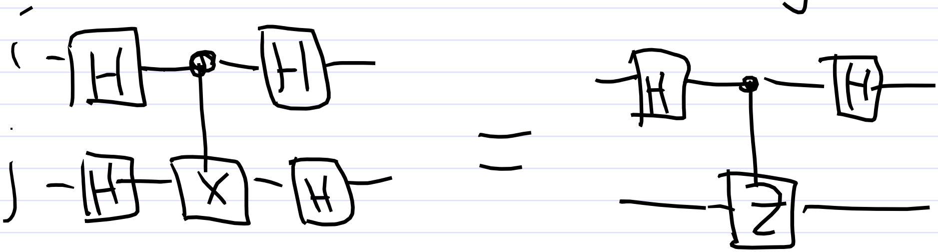
$C^Z_{11}|1\rangle|1\rangle = |1\rangle Z|1\rangle = -|1\rangle|1\rangle$

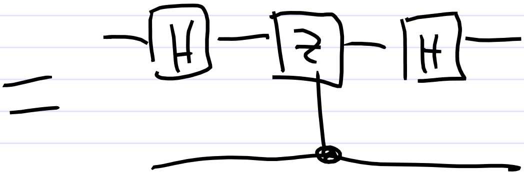
$$C^Z_{10} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix} = C^Z_{01}$$

Recall $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ $HXH = Z$

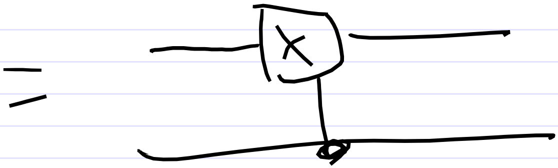
$H^2 = 1$

Consider $H_i H_j C_{ij} H_i H_j = C_{ji}$





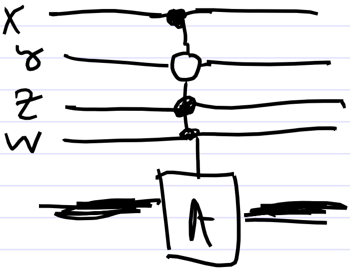
$C^z_{ij} = C^z_{ji}$



reverses control and target bits!

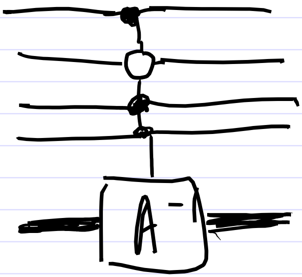
i^{th} bit

$1 \otimes 1 \otimes \cdots \otimes H \otimes 1 \cdots \otimes 1 = H_i$



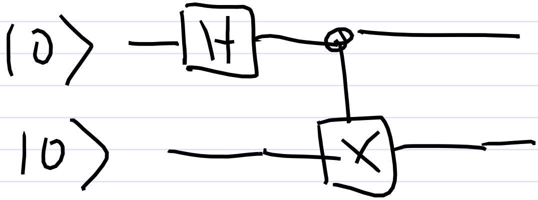
generalization to multiple control bits and multi-qubit operator A

iff $x \wedge \bar{y} \wedge z \wedge w = 1$



Someone asked if reversible,
answer: yes
if A is reversible

CNOT = general way of creating entanglement



$$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$$C_{01} H_0 |00\rangle$$

$$= C_{01} |0\rangle \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

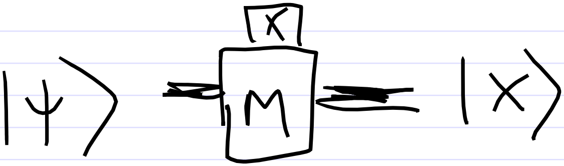
$$= \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$ "Bell pair"

Measurements:

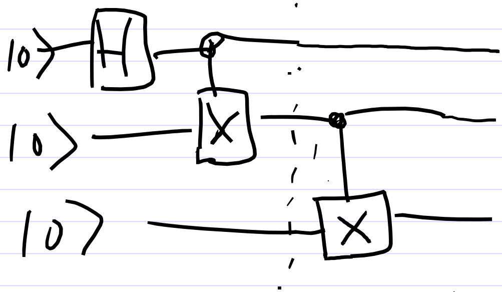
$$p(X_0=0, X_1=0) = \frac{1}{2} = p(X_0=1, X_1=1)$$

$$p(X_0=0, X_1=1) = 0 = p(X_0=1, X_0=0)$$

$$|\psi\rangle = \sum_x \alpha_x |x\rangle$$



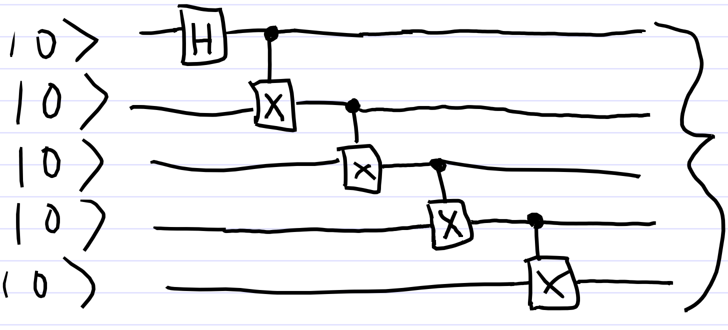
$$p = |\alpha_x|^2 = |\langle x|\psi\rangle|^2$$



$$C_{12} \tfrac{1}{\sqrt{2}} |0\rangle (|00\rangle + |11\rangle)$$
$$= \tfrac{1}{\sqrt{2}} (|000\rangle + |111\rangle)$$

"GHZ state"

see arXiv:1402.4848 fig. 4



$$\tfrac{1}{\sqrt{2}} (|00000\rangle + |11111\rangle)$$

might be tempted to think that

$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ is $|0\rangle$ 50% of time and $|1\rangle$ other 50%, but

difference between superposition and classical mixture

$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \to |0\rangle$ 50% $\to \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ → ~~25% $|0\rangle$~~, ~~25% $|1\rangle$~~

$\qquad\qquad\qquad\;\; \to |1\rangle$ 50% $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ → ~~25% $|0\rangle$~~, ~~25% $|1\rangle$~~

Not what happens

$H \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ ~~$|0\rangle$ 50%~~ ~~$|1\rangle$ 50%~~

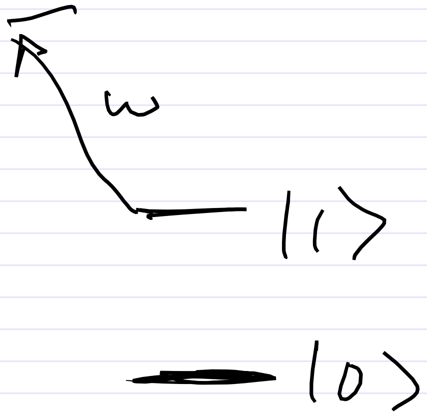
but no, it's $= |0\rangle$ 100% $|0\rangle$

0% $|1\rangle$

Projective measurement.

$$p = |\langle x | \psi \rangle|^2 \quad |x\rangle$$

$$|\psi\rangle = \sum_x \alpha_x |x\rangle$$



e.g.
Ion trap
transition energetically permitted from one state so shine laser and only $|1\rangle$ flouresces in readout

$|\Psi\rangle = \sum_x \alpha_x |x\rangle$

how many degrees of freedom?

2^n $\alpha_{00\ldots0} \ldots\ldots \alpha_{1\ldots10}$ $\alpha_{1\ldots1}$

$2\cdot 2^n$ real degrees of freedom

Two constraints, so

(i) $\sum_x |\alpha_x|^2 = 1$ $\quad 2(2^n - 1)$

(ii) overall phase $e^{i\varphi}|\Psi\rangle$

Consider single qubit

$|\Psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$

2 real degrees of freedom

Suppose n qubits,
product state
$|\psi_{n-1}\rangle|\psi_{n-2}\rangle \cdots |\psi_1\rangle|\psi_0\rangle$

has $2n$ degrees of freedom
no multi-qubit entanglement

n	1	2	3	4	
$2n$	2	4	6	8	...
$2(2^n-1)$	2	6	14	30	

growing exponentially compared to product state

Next time:

reversible operations on Cbits
extend by linearity to
unitary operations on qubits