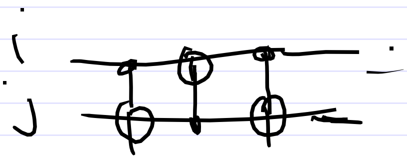
Lecture 22, 1 Dec 2020

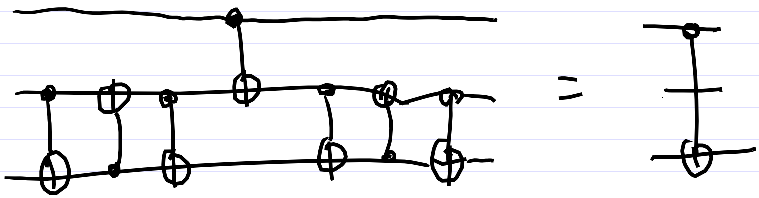
$$S = \begin{pmatrix} 1 & \\ & i \end{pmatrix} \qquad T = \begin{pmatrix} 1 & \\ & e^{i\pi/4} \end{pmatrix}$$

$$= e^{-i\pi/8 Z} \, e^{i\pi/8}$$

$$S_{ij} = C_{ij} C_{ji} C_{ij}$$

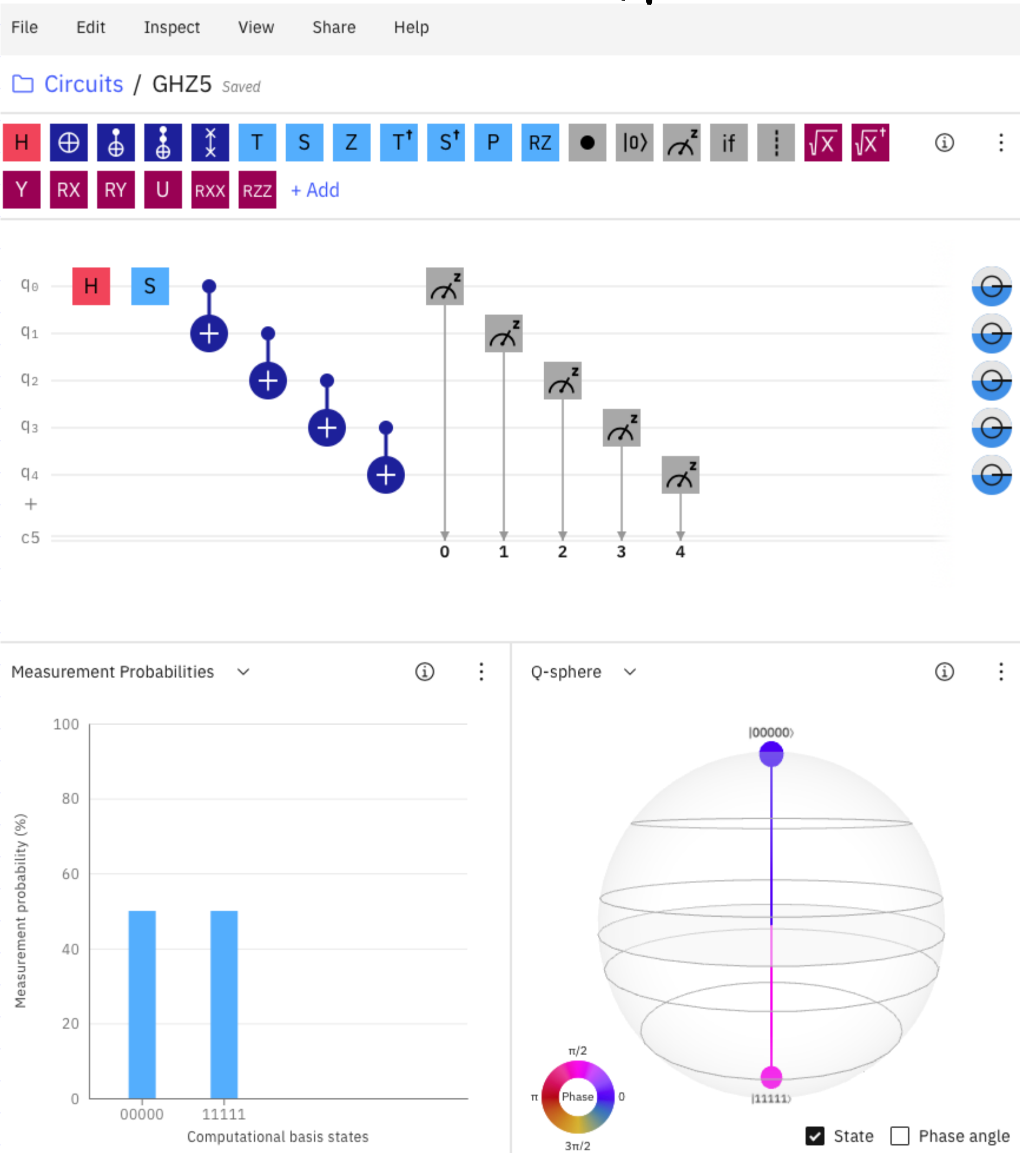


$00 \to 00$
$01 \leftrightarrow 10$
$11 \to 11$



Try $\frac{1}{\sqrt{2}}\left(|00000\rangle + i|11111\rangle\right)$:

IBM 'Quantum Experience'



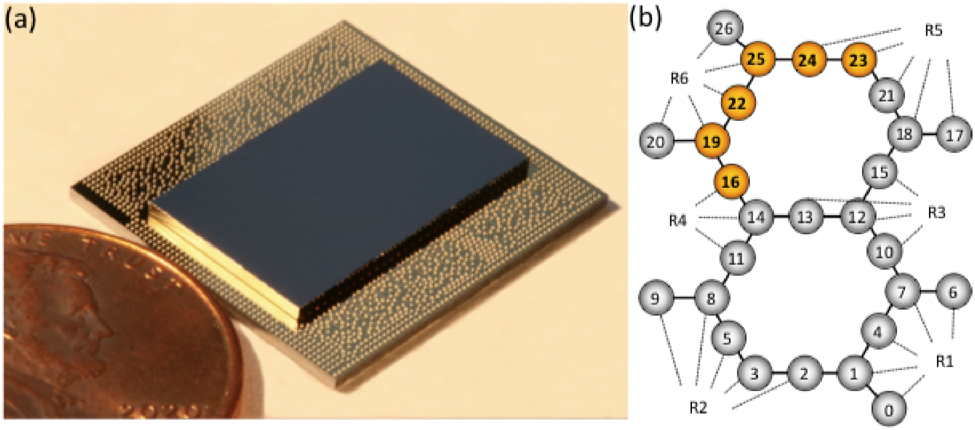


Fig. 1. (a) Image of a representative IBM Quantum Falcon processor with a penny for scale. The lattice connectivity is defined through couplings on a top qubit die which is bump-bonded to a bottom interposer die for signal delivery and readout. (b) Schematic of the 27-qubit (numbered 0 through 26) heavy-hex layout connectivity. Qubits used for the confirmed QV64 are shaded in orange. Dashed lines indicate collections of qubits that are multiplexed together for readout (labeled R1 to R6).

IBM 27-qubit chip
(not publicly available)

benchmarked in arXiv: 2008.08571

Result of running $\frac{1}{\sqrt{2}}(|00000\rangle + i|11111\rangle)$ from previous page in real class-time:

Untitled circuit job

Job Id: 5fc65d7185d6a1001a2c5a5c

Add tag +

Type	Provider	No. Circuits	Created
Composer	ibm-q/open/main	1	Dec 01, 2020 10:12 AM

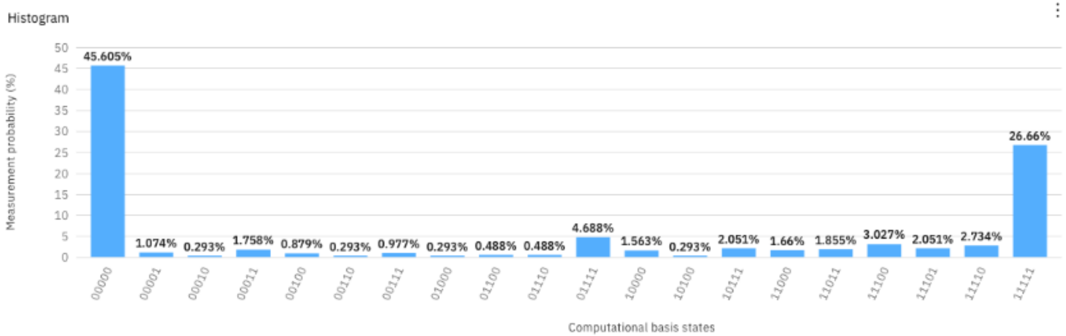
Download Delete

Created	Transpiling	Validating	In queue	Running	Completed
	711ms	802ms	35m 17.6s	7.7s	

Run details

Backend	Run mode	Shots	Status:	Time taken	Last Update
ibmq_valencia	fairshare	1024	COMPLETED	35m 31.4s	Dec 01, 2020 10:48 AM

Result



Also a qiskit python interface

```
In [1]: import numpy as np
        from qiskit import *
        %matplotlib inline
```

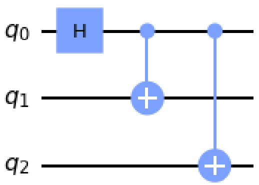
```
In [2]: circ = QuantumCircuit(3)
```

```
In [3]: #GHZ
        # Add a H gate on qubit 0, putting this qubit in superposition.
        circ.h(0)
        # Add a CX (CNOT) gate on control qubit 0 and target qubit 1, putting
        # the qubits in a Bell state.
        circ.cx(0, 1)
        # Add a CX (CNOT) gate on control qubit 0 and target qubit 2, putting
        # the qubits in a GHZ state.
        circ.cx(0, 2)
```

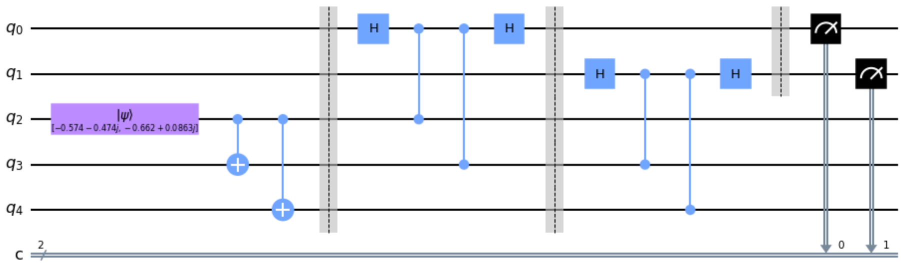
```
Out[3]: <qiskit.circuit.instructionset.InstructionSet at 0x1a2c162978>
```

```
In [4]: circ.draw('mpl')
```

Out[4]:



Bell pair



3-qubit bitflip error correction

$$|\tilde{x}\rangle = U_{FT}|x\rangle$$

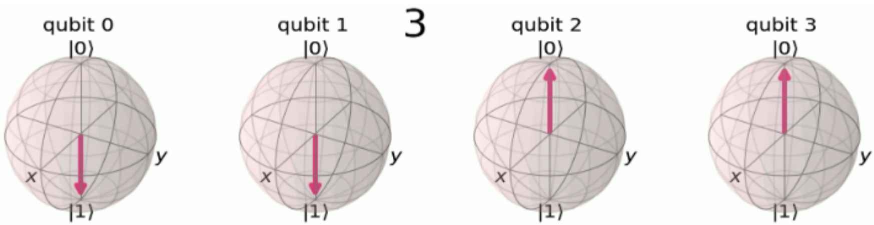
$$= \frac{1}{2^{n/2}} \sum_{y=0}^{2^n-1} e^{2\pi i x y / 2^n} |x\rangle$$

See visualizations:

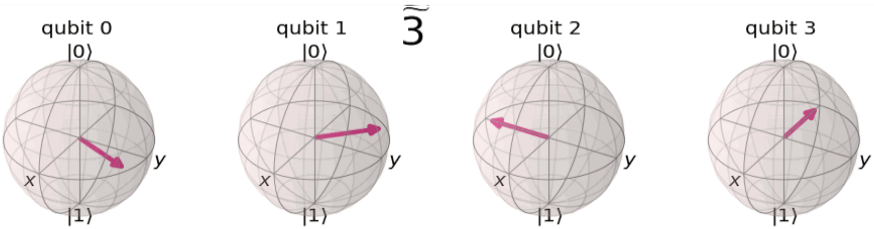
https://qiskit.org/textbook/ch-algorithms/quantum-fourier-transform.html#counting-fourier

2.1 Counting in the Fourier basis:

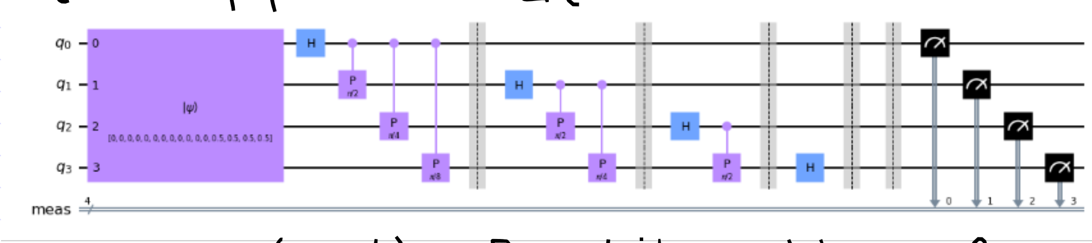
In the computational basis, we store numbers in binary using the states $|0\rangle$ and $|1\rangle$:



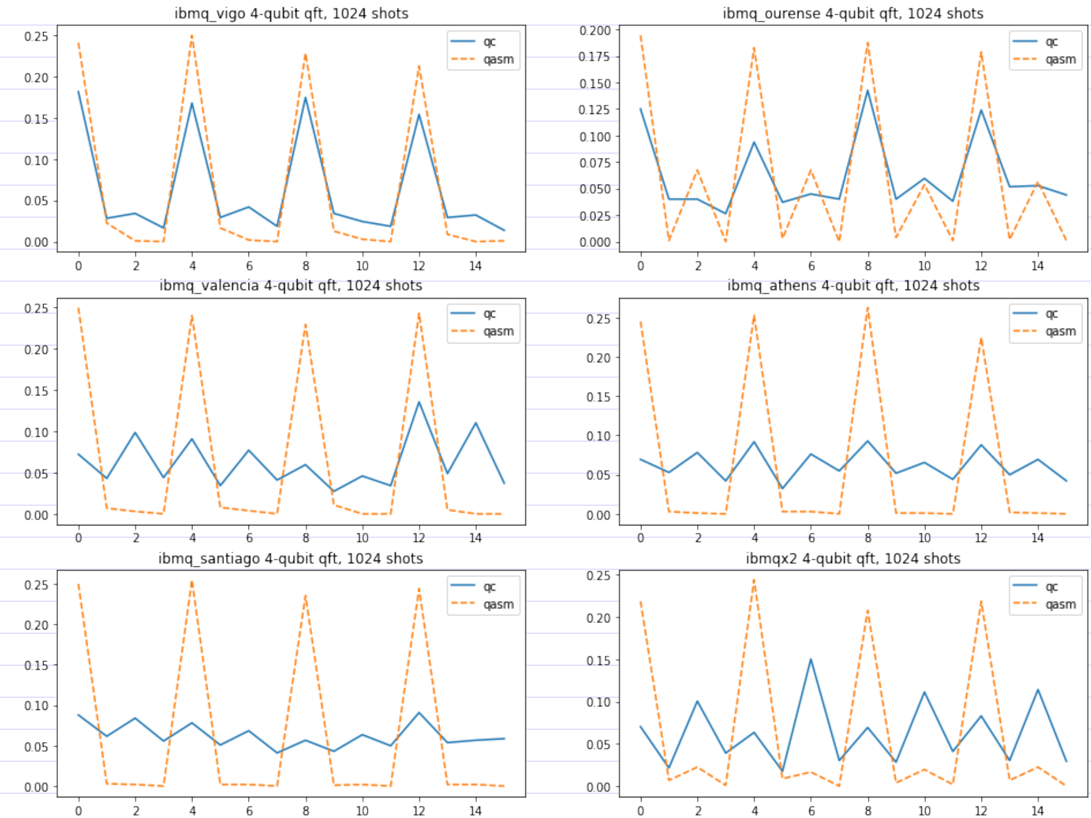
Note the frequency with which the different qubits change; the leftmost qubit flips with every increment in the number, the next with every 2 increments, the third with every 4 increments, and so on. In the Fourier basis, we store numbers using different rotations around the Z-axis:



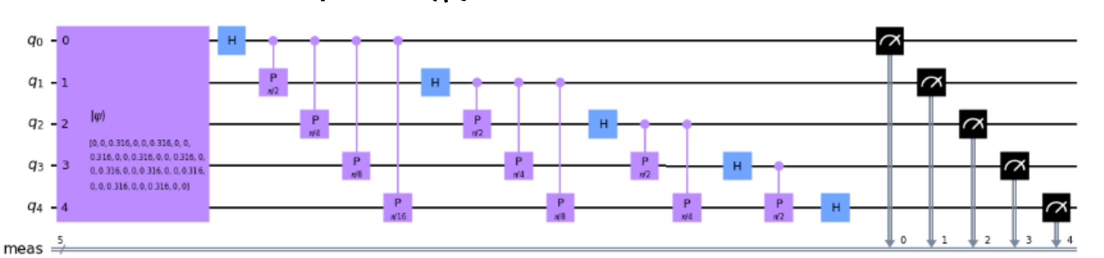
4-qubit U_{FT} on $|\psi\rangle = \frac{1}{2}(|3\rangle + |7\rangle + |11\rangle + |15\rangle)$



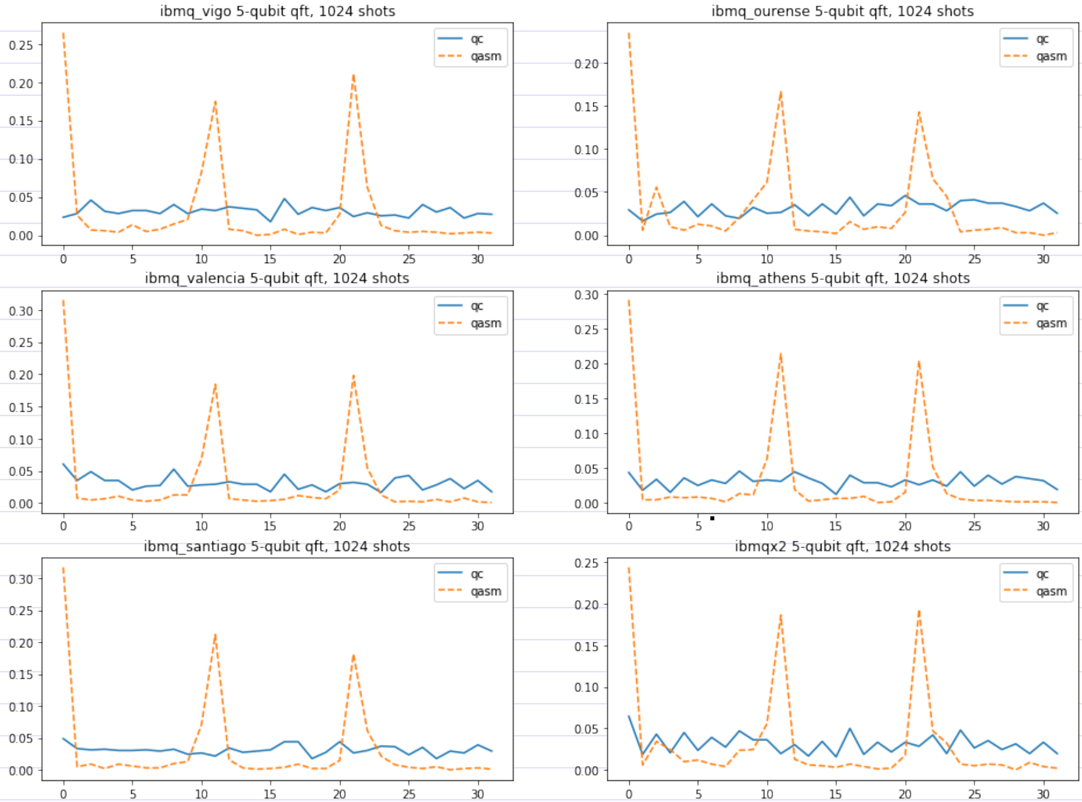
Run on 6 public 5-qubit machines, along with qasm-simulations using machine-specific noise models (too optimistic):



5-qubit U_{FT} on $|\psi\rangle = \frac{1}{\sqrt{10}}\left(|2\rangle + |5\rangle + \ldots |26\rangle + |29\rangle\right)$



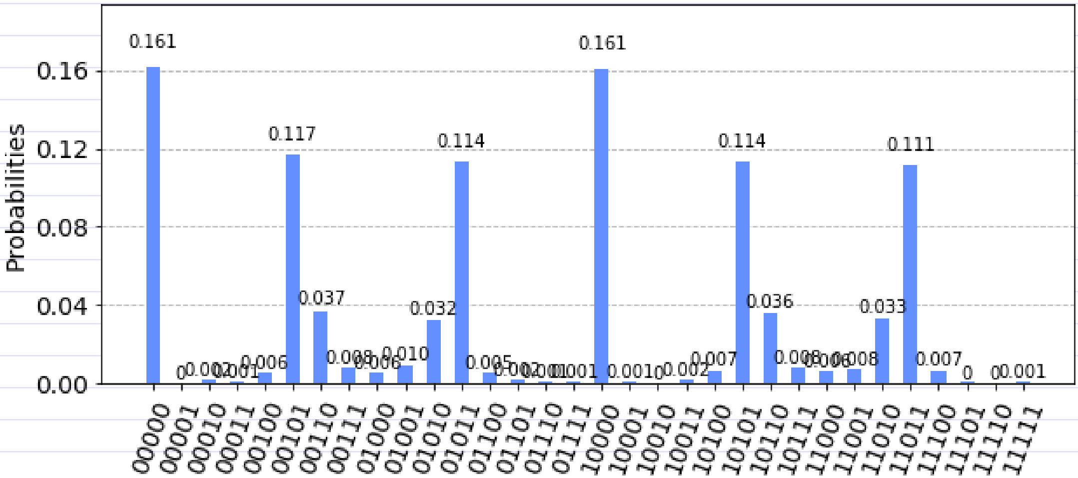
error model simulations too optimistic, just noise on all six machines. Too complicated $|\psi\rangle$?



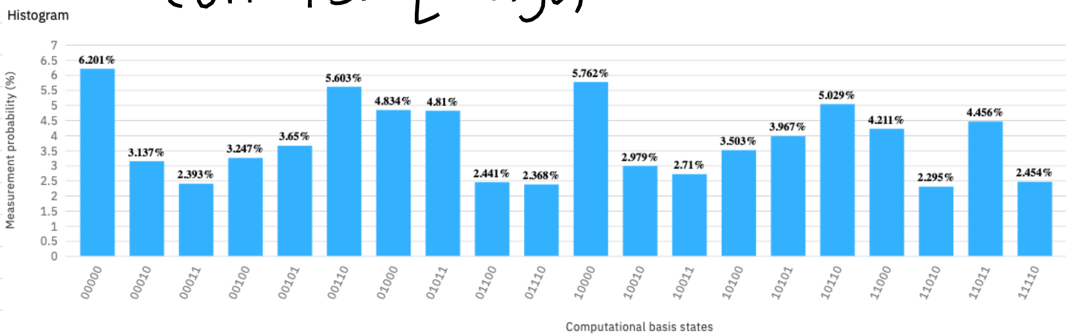
Try instead

$$|\psi\rangle = \frac{1}{\sqrt{5}}\left(|2\rangle + |8\rangle + |14\rangle + |20\rangle + |26\rangle\right)$$

zero noise simulation:



Now there's at least some signal (on ibmq-vigo)



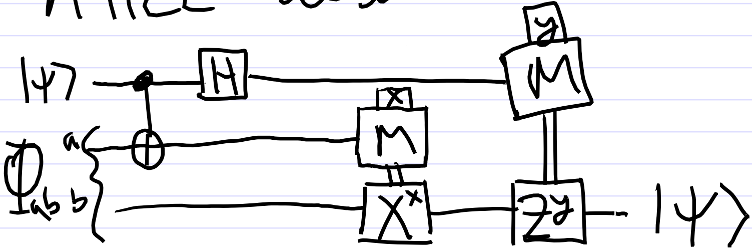
Quantum Teleportation

Sec 6.5

Alice, Bob share a Bell state

$$\Phi_{ab} = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

Alice also has some (unknown) $|\psi\rangle$



$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

(See next page)

If Alice measures:	To recover $|\psi\rangle$, Bob needs to apply:
$x, y = 0, 0$	$\mathbb{1}$
$0, 1$	X
$1, 0$	Z
$1, 1$	ZX

$$|\psi\rangle_a |\Phi_{ab}\rangle = (\alpha|0\rangle + \beta|1\rangle)\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

apply CNOT $\Rightarrow$

$$\alpha|0\rangle_a \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) + \beta|1\rangle_a \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$$

$\frac{(|0\rangle+|1\rangle)}{\sqrt{2}}$ apply H gives: $\frac{|0\rangle-|1\rangle}{\sqrt{2}}$

To correct Bob needs to apply

$$\frac{1}{2}|0\rangle_a|0\rangle_a(\alpha|0\rangle + \beta|1\rangle)_b \quad : \quad \mathbb{1}$$

$$+\frac{1}{2}|0\rangle|1\rangle(\alpha|1\rangle + \beta|0\rangle)_b \quad : \quad X$$

$$+\frac{1}{2}|1\rangle|0\rangle(\alpha|0\rangle - \beta|1\rangle)_b \quad : \quad Z$$

$$+\frac{1}{2}|1\rangle|1\rangle\underbrace{(\alpha|1\rangle - \beta|0\rangle)_b}_{|\psi\rangle_b} \quad : \quad ZX$$