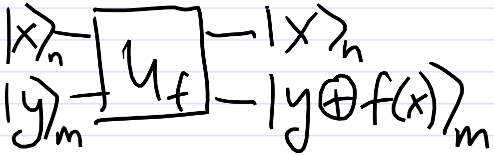
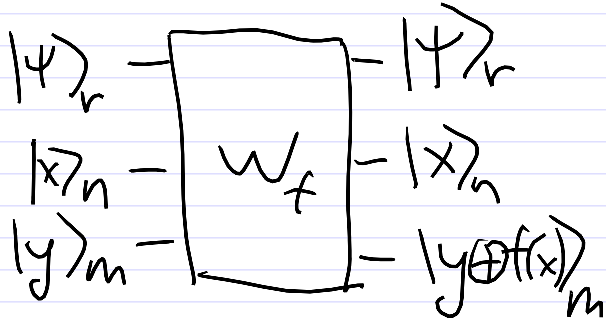
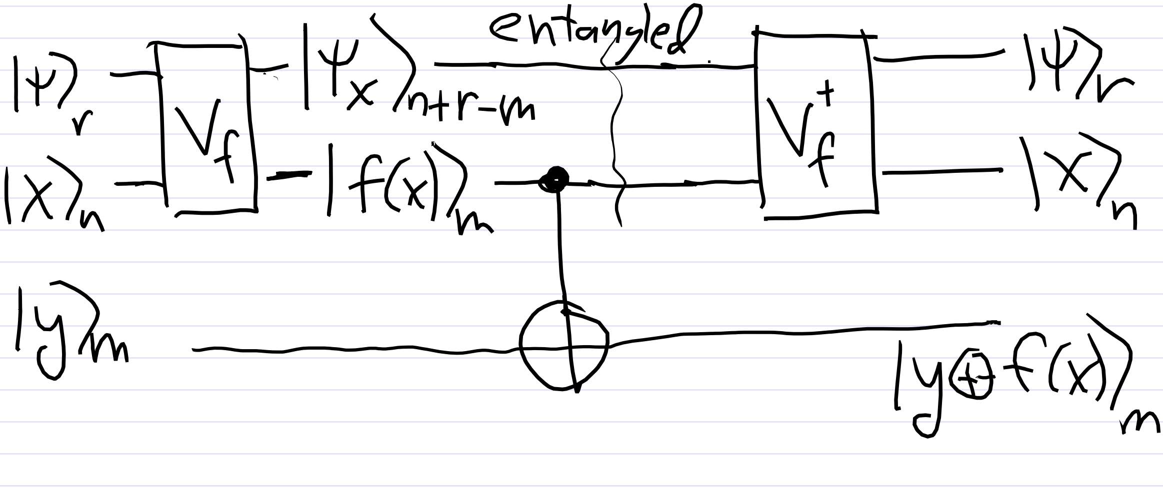
Lecture 10, 6 Oct 2020

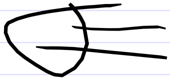
why extra qubits don't mess things up:

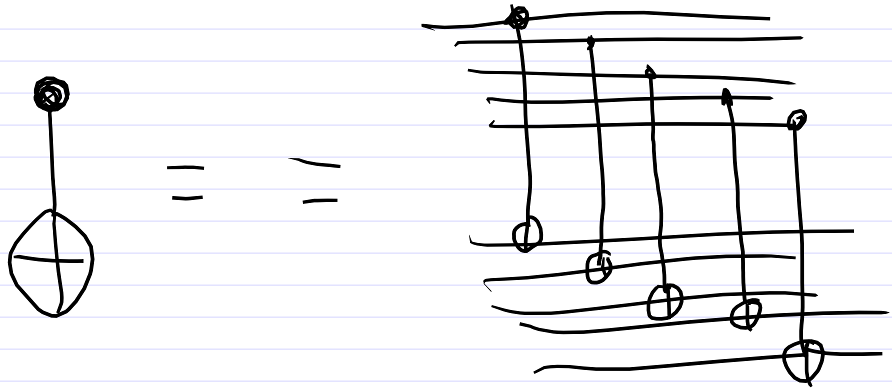






$$W_f = \begin{pmatrix} 1 & & & \\ & \ddots & & \\ & & 1 & \\ & & & \left[\, U_f \,\right] \end{pmatrix}$$





$$W_f H^{\otimes n}|0\rangle|y\rangle|\psi\rangle_r$$

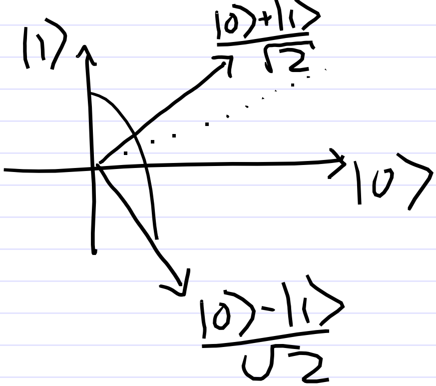
$$= \left(\frac{1}{2^{n/2}}\sum_{0\le x<2^n} |x\rangle|y\oplus f(x)\rangle\right)\otimes|\psi\rangle_r$$

$$= \left(\frac{1}{2^{n/2}}\sum_{x} U|x\rangle|y\rangle\right)\otimes|\psi\rangle_r$$

Reminder that every u has two actions.

1) act on 2-dim complex vectors

$$|\psi\rangle \to u|\psi\rangle \qquad H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$



reflection about axis at $\pi/8$ from horizontal

2) every u also acts on $\vec{a}$

$$u\,\vec{a}\cdot\vec{\sigma}\,u^\dagger = \vec{a}'\cdot\vec{\sigma}$$

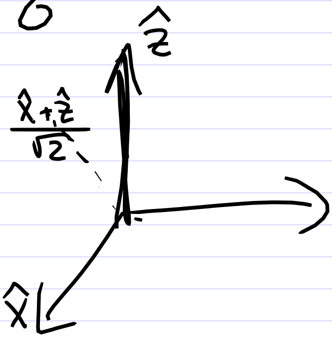
$$\vec{a}' = R_u\vec{a}$$

$HXH = Z$

$HZH = X$

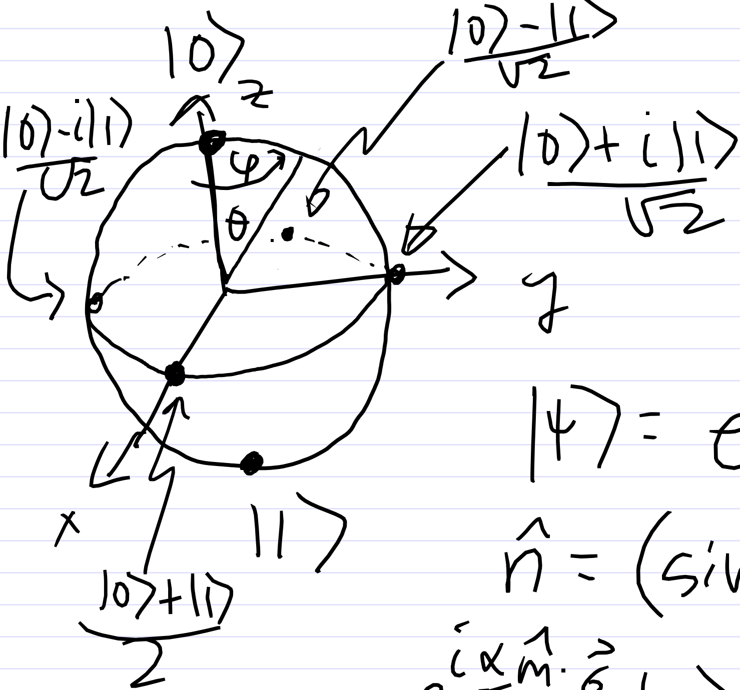
$\hat{x}\cdot\vec{\sigma} = X$

$\hat{z}\cdot\vec{\sigma} = Z$



$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

For 1-qubit state θ, φ act as polar coordinates on "Bloch sphere":



$$|\psi\rangle = e^{i\frac{\theta}{2}\hat{n}\cdot\vec{\sigma}}|0\rangle$$

$$\hat{n} = (\sin\varphi, -\cos\varphi, 0)$$

$e^{i\frac{\alpha}{2}\hat{m}\cdot\vec{\sigma}}|\psi\rangle$ rotates θ, φ coords by α about $\hat{m}$

$$\langle Z\rangle \equiv \langle\psi|Z|\psi\rangle \qquad Z=\begin{pmatrix}1 & \\ & -1\end{pmatrix}$$

$$= \langle 0|\cos^2\tfrac{\theta}{2}|0\rangle - \langle 1|\sin^2\tfrac{\theta}{2}|1\rangle$$

$$= \cos^2\tfrac{\theta}{2} - \sin^2\tfrac{\theta}{2} = \cos\theta$$

$$\langle X\rangle = \langle\psi|HZH|\psi\rangle$$

$$= \sin\theta\cos\varphi$$

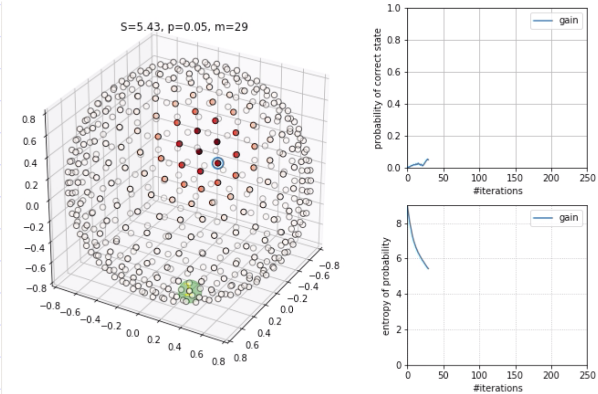
$$\langle Y\rangle = \sin\theta\sin\varphi$$

$$= \langle\psi|\tilde{H}Z\tilde{H}|\psi\rangle$$

$$\tilde{H} = \frac{1}{\sqrt{2}}(Y+Z) = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -i \\ i & 1\end{pmatrix}$$

$$\tilde{H}Y\tilde{H} = Z \qquad \tilde{H}Z\tilde{H} = Y$$

[See simulations linked from course web page]



Next time.

GHZ states $\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$

(exclude Einstein elements of reality)

start RSA and quantum factoring