Here was the attempted clarification of eq.(2.32) from text in class:

$$\begin{aligned}
\frac{1}{2^n}\sum_{0\le x<2^n}(-1)^{x\cdot z} &= \frac{1}{2^n}\sum_{x_{n-1}=0}^{1}\sum_{x_{n-1}=0}^{1}\cdots\sum_{x_0=0}^{1}(-1)^{x_{n-1}z_{n-1}+\ldots+x_0z_0}\\
&= \frac{1}{2}\sum_{x_{n-1}=0}^{1}(-1)^{x_{n-1}z_{n-1}}\cdots\frac{1}{2}\sum_{x_0=0}^{1}(-1)^{x_0z_0}\\
&= \frac{1}{2}\big(1+(-1)^{z_{n-1}}\big)\cdots\frac{1}{2}\big(1+(-1)^{z_0}\big)\\
&= \delta_{z_{n-1},0}\cdots\delta_{z_0,0}\\
&= \delta_{z,0}
\end{aligned}$$

Equivalently,

$$\begin{aligned}
\frac{1}{2^n}\sum_{0\le x<2^n}(-1)^{x\cdot z} &= \frac{1}{2^n}\sum_{x_{n-1}=0}^{1}\sum_{x_{n-1}=0}^{1}\cdots\sum_{x_0=0}^{1}(-1)^{x_{n-1}z_{n-1}+\ldots+x_0z_0}\\
&= \prod_{i=0}^{n-1}\frac{1}{2}\sum_{x_i=0}^{1}(-1)^{x_iz_i}\\
&= \prod_{i=0}^{n-1}\frac{1}{2}\big(1+(-1)^{z_i}\big)\\
&= \prod_{i=0}^{n-1}\delta_{z_i,0}\\
&= \delta_{z,0}
\end{aligned}$$

For $n=2$, that looks like

$$\begin{aligned}
\frac{1}{2^2}\sum_{0\le x<2^2}(-1)^{x_0z_0+x_1z_1} &= \frac{1}{4}\big((-1)^0+(-1)^{z_0}+(-1)^{z_1}+(-1)^{z_0+z_1}\big)\\
&= \frac{1}{4}\sum_{x_1=0}^{1}\sum_{x_0=0}^{1}(-1)^{x_1z_1+x_0z_0}\\
&= \frac{1}{2}\sum_{x_1=0}^{1}(-1)^{x_1z_1}\frac{1}{2}\sum_{x_0=0}^{1}(-1)^{x_0z_0}\\
&= \frac{1}{2}\big(1+(-1)^{z_1}\big)\frac{1}{2}\big(1+(-1)^{z_0}\big)\\
&= \delta_{z_1,0}\delta_{z_0,0}\\
&= \delta_{z,0}
\end{aligned}$$