

7. First find the heat that must be removed to (1) cool the water to 0°C , (2) freeze it, and then (3) cool the ice to -20°C .

$$Q_1 = mc_{\text{water}} \Delta T = 1.20 \text{ kg} \times 4.186 \text{ kJ}/(\text{kg}\cdot\text{K}) \times (-20.0^{\circ}\text{C}) = -100.46 \text{ kJ.}$$

$$Q_2 = -mL_f = 1.20 \text{ kg} \times 333.7 \text{ kJ}/\text{kg} = -400.44 \text{ kJ.}$$

$$Q_3 = mc_{\text{ice}} \Delta T = 1.20 \text{ kg} \times 2.10 \text{ kJ}/(\text{kg}\cdot\text{K}) \times (-20.0^{\circ}\text{C}) = -50.4 \text{ kJ.}$$

$Q_1 + Q_2 + Q_3 = -551 \text{ kJ}$. This is the total heat removed so $Q_c = 551.3 \text{ kJ}$. (Remember our convention that Q_c is positive.)

It's reversible so the total entropy change of the environment is zero:

$$\Sigma \Delta S = Q_H/T_H - Q_c/T_c = 0. \text{ Solving,}$$

$$Q_H = Q_c T_h/T_c .$$

Conservation of energy (1st law):

$$W = Q_H - Q_C = Q_c T_h/T_c - Q_c = Q_c(T_h/T_c - 1) = 551.3 \text{ kJ} \times [(293.15 \text{ K})/(253.15 \text{ K}) - 1] \\ = 87.1 \text{ kJ.}$$

Alternative solution: Use the efficiency. Because it is reversible,

$$e = W/Q_H = 1 - T_c/T_h$$

$$\text{Solving for } Q_H: Q_H = W/(1 - T_c/T_h)$$

$$\text{Substitute into the 1}^{\text{st}} \text{ law: } W = Q_H - Q_C = W/(1 - T_c/T_h) - Q_C$$

$$\text{Solve for } W: W[1 - 1/(1 - T_c/T_h)] = -Q_C$$

$$W = -Q_C/[1 - 1/(1 - T_c/T_h)]$$

This simplified algebraically to $W = Q_c(T_h/T_c - 1)$, as above.