Problem 9.1 (Electron velocity saturation at high fields in semiconductors)

\[ \frac{dE}{dt} = eFv - \frac{h\omega_p}{\tau_E} \]

(A) Rate of change of electron energy due to gain from electric field and loss to phonons.

\[ \frac{dp}{dt} = eF - \frac{p}{\tau_m} \]

(B) Rate of change of electron momentum due to electric force and momentum scattering.

\[ \text{steady state, } \frac{d(...)}{dt} = 0 \]  \[ \text{A gives } eFv = \frac{h\omega_p}{\tau_E} \]

\[ (p = mv) \]  \[ \text{B gives } eF = \frac{mv}{\tau_m} \]

\[ v^2 = \frac{h\omega_p}{m^*} \frac{1}{\tau_E} \Rightarrow v = \left( \frac{h\omega_p}{m^*} \frac{1}{\tau_E} \right)^{1/2} \]
Problem 9.2 (Electron Scattering and Mobility)

(a) Acoustic phonon scattering:

**Phonons in Semiconductors**

\[ u_s(x, t) = u_0 e^{i(\beta x - \omega t)} \]

\[ |u_s|^2 = 4u_0^2 \cos(\beta x - \omega t) \]

\[ KE = \frac{1}{2}M(\frac{du_s}{dt})^2 = 2M \omega^2 u_0^2 \sin^2(\beta x - \omega t) \]

\[ PE = \frac{1}{2}Ku_s^2 = 2Ku_0^2 \cos^2(\beta x - \omega t) \]

\[ \text{but...} \omega^2 = \frac{K}{M} \rightarrow\]

\[ KE + PE = 2M \omega^2 u_0^2 = N_\omega \cdot h\omega \rightarrow\]

since...\[ M = \rho V, \]

\[ u_0^2 = \frac{h}{2\omega pV} \cdot N_\omega \]

\[ N_\omega(T) = \frac{1}{e^{\frac{h\omega}{kT}} - 1} \]

The above is the way you can find the amplitude of phonon vibrations at a temperature \( T \) for feeding into the golden rule calculation of the scattering rates.
This is how the deformation potential scattering potential is found.
The left method is how the scattering rate is found from the scattering potential.
(b) Ionized impurity scattering rate:

\[
V(r) = -(Z |e|/4\pi \kappa_0 r) \exp(-r/L_D)
\]

\[
|k-k'| \approx 2k \sin(\theta/2)
\]

\[
\beta_{BH} = 2 \frac{m}{\hbar} \left( \frac{2}{3} k_B T \right)^{1/2} L_D
\]

\[
\beta_{BH} = 16^{1/2} \frac{T}{100 \text{ K}} \left( \frac{m}{m_0} \right)^{1/2} \left( \frac{2.08 \times 10^{18} \text{ cm}^{-3}}{n} \right)^{1/2}
\]

Screened coulomb scattering potential

\[
H_{k,k'} = \frac{Z e^2}{V \kappa_0} \int_0^\infty \exp(-r/L_D) \sin(|k-k'| r) dr
\]

\[
= \frac{Z e^2}{V \kappa_0} \frac{1}{|k-k'|^2 + L_D^2} \approx \frac{Z e^2}{V \kappa_0 4k^2 \sin^2(\theta/2) + (2kL_D)^2}.
\]

\[
\beta_{BH} = 2 \frac{m}{\hbar} \left( \frac{2}{3} k_B T \right)^{1/2} L_D
\]

The mobility \( \mu = (e/m) \langle \tau_m \rangle \) is given by

\[
\mu = \frac{2^{7/2} (4\pi \kappa_0)^2 (k_B T)^{3/2}}{\pi^{3/2} Z^2 e^3 m^{1/2} n_1 [\ln(1 + \beta_{BH}^2) - \beta_{BH}^2/(1 + \beta_{BH}^2)]}
\]

which in units of cm²/V s is

\[
\mu = 3.68 \times 10^{30} \text{ cm}^{-3} \left( \frac{\kappa}{16} \right)^2 \left( \frac{T}{100 \text{ K}} \right)^{1.5}
\]

\[
\left[ \frac{\mu}{N_1} \right]^{1/2} \left[ \ln(1 + \beta_{BH}^2) - 0.434 \beta_{BH}^2/(1 + \beta_{BH}^2) \right]
\]

and the log is to the base 10.
Problem 9.3 (Optical absorption in graphene)

a+b) Use the expression from the handouts except that integration is now over 2D k-space and an extra factor of two comes in because of the two pockets in the FBZ:

\[
R_\uparrow = \frac{2\pi}{\hbar} \left( \frac{eA_0}{2m} \right)^2 \left< \hat{P}_{cv} \cdot \hat{n} \right>^2 4 \times \int \frac{d^2 k}{(2\pi)^2} \delta(E_c(k) - E_v(k) - \hbar \omega)
\]

\[
= \frac{2\pi}{\hbar} \left( \frac{e}{2m} \right)^2 \left( \frac{2\eta_o l_{inc}}{\omega^2} \right) \left< \hat{P}_{cv} \cdot \hat{n} \right>^2 4 \times \int \frac{d^2 k}{(2\pi)^2} \delta(E_c(k) - E_v(k) - \hbar \omega)
\]

\[
= \frac{2\pi}{\hbar} \left( \frac{e}{2m} \right)^2 \left( \frac{2\eta_o l_{inc}}{\omega^2} \right) \frac{m^2 v^2}{2} 4 \times \int_0^{\infty} \frac{k \, dk}{(2\pi)} \delta(2\hbar v k - \hbar \omega)
\]

\[
= \frac{e^2}{4\hbar} \eta_o \left( \frac{l_{inc}}{\hbar \omega} \right)
\]

c) Incident photon flux per unit area is, \( I_{inc}/\hbar \omega \). The photon absorption rate per unit area is \( R_\uparrow \). Therefore, the fraction of incident photons absorbed in the graphene sheet is,

\[
\eta = \frac{\hbar \omega R_\uparrow}{I_{inc}} = \left( \frac{e^2}{4\hbar} \right) \eta_o = 0.23
\]

It follows that \( \approx 2.3\% \) of the incident photons are absorbed by graphene through interband transitions (irrespective of the wavelength!).
Problem 9.4 (Population inversion, optical gain, and lasing)

(a) The above shows the equilibrium absorption coefficient of a 3D semiconductor.

The above is the absorption spectrum of a 2D quantum well. Because of the quantization, and the constant 2D DOS, the joint optical DOS is in steps for every subband of the quantum well.
(b, c, d):

\[
\alpha(h\omega) = C_0 |\hat{e} \cdot \mathbf{p}_{ev}|^2 \times (g_sg_e) \times \int \frac{d^3k}{(2\pi)^3} \delta [E_g + \frac{\hbar^2 k^2}{2m_e^*} - h\omega] \times [f_v(k) - f_c(k)]
\]

\[
f_v(k_0) - f_c(k_0) = \frac{\exp \left(\frac{E_g - F_v}{kT}\right) - \exp \left(\frac{E_g - F_c}{kT}\right)}{1 + \exp \left(\frac{E_g - F_v}{kT}\right)\exp \left(\frac{E_g - F_c}{kT}\right)} < 0 \Rightarrow \exp \left(\frac{E_c - F_c}{kT}\right) < \exp \left(\frac{E_v - F_v}{kT}\right)
\]

\[
F_c - F_v > E_c - E_v = h\omega
\]

(e) One can achieve population inversion in a quantum well with a lower threshold current than in a bulk semiconductor because the DOS in a QW is lower, so filling them with less carriers raises their Fermi-levels more.
Problem 9.5 (Superconductivity)

The microscopic (BCS) theory of superconductivity turned out to essentially non-perturbative, because the Schrodinger equation solution of the Cooper pair problem needs an exact solution for the “ground state” of the system. The bound state energy of the Cooper pair problem is of the form

$$\Delta \sim \hbar \omega_D \exp\left[-\frac{1}{V_0 g(E_F)}\right],$$

where $\hbar \omega_D$ is the Debye energy of the crystal, $g(E_F)$ is the DOS at the Fermi energy, and $V_0$ is the weak attractive perturbation potential. This is of the functional form $f(x) = \exp[-\frac{1}{x}]$: this function does not have a Taylor series expansion. In that sense it is non-perturbative.