ECE 4070/MSE 5470: Physics of Semiconductor and Nanostructures

Spring 2015

Homework 9   **Due on May 15, 2015 at 5:00 PM**


Suggested Readings:
 a) Lecture notes

Problem 9.1 (Electron velocity saturation at high fields in semiconductors)

At low electric fields, the velocity of electrons in a semiconductor increases linearly with the field according to $v \approx \mu F$, where $\mu = e\tau_m / m^*$ is the mobility and F is the electric field, with τ_m being the momentum scattering time. But when the electric field is cranked up, the electron velocity saturates, because the electrons emit optical phonons each of energy $\hbar\omega_{op}$ every τ_E seconds, dumping the energy eFv they gain from the electric field every second. Setting up the equations for the conservation of momentum and energy, and solving for the steady state yields an estimate of this saturation velocity.

Show that the saturation velocity obtained by this scheme is $v_{sat} \approx \sqrt{\dfrac{\hbar\omega_{op}}{m^*}} \cdot \sqrt{\dfrac{\tau_m}{\tau_E}}$. Show that for a typical

semiconductor for which $\hbar\omega_{op} \approx 60$ meV, $m^* \sim 0.2 m_0$, and $\tau_m \sim \tau_E$, the electron saturation velocity is of the order of $\sim 10^7$ cm/s. This is a good rough number for the saturation velocity of most semiconductors.

Problem 9.2 (Electron Scattering and Mobility)
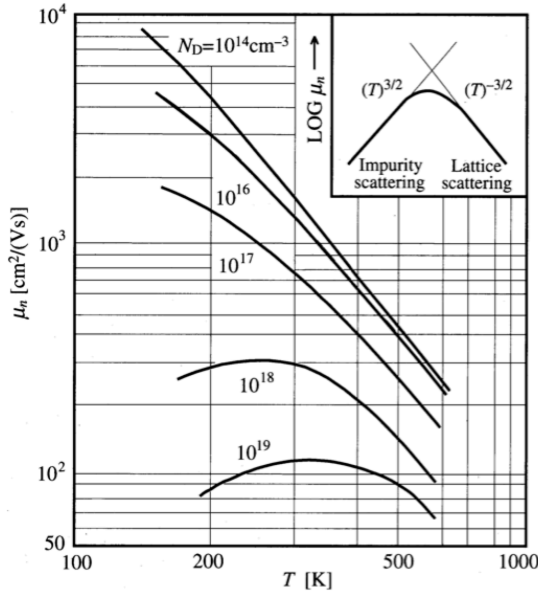
In this problem, we will explain the temperature dependence of the electron mobility in some (not all!) doped 3D semiconductors. The adjacent figure shows the experimental result: at low temperatures, the electron mobility goes as $\mu(T) \sim T^{\frac{3}{2}}$, and at high temperature it goes as $\mu(T) \sim T^{-\frac{3}{2}}$. We first connect the mobility to the scattering times via the Drude-like result $\mu = e\langle\tau\rangle / m^*$.

(a) Phonon scattering: We showed in class that the scattering rate of electrons due to acoustic phonons in semiconductors is given by Fermi's golden rule result for time-dependent oscillating perturbations

$$\frac{1}{\tau(\mathbf{k} \to \mathbf{k}')} = \frac{2\pi}{\hbar} |<\mathbf{k}'|W(\mathbf{r})|\mathbf{k}>|^2 \, \delta(E_{\mathbf{k}} - E_{\mathbf{k}'} \pm \hbar\omega_q),$$

where the acoustic phonon dispersion for low energy (or long wavelength) is $\omega_q \sim v_s q$, and the scattering potential is $W(\mathbf{r}) = D_c \nabla_r \cdot \mathbf{u}(r)$. Here D_c is the deformation potential (units: eV), and $\mathbf{u}(r) = \mathbf{n} u_0 \exp[i\mathbf{q}\cdot\mathbf{r}]$ is the spatial part of the phonon displacement

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wave, **n** is the unit vector in the direction of atomic vibration and **q** points in the direction of wave propagation. We also justified why the amplitude of vibration $2M\omega_q^2 u_0^2 \approx N_{ph} \times \hbar\omega$, where $N_{ph} = 1/[\exp(\hbar\omega_q / k_B T) - 1]$ is the Bose-number of phonons, and the mass of a unit cell of volume Ω is $M = \rho\Omega$, where ρ is the mass density (units: kg.m^{-3}). Show that a transverse acoustic (TA) phonon does *not* scatter electrons, but longitudinal acoustic (LA) phonons do. Now show using your result of HW 8, Problem 8.4 for the ensemble averaged $\langle\tau\rangle$ that the electron mobility in three dimensions due to LA phonon scattering is $\mu_{LA} = \dfrac{2\sqrt{2\pi}}{3} \dfrac{e\hbar^4 \rho v_s^2}{\left(m^*\right)^{\frac{5}{2}} D_c^2 (k_B T)^{\frac{3}{2}}} \sim T^{-\frac{3}{2}}$. This is a *very* useful result.

(b) Impurity scattering: Using Fermi's golden rule, calculate the scattering rate for electrons due to a *screened* Coulombic charged impurity potential $V(r) = -\dfrac{Ze^2}{4\pi\varepsilon_s r}\exp[-\dfrac{r}{L_D}]$, where Ze is the charge of the impurity, ε_s is the dielectric constant of the semiconductor, and $L_D = \sqrt{\dfrac{\varepsilon_s k_B T}{ne^2}}$ is the Debye screening length and n is the free carrier density. This is the scattering rate for just one impurity. Show using the result in HW8, Problem 8.5 with a $(1 - \cos\theta)$ angular factor for mobility that if the charged-impurity density is N_D, the mobility for 3D carriers is $\mu_I = \dfrac{2^{\frac{7}{2}}(4\pi\varepsilon_s)^2 (k_B T)^{\frac{3}{2}}}{\pi^{\frac{3}{2}} Z^2 e^3 \left(m^*\right)^{\frac{1}{2}} N_D F[\beta]} \sim \dfrac{T^{\frac{3}{2}}}{N_D}$. Here $\beta = 2\sqrt{\dfrac{2m^*(3k_B T)}{\hbar^2}} L_D$ is a dimensionless parameter, and $F[\beta] = \ln[1 + \beta^2] - \dfrac{\beta^2}{1 + \beta^2}$ is a weakly varying function. This famous result is named after Brooks and Herring who derived it first.

(c) Now combine your work from parts (a) and (b) of the problem to explain the experimental dependence of mobility vs temperature and as a function of impurity density as seen in the Figure above.

Problem 9.3 (Interband optical absorption in graphene)
Graphene has two complete carrier pockets (or six 1/3rd carrier pockets) in the FBZ, as shown below.
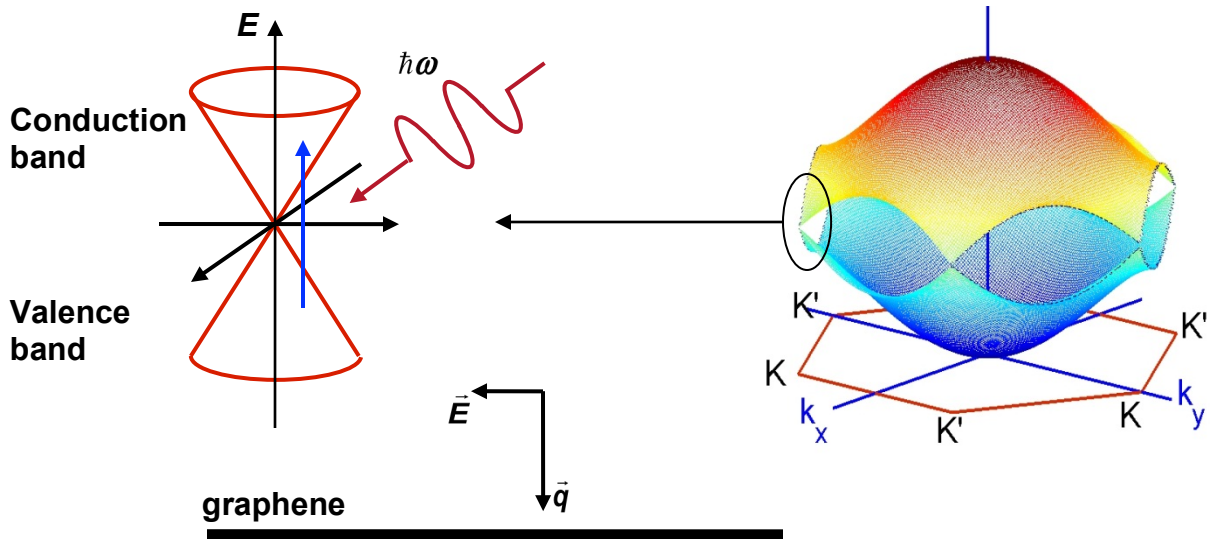
In each carrier pocket, the conduction and valence band dispersions are:

$$E_c(\vec{k}) = +\hbar \, v \, k = +\hbar \, v \sqrt{k_x^2 + k_y^2}$$

$$E_v(\vec{k}) = -\hbar \, v \, k = -\hbar \, v \sqrt{k_x^2 + k_y^2}$$

where the wavevector is measured, for simplicity, from the pocket center (as opposed to from the zone center) and the zero of energy is also chosen to coincide with E_p. Assume that the temperature is close to zero (i.e. $T \approx 0K$) and the valence band is full and the conduction band is empty. Light of frequency ω is incident normally on the graphene sheet, as shown below.
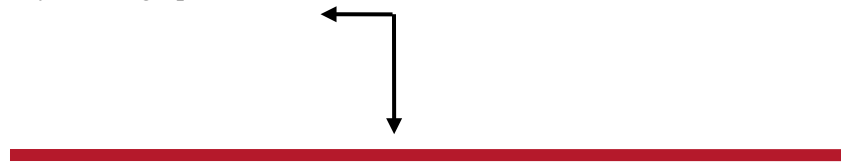
The average value of the momentum matrix element is:

$$\left\langle \left| \vec{P}_{vc} \cdot \hat{n} \right|^2 \right\rangle = \frac{m_o^2 \, v^2}{2}$$

Provided that the polarization unit vector of the incident field is in the plane of the graphene sheet. Assume that the intensity of the incident light is I_{inc}. Assume that for all practical purposes the photon momentum is small enough to be taken as zero (i.e. $\vec{q} = 0$).

a) Write an expression for the rate of stimulated absorption per unit area $R_\uparrow$ (units: $1/m^2$-sec) in graphene in terms of the incident light Intensity I_{inc}. Make sure you include contributions from both spins and both carrier pockets. Write your answer as an integral over k-space.

b) Evaluate your integral in part (a) and show that $R_\uparrow$ can be written as:

$$R_\uparrow = \text{constant} \; \eta_o \left(\frac{I_{inc}}{\hbar \omega} \right) \qquad \left\{ \eta_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} \right.$$

Find the value of the "constant" in the expression above and also specify the units of this "constant".

c) As the light crosses the graphene sheet, some photons are lost because of absorption in the graphene sheet. From your knowledge of $R_\uparrow$ and the incident light Inensity I_{inc} find out what fraction of the incident photon flux is absorbed in the graphene sheet. You will find, to your amazement perhaps, that this fraction is *independent* of the light frequency as well as of any material parameter value and depends only on few fundamental constants of physics. **Find a numerical value for this fraction.**

Problem 9.4 (Population inversion, optical gain, and lasing)

In class, we derived that the equilibrium optical absorption coefficient of a semiconductor is

$$\alpha_0(\hbar\omega) = \left(\frac{e}{m_0}\right)^2 \frac{2\pi \left\langle |\mathbf{P}_{cv} \cdot \hat{n}|^2 \right\rangle}{\varepsilon_0 n\omega c} \cdot g_J(\hbar\omega - E_g),$$ where $g_J(\hbar\omega - E_g)$ is the joint electron-photon DOS, and all

symbols have their usual meanings. We also found that under non-equilibrium conditions, the optical absorption coefficient becomes $\alpha(\hbar\omega) = \alpha_0(\hbar\omega)[f_v(k) - f_c(k)]$, where the electron occupation functions

of the bands are given by the Fermi-Dirac distribution $f_v(k) = \dfrac{1}{1 + \exp[\dfrac{E_v(k) - F_v}{k_B T}]}$, but with a quasi-Fermi

level F_v for the corresponding band as the mathematical means to capture non-equilibrium. Consider semiconductors and heterostructures with parabolic bandstrcutures for the conduction and valence bands
$$E_v(k) = -\frac{\hbar^2 k^2}{2m_v^*}, \quad E_c(k) = E_g + \frac{\hbar^2 k^2}{2m_c^*} \text{ for this problem.}$$

(a) Make a sketch of the equilibrium absorption coefficients $\alpha_0(\hbar\omega)$ for a bulk 3D semiconductor, and a 2D quantum well vs the photon energy $\hbar\omega$.

(b) Plot the Fermi difference function $f_v(k) - f_c(k)$ as a function of the photon energy $\hbar\omega$ for a few choices of the quasi-Fermi levels F_v, F_c of the valence and conduction bands. Specifically, track the photon energy at which the difference function changes sign from –ve to +ve.

(c) Now combine (a) and (b) to plot the non-equilibrium absorption coefficient $\alpha(\hbar\omega)$ for the choices of F_v, F_c from part (b). Discuss the significance of a *negative* absorption coefficient.

(d) Show that the requirement for population inversion is $F_c - F_c > E_c - E_v = \hbar\omega$. This is the famous Bernard-Duraffourg condition for population inversion in semiconductor lasers.

(e) Based on your results above, explain why semiconductor heterostructure quantum wells have lower injection thresholds for lasing than 3D bulk semiconductors. This idea and demonstration won Herb Kroemer and Zhores Alferov the 2000 Nobel prize in physics.

Problem 9.5 (Superconductivity)

Briefly explain the basic idea behind the BCS theory of superconductivity, and why it was not possible to obtain its microscopic theory from perturbative methods of quantum mechanics.