ECE 407 Homework #2 Solutions (Farhan Rana)

2.1.

From lecture notes:

$$u = \frac{3}{5} n E_F. \qquad E_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}$$

$$\Rightarrow U = Vu = \frac{3}{5} N E_F = \frac{3}{5} N \frac{\hbar^2}{2m}\left(3\pi^2 \frac{N}{V}\right)^{2/3}$$

$$\Rightarrow P = -\left.\frac{\partial U}{\partial V}\right|_{S,N} = \frac{2}{3}\frac{U}{V} = \frac{2}{3} u$$

In a classical gas, the average K.E. of each particle is $\frac{3}{2}kT$ and goes to zero as $T \to 0$. For a Fermi gas, Pauli exclusion principle dictates that a single quantum state is filled by one electron only. Consequently, electrons have kinetic energies from zero all the way up to E_F even at $T=0$ and this results in a non-zero pressure at $T=0$.

2.2.

$$2 \times \int \frac{d^3\vec{k}}{(2\pi)^3} \longrightarrow \sqrt{\frac{m_x m_y m_z}{m^3}}\; 2 \times \int \frac{d^3\vec{q}}{(2\pi)^3}$$

Now since $E(\vec{q}) = \frac{\hbar^2 q^2}{2m} = \frac{\hbar^2}{2m}(q_x^2 + q_y^2 + q_z^2)$

$$\Rightarrow \sqrt{\frac{m_x m_y m_z}{m^3}}\; 2 \times \int \frac{d^3\vec{q}}{(2\pi)^3} \longrightarrow \sqrt{\frac{m_x m_y m_z}{m^3}} \int_0^\infty \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \sqrt{E}\; dE$$

$$\Rightarrow g(E) = \frac{1}{2\pi^2}\left(\frac{2m_d}{\hbar^2}\right)^{3/2}\sqrt{E} \qquad \text{where} \qquad m_d = (m_x m_y m_z)^{1/3}$$

= is called the density of states effective mass.

2.3.

$$2 \times \int \frac{d^2\vec{k}}{(2\pi)^2} \longrightarrow \sqrt{\frac{m_x m_y}{m^2}}\; 2 \times \int \frac{d^2\vec{q}}{(2\pi)^3} \qquad \text{and} \qquad E(\vec{q}) = \frac{\hbar^2 q^2}{2m}$$

$$\Rightarrow \sqrt{\frac{m_x m_y}{m^2}}\; 2 \times \int \frac{d^2\vec{q}}{(2\pi)^2} \longrightarrow \sqrt{\frac{m_x m_y}{m^2}} \int_0^E \frac{m}{\pi \hbar^2}\, dE$$

$$\Rightarrow g_{2D}(E) = \frac{m_d}{\pi \hbar^2}$$

where $m_d = \sqrt{m_x m_y}$

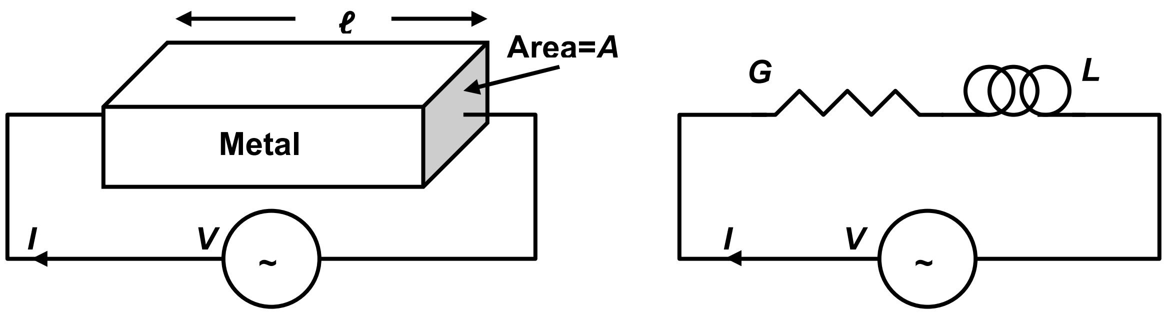
= density of states effective mass.

2.4 (Drude model)

The frequency dependent conductivity of an electron gas (in 3D) in a metal is given by the Drude model,

$$\vec{J}(\omega) = \sigma(\omega)\,\vec{E}(\omega)$$

$$\sigma(\omega) = \frac{ne^2\tau/m}{1 - i\,\omega\tau}$$



For the equivalent circuit model,

$$I(\omega) = \frac{V(\omega)}{1/G - i\omega L}$$

For the electron gas,

$$J(\omega) = \sigma(\omega)\,E(\omega)$$

$$\Rightarrow I(\omega) = AJ(\omega) = A\sigma(\omega)\,E(\omega) = \frac{A\sigma(\omega)}{\ell}\,\vec{E}(\omega)\ell = \frac{A\sigma(\omega)}{\ell}V(\omega)$$

$$\Rightarrow \frac{\ell}{A\sigma(\omega)} = \frac{1}{G} - i\omega L \quad \Rightarrow \frac{\ell}{A}\frac{1 - i\omega\tau}{ne^2\tau/m} = \frac{1}{G} - i\omega L$$

$$\Rightarrow G = \frac{A}{\ell}\frac{ne^2\tau}{m} \quad L = \frac{\ell}{A}\frac{m}{ne^2}$$