ECE 4070/MSE 5470: Physics of Semiconductor and Nanostructures

Spring 2015

Homework 2 **Due on Feb. 09, 2015 at 5:00 PM**

Suggested Readings:

a) Lecture notes
b) Chapter 1 and Chapter 2 in Kittel (Introduction to Solid State Physics)

Problem 2.1 (Pressure of a free electron gas in 3D)

For a classical gas the pressure, given by: $P = nKT$, goes to zero when the temperature goes to zero. For a Fermi gas of free electrons this is not the case. The expression for the pressure is:

$$P = -\left.\frac{\partial U}{\partial V}\right|_{S,N}$$

where the derivative is taken keeping the TOTAL electron number and TOTAL entropy constant. One can find the pressure of a Fermi gas relatively easily at zero temperature since the entropy will remain constant and one need only worry about keeping the total electron number constant. Find the pressure of a free electron gas (in 3D). Explain physically why the pressure of an electron gas is not zero at zero temperature (as is the case in a classical gas).

Problem 2.2 (Free electron gas with an anisotropic dispersion in 3D)

This problem is an exercise in calculating density of states functions that will be very useful later in the course. Suppose one has a free electron gas in 3D where the electron energy-vs-wavevector relation is given by:

$$E(\vec{k}) = \frac{\hbar^2 k_x^2}{2m_x} + \frac{\hbar^2 k_y^2}{2m_y} + \frac{\hbar^2 k_z^2}{2m_z}$$

Note that the electron has a different "mass" associated with its kinetic energy when moving in different directions. This happens in materials as a result of the interaction of the electrons with the atoms, as you will see later in the course. Find the density of states function $g(E)$ so that an integral over k-space can be converted into an integral over energy as follows:

$$2 \times \int \frac{d^3\vec{k}}{(2\pi)^3} \quad \rightarrow \quad \int_0^\infty dE \;\; g(E)$$

Hint: You can start by first defining a new wavevector $\vec{q}$ as follows:

$$q_x = \sqrt{\frac{m}{m_x}} k_x \qquad q_y = \sqrt{\frac{m}{m_y}} k_y \qquad q_z = \sqrt{\frac{m}{m_z}} k_z$$

Write the energy dispersion in terms of the wavevector components of $\vec{q}$. Convert the k-space integral into q-space integral and then convert the q-space integral into an energy integral.

Problem 2.3 (Free electron gas with an anisotropic dispersion in 2D)

Now suppose one has a free electron gas in 2D where the electron energy-vs-wavevector relation is given by:

$$E(\vec{k}) = \frac{\hbar^2 k_x^2}{2m_x} + \frac{\hbar^2 k_y^2}{2m_y}$$

Find the density of states function $g_{2D}(E)$ so that an integral over k-space can be converted into an integral over energy as follows:

$$2 \times \int \frac{d^2\vec{k}}{(2\pi)^2} \quad \rightarrow \quad \int_0^\infty dE \;\; g_{2D}(E)$$

Problem 2.4 (Drude model)

The frequency dependent conductivity of an electron gas (in 3D) in a metal is to a very good approximation given by the Drude model,

$$\vec{J}(\omega) = \sigma(\omega)\,\vec{E}(\omega)$$

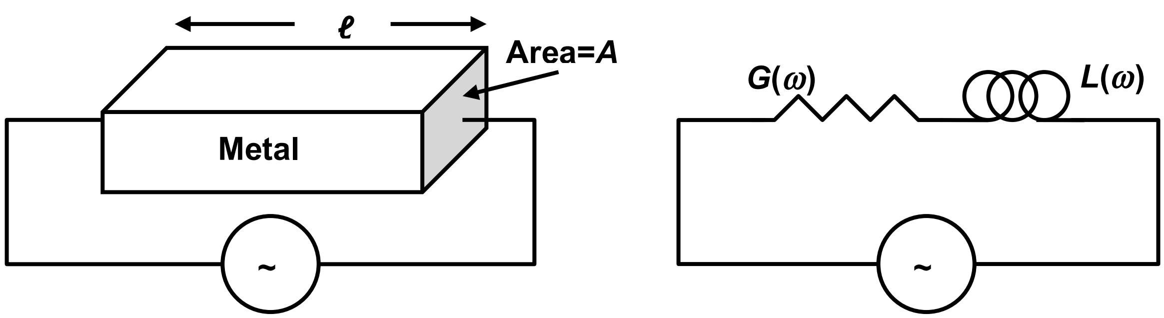
$$\sigma(\omega) = \frac{ne^2\tau/m}{1 - i\,\omega\tau}$$

The conductivity has both real and an imaginary part,

$$\text{Real}\{\sigma(\omega)\} = \frac{ne^2\tau/m}{1 + (\omega\tau)^2}$$

$$\text{Imag}\{\sigma(\omega)\} = \frac{ne^2\tau/m\;(\omega\tau)}{1 + (\omega\tau)^2}$$

Consider a slab of a metal connected in a circuit shown below. At a high frequency ω, the equivalent circuit model consists of a resistor and an inductor, as also shown below. Find the values of the conductance G (units: Siemens) and the inductance L (units: Henry) in the circuit model in terms of the given parameters.



Note added: This problem shows that the response of an electron gas at high frequencies ($\omega\tau \geq 1$) has a significant inductive component.