Handout 3

Free Electron Gas in 2D and 1D

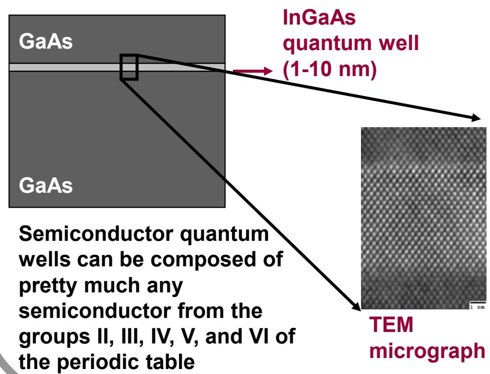
In this lecture you will learn:

- Free electron gas in two dimensions and in one dimension
- Density of States in k-space and in energy in lower dimensions

Electron Gases in 2D

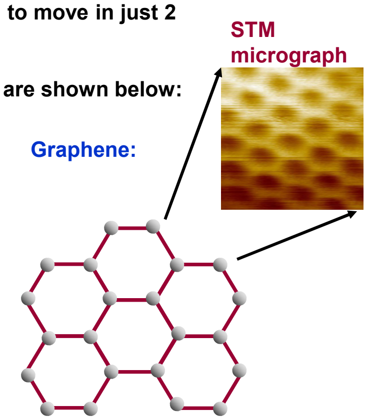
- In several physical systems electron are confined to move in just 2 dimensions
- Examples, discussed in detail later in the course, are shown below:

Semiconductor Quantum Wells:



Semiconductor quantum wells can be composed of pretty much any semiconductor from the groups II, III, IV, V, and VI of the periodic table

Graphene:



Graphene is a single atomic layer of carbon atoms arranged in a honeycomb lattice

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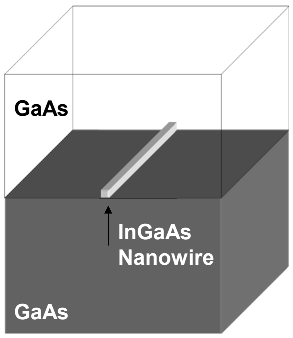
Electron Gases in 1D

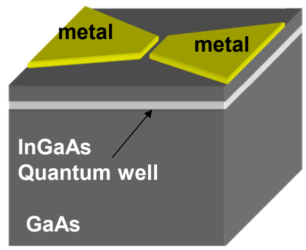
• In several physical systems electron are confined to move in just 1 dimension

• Examples, discussed in detail later in the course, are shown below:

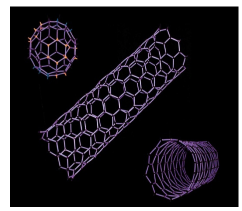
Semiconductor Quantum Wires (or Nanowires):



Semiconductor Quantum Point Contacts (Electrostatic Gating):



Carbon Nanotubes (Rolled Graphene Sheets):

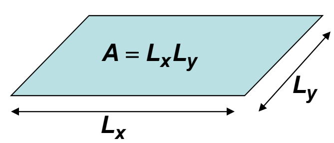


Electrons in 2D Metals: The Free Electron Model

The quantum state of an electron is described by the time-independent Schrodinger equation:

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r})+V(\vec{r})\psi(\vec{r})=E\,\psi(\vec{r})$$

Consider a large metal sheet of area $A= L_x\, L_y$:



Use the Sommerfeld model:

• The electrons inside the sheet are confined in a two-dimensional infinite potential well with zero potential inside the sheet and infinite potential outside the sheet

$$V(\vec{r})=0 \quad \text{for } \vec{r} \text{ inside the sheet}$$
$$V(\vec{r})=\infty \quad \text{for } \vec{r} \text{ outside the sheet}$$

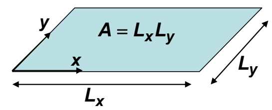
free electrons (experience no potential when inside the sheet)

• The electron states inside the sheet are given by the Schrodinger equation

Born Von Karman Periodic Boundary Conditions in 2D

Solve: $-\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r}) = E\,\psi(\vec{r})$

Use periodic boundary conditions:



$$\left.\begin{aligned}\psi(x+L_x,y,z)&=\psi(x,y,z)\\ \psi(x,y+L_y,z)&=\psi(x,y,z)\end{aligned}\right\}$$

These imply that each edge of the sheet is folded and joined to the opposite edge

Solution is: $\psi(\vec{r}) = \sqrt{\frac{1}{A}}\; e^{i\,\vec{k}\,.\,\vec{r}} = \sqrt{\frac{1}{A}}\; e^{i\left(k_x x + k_y y\right)}$

The boundary conditions dictate that the allowed values of k_x, and k_y are such that:

$$e^{i\left(k_x L_x\right)} = 1 \quad \Rightarrow \quad k_x = n\frac{2\pi}{L_x} \qquad n = 0, \pm1, \pm2, \pm3, \ldots\ldots.$$

$$e^{i\left(k_y L_y\right)} = 1 \quad \Rightarrow \quad k_y = m\frac{2\pi}{L_y} \qquad m = 0, \pm1, \pm2, \pm3, \ldots\ldots.$$

Born Von Karman Periodic Boundary Conditions in 2D

Labeling Scheme:

All electron states and energies can be labeled by the corresponding k-vector

$$\psi_{\vec{k}}(\vec{r}) = \sqrt{\frac{1}{A}}\; e^{i\,\vec{k}\,.\,\vec{r}} \qquad E(\vec{k}) = \frac{\hbar^2 k^2}{2\,m}$$

Normalization: The wavefunction is properly normalized: $\int d^2\vec{r}\; \left|\psi_{\vec{k}}(\vec{r})\right|^2 = 1$

Orthogonality: Wavefunctions of two different states are orthogonal:

$$\int d^2\vec{r}\; \psi^*_{\vec{k}'}(\vec{r})\,\psi_{\vec{k}}(\vec{r}) = \int d^2\vec{r}\; \frac{e^{i\left(\vec{k}-\vec{k}'\right).\,\vec{r}}}{A} = \delta_{\vec{k}',\,\vec{k}}$$

Momentum Eigenstates:

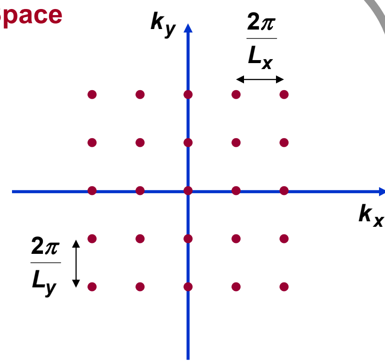
Another advantage of using the plane-wave energy eigenstates (as opposed to the "sine" energy eigenstates) is that the plane-wave states are also momentum eigenstates

Momentum operator: $\hat{p} = \frac{\hbar}{i}\nabla \quad \Rightarrow \quad \hat{p}\,\psi_{\vec{k}}(\vec{r}) = \frac{\hbar}{i}\nabla\psi_{\vec{k}}(\vec{r}) = \hbar\vec{k}\;\psi_{\vec{k}}(\vec{r})$

Velocity:

Velocity of eigenstates is: $\vec{v}(\vec{k}) = \frac{\hbar\vec{k}}{m} = \frac{1}{\hbar}\nabla_{\vec{k}}\,E(\vec{k})$

States in 2D k-Space



k-space Visualization:

The allowed quantum states states can be visualized as a 2D grid of points in the entire "k-space"

$$k_x = n\frac{2\pi}{L_x} \qquad k_y = m\frac{2\pi}{L_y}$$

$n, m = 0, \pm 1, \pm 2, \pm 3, \ldots\ldots$

Density of Grid Points in k-space:

Looking at the figure, in k-space there is only one grid point in every small area of size:

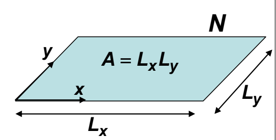
$$\left(\frac{2\pi}{L_x}\right)\left(\frac{2\pi}{L_y}\right) = \frac{(2\pi)^2}{A}$$

$\Rightarrow$ There are $\frac{A}{(2\pi)^2}$ grid points per unit area of k-space } **Very important result**

The Electron Gas in 2D at Zero Temperature - I

• Suppose we have N electrons in the sheet.

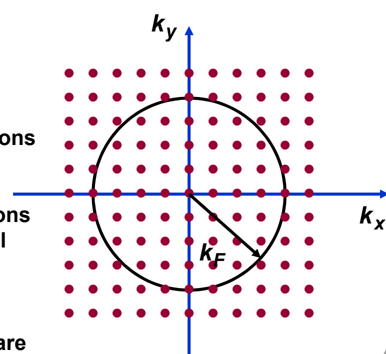
• **Then how do we start filling the allowed quantum states?**



• Suppose T~0K and we are interested in a filling scheme that gives the lowest total energy.

The energy of a quantum state is:

$$E(\vec{k}) = \frac{\hbar^2\left(k_x^2 + k_y^2\right)}{2m} = \frac{\hbar^2 k^2}{2m}$$



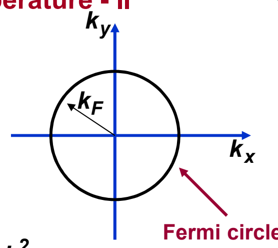
Strategy:

• Each grid-point can be occupied by two electrons (spin up and spin down)

• Start filling up the grid-points (with two electrons each) in circular regions of increasing radii until you have a total of N electrons

• When we are done, all filled (i.e. occupied) quantum states correspond to grid-points that are inside a circular region of radius k_F

The Electron Gas in 2D at Zero Temperature - II



- Each grid-point can be occupied by two electrons (spin up and spin down)
- All filled quantum states correspond to grid-points that are inside a circular region of radius k_F

Area of the circular region = πk_F^2

Number of grid-points in the circular region = $\frac{A}{(2\pi)^2} \times \pi k_F^2$

Number of quantum states (including spin) in the circular region = $2 \times \frac{A}{(2\pi)^2} \times \pi k_F^2 = \frac{A}{2\pi} k_F^2$

But the above must equal the total number *N* of electrons inside the box:

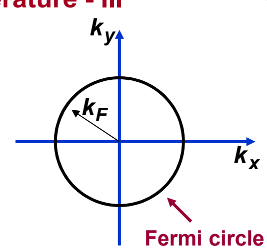
$$N = \frac{A}{2\pi} k_F^2$$

$$\Rightarrow \quad n = \text{electron density} = \frac{N}{A} = \frac{k_F^2}{2\pi}$$

$$\Rightarrow \quad k_F = (2\pi n)^{\frac{1}{2}}$$

Units of the electron density *n* are #/cm^2

The Electron Gas in 2D at Zero Temperature - III



- All quantum states inside the Fermi circle are filled (i.e. occupied by electrons)
- All quantum states outside the Fermi circle are empty

Fermi Momentum:

The largest momentum of the electrons is: $\hbar k_F$

This is called the Fermi momentum

Fermi momentum can be found if one knows the electron density:

$$k_F = (2\pi n)^{\frac{1}{2}}$$

Fermi Energy:

The largest energy of the electrons is: $\frac{\hbar^2 k_F^2}{2m}$

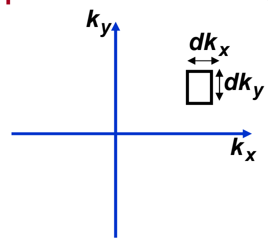
This is called the Fermi energy E_F: $E_F = \frac{\hbar^2 k_F^2}{2m}$

Also: $E_F = \frac{\hbar^2 \pi n}{m}$ or $n = \frac{m}{\pi \hbar^2} E_F$

Fermi Velocity:

The largest velocity of the electrons is called the Fermi velocity v_F: $v_F = \frac{\hbar k_F}{m}$

The Electron Gas in 2D at Non-Zero Temperature - I



Recall that there are $\frac{A}{(2\pi)^2}$ grid points per unit area of k-space

$\Rightarrow$ So in area $dk_x\, dk_y$ of k-space the number of grid points is:

$$\frac{A}{(2\pi)^2} dk_x\, dk_y = \frac{A}{(2\pi)^2} d^2\vec{k}$$

$\Rightarrow$ The summation over all grid points in k-space can be replaced by an area integral

$$\sum_{\text{all } \vec{k}} \rightarrow A \int \frac{d^2\vec{k}}{(2\pi)^2}$$

Therefore:

$$N = 2 \times \sum_{\text{all } \vec{k}} f(\vec{k}) = 2 \times A \int \frac{d^2\vec{k}}{(2\pi)^2} f(\vec{k})$$

$f(\vec{k})$ is the occupation probability of a quantum state

The Electron Gas in 2D at Non-Zero Temperature - II

The probability $f(\vec{k})$ that the quantum state of wavevector $\vec{k}$ is occupied by an electron is given by the Fermi-Dirac distribution function:

$$f(\vec{k}) = \frac{1}{1 + e^{\left(E(\vec{k}) - E_f\right)/KT}} \qquad \text{Where:} \qquad E(\vec{k}) = \frac{\hbar^2\left(k_x^2 + k_y^2\right)}{2m} = \frac{\hbar^2 k^2}{2m}$$

Therefore:

$$N = 2 \times A \int \frac{d^2\vec{k}}{(2\pi)^2} f(\vec{k}) = 2 \times A \int \frac{d^2\vec{k}}{(2\pi)^2} \frac{1}{1 + e^{\left(E(\vec{k}) - E_f\right)/KT}}$$

Density of States:

The k-space integral is cumbersome. We need to convert into a simpler form – an energy space integral – using the following steps:

$$d^2\vec{k} = 2\pi\, k\, dk \qquad \text{and} \qquad E = \frac{\hbar^2 k^2}{2m} \Rightarrow dE = \frac{\hbar^2 k}{m} dk$$

Therefore:

$$2 \times A \int \frac{d^2\vec{k}}{(2\pi)^2} \rightarrow A \int_0^\infty \frac{k\, dk}{\pi} \rightarrow A \int_0^\infty \frac{m}{\pi \hbar^2} dE$$

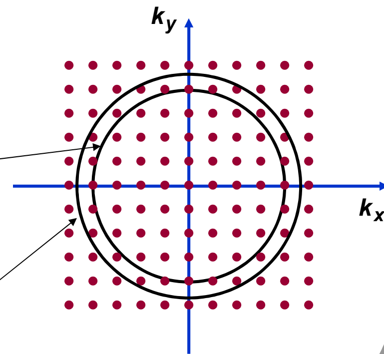
The Electron Gas in 2D at Non-Zero Temperature - III

$$N = 2\times A\int\frac{d^2\vec{k}}{(2\pi)^2}\,\frac{1}{1+e^{\left(E(\vec{k})-E_f\right)/KT}} = A\int_0^\infty dE\;\; g_{2D}(E)\,\frac{1}{1+e^{(E-E_f)/KT}}$$

Where: $g_{2D}(E) = \frac{m}{\pi\hbar^2}$ ← Density of states function is constant (independent of energy) in 2D

$g_{2D}(E)$ has units: # / Joule-cm^2

The product $g(E)\,dE$ represents the number of quantum states available in the energy interval between E and $(E+dE)$ per cm^2 of the metal



Suppose E corresponds to the inner circle from the relation:

$$E = \frac{\hbar^2 k^2}{2m}$$

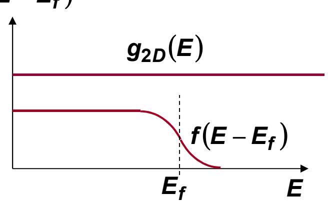
And suppose $(E+dE)$ corresponds to the outer circle, then $g_{2D}(E)\,dE$ corresponds to twice the number of the grid points between the two circles

The Electron Gas in 2D at Non-Zero Temperature - IV

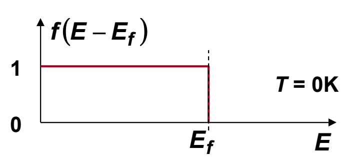
$$N = A\int_0^\infty dE\;\; g_{2D}(E)\,\frac{1}{1+e^{(E-E_f)/KT}} = A\int_0^\infty dE\;\; g_{2D}(E)\;\; f(E-E_f)$$

Where: $g_{2D}(E) = \frac{m}{\pi\hbar^2}$

The expression for N can be visualized as the integration over the product of the two functions:



Check: Suppose T=0K:



$T = 0\text{K}$

$$N = A\int_0^\infty dE\;\; g_{2D}(E)\;\; f(E-E_f) = A\int_0^{E_f} dE\;\; g_{2D}(E)$$

$$= A\,\frac{m}{\pi\hbar^2}E_f$$

$$\Rightarrow \quad n = \frac{m}{\pi\hbar^2}E_f$$

Compare with the previous result at T=0K:

$$n = \frac{m}{\pi\hbar^2}E_F \quad \Rightarrow$$

At T=0K (and only at T=0K) the Fermi level E_f is the same as the Fermi energy E_F

The Electron Gas in 2D at Non-Zero Temperature - V

For $T \neq 0$K:

Since the carrier density is known, and does not change with temperature, the Fermi level at temperature T is found from the expression

$$n = \int_0^{\infty} dE \;\; g_{2D}(E) \; \frac{1}{1+e^{(E-E_f)/KT}} = \frac{m}{\pi \hbar^2} KT \log\left[1+e^{\frac{E_f}{KT}}\right]$$

In general, the Fermi level E_f is a function of temperature and decreases from E_F as the temperature increases. The exact relationship can be found by inverting the above equation and recalling that:

$$n = \frac{m}{\pi \hbar^2} E_F$$

to get:

$$E_f(T) = KT \log\left[e^{\frac{E_F}{KT}} - 1\right]$$

Total Energy of the 2D Electron Gas

The total energy U of the electron gas can be written as:

$$U = 2 \times \sum_{\text{all } \vec{k}} f(\vec{k}) E(\vec{k}) = 2 \times A \int \frac{d^2\vec{k}}{(2\pi)^2} f(\vec{k}) E(\vec{k})$$

Convert the k-space integral to energy integral: $U = A \int_0^{\infty} dE \;\; g_{2D}(E) \; f(E - E_f) \, E$

The energy density u is: $u = \frac{U}{A} = \int_0^{\infty} dE \;\; g_{2D}(E) \; f(E - E_f) \, E$

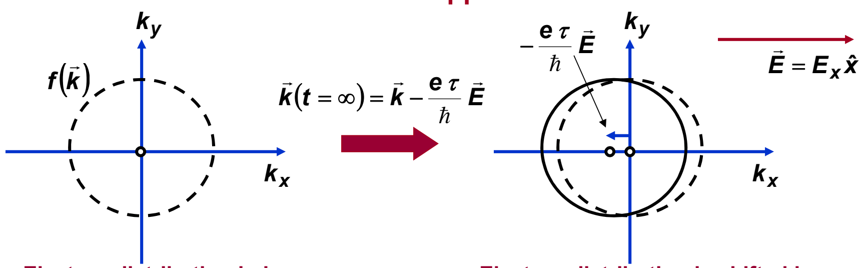
Suppose T=0K:

$$u = \int_0^{E_F} dE \;\; g_{2D}(E) \; E = \frac{m}{2\pi \hbar^2} E_F^2$$

Since: $n = \frac{m}{\pi \hbar^2} E_F$

We have: $u = \frac{1}{2} n E_F$

2D Electron Gas in an Applied Electric Field - I



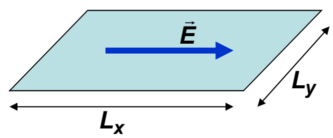
Electron distribution in k-space when E-field is zero

Distribution function: $f(\vec{k})$

Electron distribution is shifted in k-space when E-field is not zero

Distribution function: $f\left(\vec{k}+\frac{e\,\tau}{\hbar}\vec{E}\right)$

Since the wavevector of each electron is shifted by the same amount in the presence of the E-field, the net effect in k-space is that the entire electron distribution is shifted as shown



2D Electron Gas in an Applied Electric Field - II

Current density (units: A/cm)

$$\vec{J}=-2\,e\times\int\frac{d^2\vec{k}}{(2\pi)^2}\,f\left(\vec{k}+\frac{e\,\tau}{\hbar}\vec{E}\right)\vec{v}(\vec{k})$$

Do a shift in the integration variable:

$$\vec{J}=-2\,e\times\int\frac{d^2\vec{k}}{(2\pi)^2}\,f(\vec{k})\,\vec{v}\left(\vec{k}-\frac{e\,\tau}{\hbar}\vec{E}\right)$$

$$\vec{J}=-2\,e\times\int\frac{d^2\vec{k}}{(2\pi)^2}\,f(\vec{k})\,\frac{\hbar\left(\vec{k}-\frac{e\,\tau}{\hbar}\vec{E}\right)}{m}$$

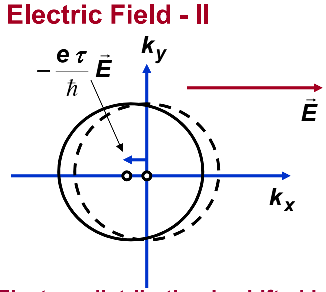
$$\vec{J}=\frac{e^2\tau}{m}\left[2\times\int\frac{d^2\vec{k}}{(2\pi)^2}\,f(\vec{k})\right]\vec{E}$$

electron density = n (units: #/cm²)

$$\vec{J}=\frac{n\,e^2\tau}{m}\vec{E}=\sigma\,\vec{E}$$

Where: $\sigma=\frac{n\,e^2\tau}{m}$

Same as the Drude result - but units are different. Units of σ are Siemens in 2D



Electron distribution is shifted in k-space when E-field is not zero

Distribution function: $f\left(\vec{k}+\frac{e\,\tau}{\hbar}\vec{E}\right)$

Electrons in 1D Metals: The Free Electron Model

The quantum state of an electron is described by the time-independent Schrodinger equation:

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x)+V(x)\psi(x)=E\,\psi(x)$$

Consider a large metal wire of length L :

Use the Sommerfeld model:

• The electrons inside the wire are confined in a one-dimensional infinite potential well with zero potential inside the wire and infinite potential outside the wire

$$V(x)=0 \quad \text{for } x \text{ inside the wire}$$
$$V(x)=\infty \quad \text{for } x \text{ outside the wire}$$

free electrons (experience no potential when inside the wire)

• The electron states inside the wire are given by the Schrodinger equation

Born Von Karman Periodic Boundary Conditions in 1D

Solve: $$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x)=E\,\psi(x)$$

Use periodic boundary conditions:

$$\psi(x+L,y,z)=\psi(x,y,z)$$

These imply that each facet of the sheet is folded and joined to the opposite facet

Solution is: $$\psi(x)=\sqrt{\frac{1}{L}}\;e^{i(k_x x)}$$

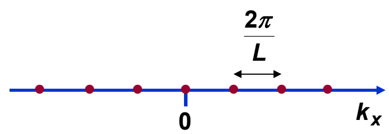
The boundary conditions dictate that the allowed values of k_x are such that:

$$e^{i(k_x L)}=1 \quad \Rightarrow \quad k_x=n\frac{2\pi}{L} \qquad n = 0, \pm1, \pm2, \pm3, \ldots\ldots$$

States in 1D k-Space

k-space Visualization:

The allowed quantum states states can be visualized as a 1D grid of points in the entire "k-space"



$$k_x = n\frac{2\pi}{L}$$

$n = 0, \pm 1, \pm 2, \pm 3, \ldots\ldots$

Density of Grid Points in k-space:

Looking at the figure, in k-space there is only one grid point in every small length of size:

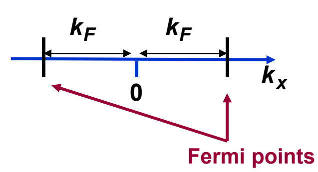
$$\left(\frac{2\pi}{L}\right)$$

$\Rightarrow$ There are $\frac{L}{2\pi}$ grid points per unit length of k-space } Very important result

The Electron Gas in 1D at Zero Temperature - I

• Each grid-point can be occupied by two electrons (spin up and spin down)

• All filled quantum states correspond to grid-points that are within a distance k_F from the origin



Length of the region = $2k_F$

Number of grid-points in the region = $\frac{L}{2\pi} \times 2k_F$

Number of quantum states (including spin) in the region = $2 \times \frac{L}{2\pi} \times 2k_F$

But the above must equal the total number N of electrons in the wire:

$$N = L\frac{2k_F}{\pi}$$

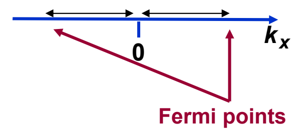
$$\Rightarrow \quad n = \text{electron density} = \frac{N}{L} = \frac{2k_F}{\pi}$$

{ Units of the electron density n are #/cm

$$\Rightarrow \quad k_F = \frac{\pi n}{2}$$

The Electron Gas in 1D at Zero Temperature - II

- All quantum states between the Fermi points are filled (i.e. occupied by electrons)
- All quantum states outside the Fermi points are empty



Fermi Momentum:

The largest momentum of the electrons is: $\hbar k_F$

This is called the Fermi momentum

Fermi momentum can be found if one knows the electron density:

$$k_F = \frac{\pi n}{2}$$

Fermi Energy:

The largest energy of the electrons is: $\frac{\hbar^2 k_F^2}{2m}$

This is called the Fermi energy E_F : $E_F = \frac{\hbar^2 k_F^2}{2m}$

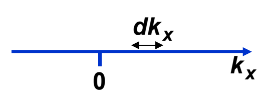
Also: $$E_F = \frac{\hbar^2 \pi^2 n^2}{8\,m}$$ or $$n = \frac{\sqrt{8m}}{\pi \hbar}\sqrt{E_F}$$

Fermi Velocity:

The largest velocity of the electrons is called the Fermi velocity v_F : $v_F = \frac{\hbar k_F}{m}$

The Electron Gas in 1D at Non-Zero Temperature - I

Recall that there are $\frac{L}{2\pi}$ grid points per unit length of k-space



$\Rightarrow$ So in length dk_x of k-space the number of grid points is:

$$\frac{L}{2\pi} dk_x$$

$\Rightarrow$ The summation over all grid points in k-space can be replaced by an integral

$$\sum_{\text{all } \vec{k}} \rightarrow L \int_{-\infty}^{\infty} \frac{dk_x}{2\pi}$$

Therefore:

$$N = 2 \times \sum_{\text{all } \vec{k}} f(k_x) = 2 \times L \int_{-\infty}^{\infty} \frac{dk_x}{2\pi} f(k_x)$$

$f(k_x)$ is the occupation probability of a quantum state

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The Electron Gas in 1D at Non-Zero Temperature - II

The probability $f(k_x)$ that the quantum state of wavevector k_x is occupied by an electron is given by the Fermi-Dirac distribution function:

$$f(k_x) = \frac{1}{1+e^{(E(k_x)-E_f)/KT}} \qquad \text{Where:} \qquad E(\vec{k}) = \frac{\hbar^2 k_x^2}{2m}$$

Therefore:

$$N = 2\times L \int_{-\infty}^{\infty} \frac{dk_x}{2\pi} f(k_x) = 2\times L \int_{-\infty}^{\infty} \frac{dk_x}{2\pi} \frac{1}{1+e^{(E(k_x)-E_f)/KT}}$$

Density of States:

The k-space integral is cumbersome. We need to convert into a simpler form – an energy space integral – using the following steps:

$$2\times L \int_{-\infty}^{\infty} \frac{dk_x}{2\pi} \rightarrow 2\times L\times 2\int_{0}^{\infty} \frac{dk}{2\pi} \qquad \text{and} \qquad E = \frac{\hbar^2 k^2}{2m} \Rightarrow dE = \frac{\hbar^2 k}{m} dk$$

Therefore:

$$2\times L \int_{-\infty}^{\infty} \frac{dk_x}{2\pi} \rightarrow L\int_{0}^{\infty} dE \frac{\sqrt{2m}}{\pi\hbar} \frac{1}{\sqrt{E}}$$

The Electron Gas in 1D at Non-Zero Temperature - III

$$N = 2\times L \int_{-\infty}^{\infty} \frac{dk_x}{2\pi} \frac{1}{1+e^{(E(k_x)-E_f)/KT}} = L\int_{0}^{\infty} dE\; g_{1D}(E) \frac{1}{1+e^{(E-E_f)/KT}}$$

Where: $g_{1D}(E) = \frac{\sqrt{2m}}{\pi\hbar} \frac{1}{\sqrt{E}}$ ← **Density of states function in 1D**

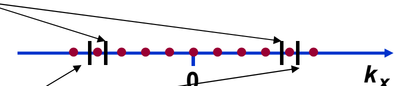
$g_{1D}(E)$ has units: # / Joule-cm

The product $g(E)\, dE$ represents the number of quantum states available in the energy interval between E and ($E+dE$) per cm of the metal

Suppose E corresponds to the inner points from the relation:

$$E = \frac{\hbar^2 k^2}{2m}$$

And suppose ($E+dE$) corresponds to the outer points, then $g_{1D}(E)\, dE$ corresponds to twice the number of the grid points between the points (adding contributions from both sides)

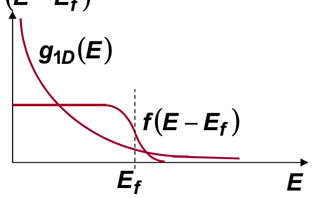


The Electron Gas in 1D at Non-Zero Temperature - IV

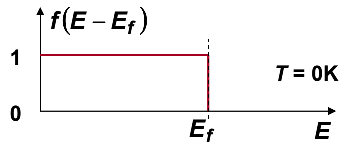
$$N = L\int_0^\infty dE\ \ g_{1D}(E)\ \frac{1}{1+e^{(E-E_f)/KT}} = L\int_0^\infty dE\ \ g_{1D}(E)\ \ f(E-E_f)$$

Where: $g_{1D}(E) = \frac{\sqrt{2m}}{\pi\hbar}\frac{1}{\sqrt{E}}$

The expression for N can be visualized as the integration over the product of the two functions:



Check: Suppose T=0K:



$$N = L\int_0^\infty dE\ \ g_{1D}(E)\ \ f(E-E_f) = L\int_0^{E_f} dE\ \ g_{1D}(E)$$

$$= L\frac{\sqrt{8m}}{\pi\hbar}\sqrt{E_f}$$

$$\Rightarrow\quad n = \frac{\sqrt{8m}}{\pi\hbar}\sqrt{E_f}$$

Compare with the previous result at T=0K:

$$n = \frac{\sqrt{8m}}{\pi\hbar}\sqrt{E_F}\quad\Rightarrow$$

At T=0K (and only at T=0K) the Fermi level E_f is the same as the Fermi energy E_F

The Electron Gas in 1D at Non-Zero Temperature - V

For $T \neq$ 0K:

Since the carrier density is known, and does not change with temperature, the Fermi level at temperature T is found from the expression

$$n = \int_0^\infty dE\ \ g_{1D}(E)\ \frac{1}{1+e^{(E-E_f)/KT}}$$

In general, the Fermi level E_f is a function of temperature and decreases from E_F as the temperature increases.

Total Energy of the 1D Electron Gas

The total energy U of the electron gas can be written as:

$$U = 2\times\sum_{\text{all } \vec{k}} f(k_x)\,E(k_x) = 2\times L\int_{-\infty}^{\infty}\frac{dk_x}{2\pi}\,f(k_x)\,E(k_x)$$

Convert the k-space integral to energy integral: $U = L\int_0^{\infty} dE\;\; g_{1D}(E)\;\; f(E-E_f)\,E$

The energy density u is $u = \dfrac{U}{L} = \int_0^{\infty} dE\;\; g_{1D}(E)\;\; f(E-E_f)\,E$

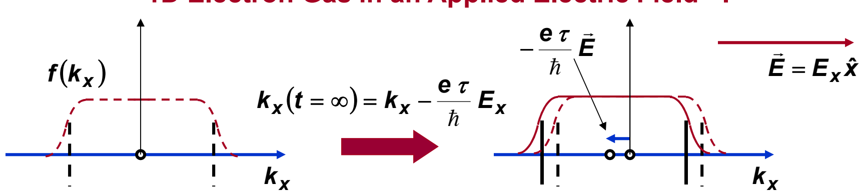
Suppose T=0K:

$$u = \int_0^{E_F} dE\;\; g_{1D}(E)\;\; E = \frac{\sqrt{8m}}{\pi\hbar}\frac{E_F^{3/2}}{3}$$

Since: $n = \dfrac{\sqrt{8m}}{\pi\hbar}\sqrt{E_F}$

We have: $u = \dfrac{1}{3}n\,E_F$

1D Electron Gas in an Applied Electric Field - I



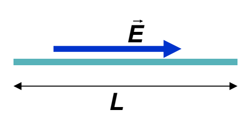
Electron distribution in k-space when E-field is zero

Distribution function: $f(k_x)$

Electron distribution is shifted in k-space when E-field is not zero

Distribution function: $f\left(k_x + \dfrac{e\tau}{\hbar}E_x\right)$

Since the wavevector of each electron is shifted by the same amount in the presence of the E-field, the net effect in k-space is that the entire electron distribution is shifted as shown



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1D Electron Gas in an Applied Electric Field - II

Current (units: A)

$$I = -2\,e \times \int_{-\infty}^{\infty} \frac{dk_x}{2\pi}\, f\left(k_x + \frac{e\,\tau}{\hbar} E_x\right) v(k_x)$$

Do a shift in the integration variable:

$$I = -2\,e \times \int_{-\infty}^{\infty} \frac{dk_x}{2\pi}\, f(k_x)\, v\left(k_x - \frac{e\,\tau}{\hbar} E_x\right)$$

$$I = -2\,e \times \int_{-\infty}^{\infty} \frac{dk_x}{2\pi}\, f(k_x)\, \frac{\hbar\left(k_x - \frac{e\,\tau}{\hbar} E_x\right)}{m}$$

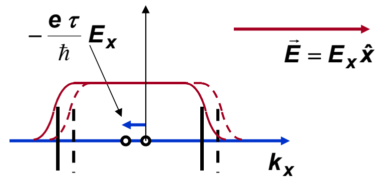
$$I = \frac{e^2\tau}{m}\left[2 \times \int_{-\infty}^{\infty} \frac{dk_x}{2\pi}\, f(k_x)\right] E_x$$

electron density = n (units: #/cm)

$$I = \frac{n\,e^2\tau}{m}\,\vec{E} = \sigma\,\vec{E}$$

Where: $\sigma = \dfrac{n\,e^2\tau}{m}$

Same as the Drude result - but units are different. Units of σ are Siemens-cm in 1D



Electron distribution is shifted in k-space when E-field is not zero

Distribution function: $f\left(k_x + \frac{e\,\tau}{\hbar} E_x\right)$

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