Handout 29

Optical Transitions in Solids, Optical Gain, and Semiconductor Lasers

In this lecture you will learn:

- Electron-photon Hamiltonian in solids
- Optical transition matrix elements
- Optical absorption coefficients
- Stimulated absorption and stimulated emission
- Optical gain in semiconductors
- Semiconductor heterostructure lasers

Interactions Between Light and Solids

The basic interactions between light and solids cover a wide variety of topics that can include:

- Interband electronic transitions in solids
- Intraband electronic transitions and intersubband electronic transitions
- Plasmons and plasmon-polaritons
- Surface plasmons
- Excitons and exciton-polaritons
- Phonon and phonon-polaritons
- Nonlinear optics
- Quantum optics
- Optical spintronics

Fermi's Golden Rule: A Review

Consider a Hamiltonian with the following eigenstates and eigenenergies:

$$\hat{H}_o\,|\psi_m\rangle = E_m\,|\psi_m\rangle \qquad \{\, m = \text{integer}$$

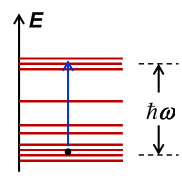
Now suppose a time dependent externally applied potential is added to the Hamiltonian:

$$\hat{H} = \hat{H}_o + \hat{V}_{\uparrow}\, e^{-i\omega t} + \hat{V}_{\downarrow}\, e^{i\omega t}$$

Suppose at time $t = 0$ an electron was in some initial state k: $|\psi(t=0)\rangle = |\psi_p\rangle$

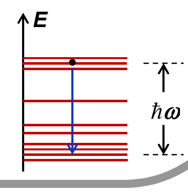
Fermi's golden rule tells that the rate at which the electron absorbs energy $\hbar\omega$ from the time-dependent potential and makes a transition to some higher energy level is given by:

$$W_{\uparrow}(p) = \frac{2\pi}{\hbar}\sum_m \left|\langle\psi_m|\hat{V}_{\uparrow}|\psi_p\rangle\right|^2 \delta\left(E_m - E_p - \hbar\omega\right)$$



The rate at which the electron gives away energy $\hbar\omega$ to the time-dependent potential and makes a transition to some lower energy level is given by:

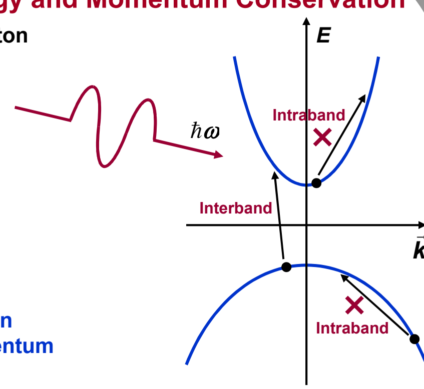
$$W_{\downarrow}(p) = \frac{2\pi}{\hbar}\sum_m \left|\langle\psi_m|\hat{V}_{\downarrow}|\psi_p\rangle\right|^2 \delta\left(E_m - E_p + \hbar\omega\right)$$



Optical Transitions in Solids: Energy and Momentum Conservation

For an electron to absorb energy from a photon energy conservation implies:

$$\underbrace{E_m(\vec{k}_f)}_{\text{Final energy}} = \underbrace{E_n(\vec{k}_i)}_{\text{Initial energy}} + \underbrace{\hbar\omega}_{\text{Photon energy}}$$



Momentum conservation implies:

$$\underbrace{\hbar\,\vec{k}_f}_{\text{Final momentum}} = \underbrace{\hbar\,\vec{k}_i}_{\text{Initial momentum}} + \underbrace{\hbar\,\vec{q}}_{\text{Photon momentum}}$$

Note that the momentum conservation principle is stated in terms of the crystal momentum of the electrons. This principle will be derived later.

Intraband photonic transitions are not possible:

For parabolic bands, it can be shown that intraband optical transitions cannot satisfy both energy and momentum conservation and are therefore not possible

Electromagnetic Wave Basics

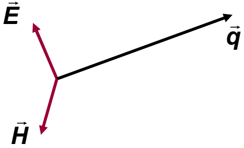
Consider an electromagnetic wave passing through a solid with electric field given by:

$$\vec{E}(\vec{r},t) = -\hat{n}\, E_o \sin(\vec{q}\,.\,\vec{r} - \omega\, t)$$

The vector potential associated with the field is:

$$\vec{E}(\vec{r},t) = -\frac{\partial \vec{A}(\vec{r},t)}{\partial t} \quad \Rightarrow \quad \vec{A}(\vec{r},t) = \hat{n}\, \frac{E_o}{\omega} \cos(\vec{q}\,.\,\vec{r} - \omega\, t)$$

$$= \hat{n}\, A_o \cos(\vec{q}\,.\,\vec{r} - \omega\, t)$$



The divergence of the field is zero:

$$\nabla\,.\,\vec{E}(\vec{r},t) = \nabla\,.\,\vec{A}(\vec{r},t) = 0$$

The power per unit area or the Intensity of the field is given by the Poynting vector:

$$\vec{I} = \left\langle \vec{S}(\vec{r},t) \right\rangle = \left\langle \vec{E}(\vec{r},t) \times \vec{H}(\vec{r},t) \right\rangle = \hat{q}\, \frac{E_o^2}{2\eta} = \hat{q}\, \frac{\omega^2 A_o^2}{2\eta} \qquad \left\{ \eta = \sqrt{\frac{\mu_o}{\varepsilon}} = \frac{\eta_o}{n} \right.$$

The photon flux per unit area is:

$$F = \frac{\left|\vec{I}\right|}{\hbar\omega} = \frac{\omega\, A_o^2}{2\eta\hbar}$$

Electron-Photon Hamiltonian in Solids

Consider electrons in a solid. The eigenstates (Bloch functions) and eigenenergies satisfy:

$$\hat{H}_o \left|\psi_{n,\vec{k}}\right\rangle = E_n(\vec{k}) \left|\psi_{n,\vec{k}}\right\rangle$$

where:

$$\hat{H}_o = \frac{\hat{\vec{P}}\,.\,\hat{\vec{P}}}{2m} + V_{lattice}(\hat{\vec{r}})$$

In the presence of E&M fields the Hamiltonian is:

$$\hat{H} = \frac{\left[\hat{\vec{P}} + e\vec{A}(\hat{\vec{r}},t)\right]^2}{2m} + V_{lattice}(\hat{\vec{r}}) \qquad \left\{ \vec{A}(\vec{r},t) = \hat{n}\, A_o \cos(\vec{q}\,.\,\vec{r} - \omega\, t) \right.$$

$$= \frac{\hat{\vec{P}}\,.\,\hat{\vec{P}}}{2m} + V_{lattice}(\hat{\vec{r}}) + \frac{e}{2m}\, \hat{\vec{P}}\,.\,\vec{A}(\hat{\vec{r}},t) + \frac{e}{2m}\, \vec{A}(\hat{\vec{r}},t)\,.\,\hat{\vec{P}} + \cancel{\frac{e^2}{2m}\, \vec{A}(\hat{\vec{r}},t)\,.\,\vec{A}(\hat{\vec{r}},t)}$$

Assume small

$$\approx \hat{H}_o + \frac{e}{2m}\, \hat{\vec{P}}\,.\,\vec{A}(\hat{\vec{r}},t) + \frac{e}{2m}\, \vec{A}(\hat{\vec{r}},t)\,.\,\hat{\vec{P}}$$

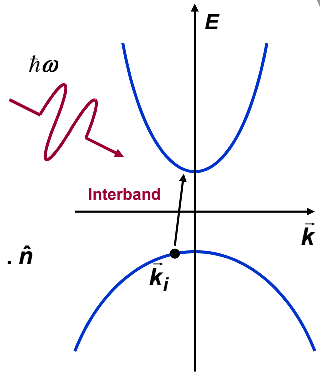
$$= \hat{H}_o + \frac{e}{m}\, \vec{A}(\hat{\vec{r}},t)\,.\,\hat{\vec{P}} \longrightarrow \left\{ \text{Provided: } \nabla\,.\,\vec{A}(\vec{r},t) = 0 \right.$$

$$= \hat{H}_o + \frac{e\, A_o \left[e^{i\vec{q}\,.\,\hat{\vec{r}} - i\,\omega\, t} + e^{-i\vec{q}\,.\,\hat{\vec{r}} + i\,\omega\, t} \right]}{2m}\, \hat{n}\cdot\hat{\vec{P}}$$

Optical Interband Transitions in Solids

$$\hat{H}_o \left|\psi_{n,\vec{k}}\right\rangle = E_n\left(\vec{k}\right)\left|\psi_{n,\vec{k}}\right\rangle$$

$$\hat{H} = \hat{H}_o + \frac{e\,A_o\left[\,e^{i\vec{q}\,.\,\hat{\vec{r}}-i\,\omega\,t} + e^{-i\vec{q}\,.\,\hat{\vec{r}}+i\,\omega\,t}\,\right]}{2m}\,\hat{\vec{P}}\,.\,\hat{n}$$



Comparison with:

$$\hat{H} = \hat{H}_o + \hat{V}_{\uparrow}\,e^{-i\omega t} + \hat{V}_{\downarrow}\,e^{i\omega t}$$

gives:

$$\hat{V}_{\uparrow} = \frac{e\,A_o e^{i\vec{q}\,.\,\hat{\vec{r}}}}{2m}\,\hat{\vec{P}}\,.\,\hat{n} \qquad \hat{V}_{\downarrow} = \frac{e\,A_o e^{-i\vec{q}\,.\,\hat{\vec{r}}}}{2m}\,\hat{\vec{P}}\,.\,\hat{n}$$

Suppose at time $t = 0$ the electron was sitting in the valence band with crystal momentum $\vec{k}_i$:

$$\left|\psi(t=0)\right\rangle = \left|\psi_{v,\vec{k}_i}\right\rangle$$

The transition rate to states in the conduction band is given by the Fermi's golden rule:

$$W_{\uparrow}\left(\vec{k}_i\right) = \frac{2\pi}{\hbar}\sum_{\vec{k}_f}\left|\left\langle\psi_{c,\vec{k}_f}\right|\hat{V}_{\uparrow}\left|\psi_{v,\vec{k}_i}\right\rangle\right|^2\,\delta\!\left(E_c\left(\vec{k}_f\right)-E_v\left(\vec{k}_i\right)-\hbar\omega\right)$$

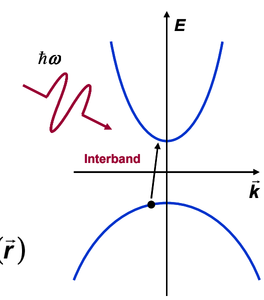
The summation is over all possible final states in the conduction band that have the same spin as the initial state. Energy conservation is enforced by the delta function.

Optical Matrix Element

$$W_{\uparrow}\left(\vec{k}_i\right) = \frac{2\pi}{\hbar}\sum_{\vec{k}_f}\left|\left\langle\psi_{c,\vec{k}_f}\right|\hat{V}_{\uparrow}\left|\psi_{v,\vec{k}_i}\right\rangle\right|^2\,\delta\!\left(E_c\left(\vec{k}_f\right)-E_v\left(\vec{k}_i\right)-\hbar\omega\right)$$

$$= \frac{2\pi}{\hbar}\left(\frac{eA_o}{2m}\right)^2\sum_{\vec{k}_f}\left|\left\langle\psi_{c,\vec{k}_f}\right|e^{i\vec{q}\,.\,\hat{\vec{r}}}\;\hat{\vec{P}}\,.\,\hat{n}\left|\psi_{v,\vec{k}_i}\right\rangle\right|^2\,\delta\!\left(E_c\left(\vec{k}_f\right)-E_v\left(\vec{k}_i\right)-\hbar\omega\right)$$

Now consider the matrix element:



$$\left\langle\psi_{c,\vec{k}_f}\right|e^{i\vec{q}\,.\,\hat{\vec{r}}}\hat{\vec{P}}\,.\,\hat{n}\left|\psi_{v,\vec{k}_i}\right\rangle$$

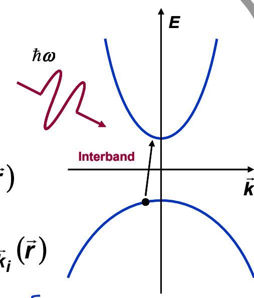
$$= \int d^3\vec{r}\;\psi^*_{c,\vec{k}_f}(\vec{r})\quad e^{i\vec{q}\,.\,\vec{r}}\;\hat{\vec{P}}\,.\,\hat{n}\quad\psi_{v,\vec{k}_i}(\vec{r})$$

$$= \int d^3\vec{r}\;e^{-i\vec{k}_f\,.\,\vec{r}}u^*_{c,\vec{k}_f}(\vec{r})\quad e^{i\vec{q}\,.\,\vec{r}}\;\hat{\vec{P}}\,.\,\hat{n}\quad e^{i\vec{k}_i\,.\,\vec{r}}u_{v,\vec{k}_i}(\vec{r})$$

$$= \int d^3\vec{r}\;e^{-i\vec{k}_f\,.\,\vec{r}}u^*_{c,\vec{k}_f}(\vec{r})\quad e^{i\vec{q}\,.\,\vec{r}}\;e^{i\vec{k}_i\,.\,\vec{r}}\left(\hat{\vec{P}}+\cancelto{0}{\hbar\vec{k}_i}\right).\,\hat{n}\quad u_{v,\vec{k}_i}(\vec{r})$$

$$= \int d^3\vec{r}\;\underbrace{e^{-i\vec{k}_f\,.\,\vec{r}}}_{\text{Slowly varying in space}}\underbrace{u^*_{c,\vec{k}_f}(\vec{r})}_{\text{Rapidly varying in space}}\quad \underbrace{e^{i\vec{q}\,.\,\vec{r}}\;e^{i\vec{k}_i\,.\,\vec{r}}}_{\text{Slowly varying in space}}\;\hat{\vec{P}}\,.\,\hat{n}\quad \underbrace{u_{v,\vec{k}_i}(\vec{r})}_{\text{Rapidly varying in space}}$$

Optical Matrix Element



$$\left\langle \psi_{c,\vec{k}_f} \middle| e^{i\vec{q}\,.\,\hat{\vec{r}}}\,\hat{\vec{P}}\,.\,\hat{n} \middle| \psi_{v,\vec{k}_i} \right\rangle$$

$$= \int d^3\vec{r}\; e^{-i\vec{k}_f\,.\,\vec{r}}\,u^*_{c,\vec{k}_f}(\vec{r}) \quad e^{i\vec{q}\,.\,\vec{r}}\; e^{i\vec{k}_i\,.\,\vec{r}}\; \hat{\vec{P}}\,.\,\hat{n} \quad u_{v,\vec{k}_i}(\vec{r})$$

$$= \sum_{\vec{R}_j} e^{i\left(\vec{k}_i+\vec{q}-\vec{k}_f\right).\,\vec{R}_j} \int_{j\text{-th primitive cell}} d^3\vec{r} \quad u^*_{c,\vec{k}_f}(\vec{r}) \;\; \hat{\vec{P}}\,.\,\hat{n} \;\; u_{v,\vec{k}_i}(\vec{r})$$

$$= \sum_{\vec{R}_j} e^{i\left(\vec{k}_i+\vec{q}-\vec{k}_f\right).\,\vec{R}_j} \int_{\text{any one primitive cell}} d^3\vec{r} \quad u^*_{c,\vec{k}_f}(\vec{r}) \;\; \hat{\vec{P}}\,.\,\hat{n} \;\; u_{v,\vec{k}_i}(\vec{r})$$

$$= N\delta_{\vec{k}_i+\vec{q}\,,\,\vec{k}_f} \int_{\text{any one primitive cell}} d^3\vec{r} \quad u^*_{c,\vec{k}_f}(\vec{r}) \;\; \hat{\vec{P}}\,.\,\hat{n} \;\; u_{v,\vec{k}_i}(\vec{r})$$

N = number of primitive cells in the crystal

$$= \delta_{\vec{k}_i+\vec{q}\,,\,\vec{k}_f} \int_{\text{entire crystal}} d^3\vec{r} \;\; u^*_{c,\vec{k}_f}(\vec{r}) \;\; \hat{\vec{P}}\,.\,\hat{n} \;\; u_{v,\vec{k}_i}(\vec{r})$$

$$= \delta_{\vec{k}_i+\vec{q}\,,\,\vec{k}_f}\; \vec{P}_{cv}\,.\,\hat{n}$$

Interband momentum matrix element

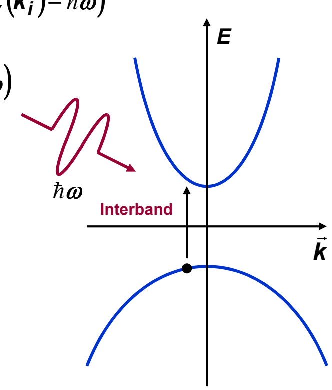
Crystal Momentum Selection Rule

$$W_{\uparrow}\left(\vec{k}_i\right) = \frac{2\pi}{\hbar}\left(\frac{eA_o}{2m}\right)^2 \sum_{\vec{k}_f} \left|\left\langle \psi_{c,\vec{k}_f} \middle| e^{i\vec{q}\,.\,\hat{\vec{r}}}\,\hat{\vec{P}}\,.\,\hat{n} \middle| \psi_{v,\vec{k}_i} \right\rangle\right|^2 \delta\left(E_c\left(\vec{k}_f\right) - E_v\left(\vec{k}_i\right) - \hbar\omega\right)$$

$$= \frac{2\pi}{\hbar}\left(\frac{eA_o}{2m}\right)^2 \sum_{\vec{k}_f} \left|\vec{P}_{cv}\,.\,\hat{n}\right|^2 \delta_{\vec{k}_i+\vec{q}\,,\,\vec{k}_f}\; \delta\left(E_c\left(\vec{k}_f\right) - E_v\left(\vec{k}_i\right) - \hbar\omega\right)$$

$$= \frac{2\pi}{\hbar}\left(\frac{eA_o}{2m}\right)^2 \left|\vec{P}_{cv}\,.\,\hat{n}\right|^2 \delta\left(E_c\left(\vec{k}_i+\vec{q}\right) - E_v\left(\vec{k}_i\right) - \hbar\omega\right)$$

Crystal Momentum Selection Rule:



We have the crystal momentum selection rule:

$$\boxed{\vec{k}_f = \vec{k}_i + \vec{q}}$$

The wave vector of the photons is very small since the speed of light is very large

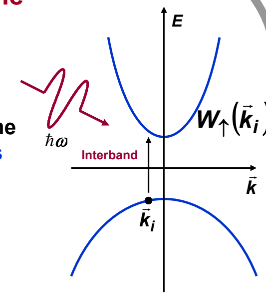
Therefore, one may assume that:

$$\boxed{\vec{k}_f \approx \vec{k}_i}$$

Optical transitions are vertical in k-space

Transition Rates per Unit Volume

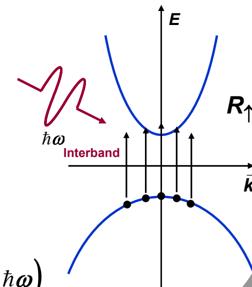
$$W_{\uparrow}(\vec{k}_i) = \frac{2\pi}{\hbar}\left(\frac{eA_o}{2m}\right)^2 \left|\vec{P}_{cv} \cdot \hat{n}\right|^2 \delta\left(E_c(\vec{k}_i) - E_v(\vec{k}_i) - \hbar\omega\right)$$



Generally one is not interested in the transition rate for any one particular initial electron state but in the number of transitions happening per unit volume of the material per second

The upward transition $R_{\uparrow}$ rate per unit volume is obtained by summing over all the possible initial states per unit volume weighed by the occupation probability of the initial state and by the probability that the final state is empty:

$$R_{\uparrow}(\omega) = \frac{2}{V} \times \sum_{\vec{k}_i} W_{\uparrow}(\vec{k}_i)\, f_v(\vec{k}_i)\left[1 - f_c(\vec{k}_i)\right]$$

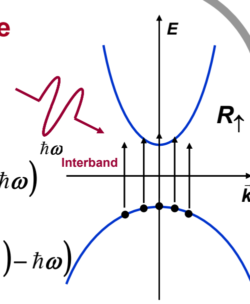


If we assume, as in an intrinsic semiconductor, that the valence band is full and the conduction band is empty of electrons, then:

$$R_{\uparrow}(\omega) = \frac{2}{V} \times \sum_{\vec{k}_i} W_{\uparrow}(\vec{k}_i)$$

$$= \frac{2}{V} \times \sum_{\vec{k}_i} \frac{2\pi}{\hbar}\left(\frac{eA_o}{2m}\right)^2 \left|\vec{P}_{cv} \cdot \hat{n}\right|^2 \delta\left(E_c(\vec{k}_i) - E_v(\vec{k}_i) - \hbar\omega\right)$$

Transition Rates per Unit Volume



$$R_{\uparrow}(\omega) = \frac{2}{V} \times \sum_{\vec{k}_i} \frac{2\pi}{\hbar}\left(\frac{eA_o}{2m}\right)^2 \left|\vec{P}_{cv} \cdot \hat{n}\right|^2 \delta\left(E_c(\vec{k}_i) - E_v(\vec{k}_i) - \hbar\omega\right)$$

$$= 2 \times \int_{FBZ} \frac{d^3\vec{k}_i}{(2\pi)^3} \frac{2\pi}{\hbar}\left(\frac{eA_o}{2m}\right)^2 \left|\vec{P}_{cv} \cdot \hat{n}\right|^2 \delta\left(E_c(\vec{k}_i) - E_v(\vec{k}_i) - \hbar\omega\right)$$

$$= \frac{2\pi}{\hbar}\left(\frac{eA_o}{2m}\right)^2 \left\langle\left|\vec{P}_{cv} \cdot \hat{n}\right|^2\right\rangle 2 \times \int_{FBZ} \frac{d^3\vec{k}_i}{(2\pi)^3} \delta\left(E_c(\vec{k}_i) - E_v(\vec{k}_i) - \hbar\omega\right)$$

$$= \frac{2\pi}{\hbar}\left(\frac{eA_o}{2m}\right)^2 \left\langle\left|\vec{P}_{cv} \cdot \hat{n}\right|^2\right\rangle \underbrace{2 \times \int_{FBZ} \frac{d^3\vec{k}}{(2\pi)^3} \delta\left(E_c(\vec{k}) - E_v(\vec{k}) - \hbar\omega\right)}_{\text{Joint density of states}}$$

The integral in the expression above is similar to the density of states integral:

Suppose: $E_c(\vec{k}) = E_c + \dfrac{\hbar^2 k^2}{2m_e}$

Then: $g_{3D}(E) = 2 \times \displaystyle\int_{FBZ} \frac{d^3\vec{k}}{(2\pi)^3} \delta\left(E_c(\vec{k}) - E\right) = \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2} \sqrt{E - E_c}$

Transition Rates per Unit Volume and Joint Density of States

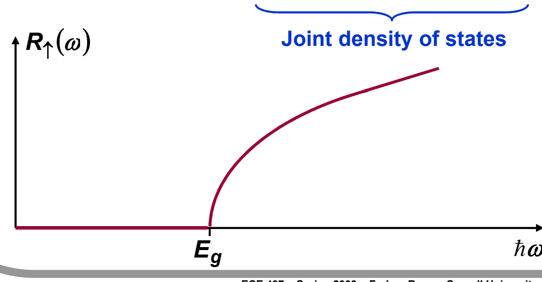
$$R_{\uparrow}(\omega) = \frac{2\pi}{\hbar}\left(\frac{eA_o}{2m}\right)^2 \left\langle \left|\vec{P}_{cv}\,.\,\hat{n}\right|^2 \right\rangle\; 2\times \int_{FBZ} \frac{d^3\vec{k}}{(2\pi)^3}\; \delta\left(E_c(\vec{k}) - E_v(\vec{k}) - \hbar\omega\right)$$

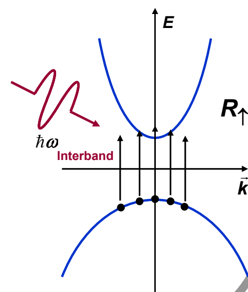
Suppose: $E_c(\vec{k}) = E_c + \dfrac{\hbar^2 k^2}{2m_e}$ $\qquad E_v(\vec{k}) = E_v - \dfrac{\hbar^2 k^2}{2m_h}$

Then: $E_c(\vec{k}) - E_v(\vec{k}) = E_c - E_v + \dfrac{\hbar^2 k^2}{2}\left(\dfrac{1}{m_e} + \dfrac{1}{m_h}\right) = E_g + \dfrac{\hbar^2 k^2}{2m_r}$

Reduced effective mass

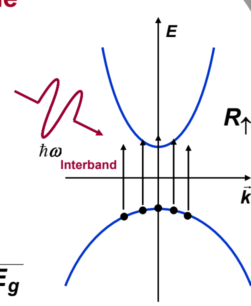
$$R_{\uparrow}(\omega) = \frac{2\pi}{\hbar}\left(\frac{eA_o}{2m}\right)^2 \left\langle \left|\vec{P}_{cv}\,.\,\hat{n}\right|^2 \right\rangle \underbrace{\frac{1}{2\pi^2}\left(\frac{2m_r}{\hbar^2}\right)^{3/2}\sqrt{\hbar\omega - E_g}}_{\text{Joint density of states}}$$





Transition Rates per Unit Volume

$$R_{\uparrow}(\omega) = \frac{2\pi}{\hbar}\left(\frac{eA_o}{2m}\right)^2 \left\langle \left|\vec{P}_{cv}\,.\,\hat{n}\right|^2 \right\rangle \frac{1}{2\pi^2}\left(\frac{2m_r}{\hbar^2}\right)^{3/2}\sqrt{\hbar\omega - E_g}$$



The power per unit area or the Intensity of the E&M wave is:

$$I = \frac{\omega^2 A_o^2}{2\eta}$$

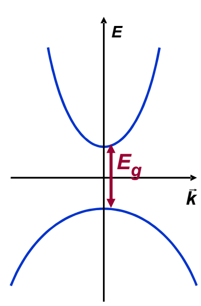
$$R_{\uparrow}(\omega) = \frac{2\pi}{\hbar}\left(\frac{e}{2m}\right)^2 \left(\frac{2\eta I}{\omega^2}\right) \left\langle \left|\vec{P}_{cv}\,.\,\hat{n}\right|^2 \right\rangle \frac{1}{2\pi^2}\left(\frac{2m_r}{\hbar^2}\right)^{3/2}\sqrt{\hbar\omega - E_g}$$

Interband Momentum Matrix Elements:

Recall the result from homework 7:

$$\frac{1}{m_r} = \left(\frac{1}{m_e} + \frac{1}{m_h}\right) = \frac{4}{m^2}\frac{\left|\vec{P}_{cv}\,.\,\hat{n}\right|^2}{E_g}$$

The above result assumed diagonal isotropic conduction and valence bands and also that no other bands are present, and is therefore oversimplified



Momentum Matrix Elements

In bulk III-V semiconductors with cubic symmetry, the average value of the momentum matrix element is usually expressed in terms of the energy E_p as,

$$\left\langle \left| \vec{P}_{cv} \cdot \hat{n} \right|^2 \right\rangle = \frac{m E_p}{6}$$

Note that the momentum matrix elements are independent of the direction of light polarization!

Parameters at 300K	GaAs	AlAs	InAs	InP	GaP
Ep (eV)	25.7	21.1	22.2	20.7	22.2

Transition Rates per Unit Volume and the Absorption or Loss Coefficient

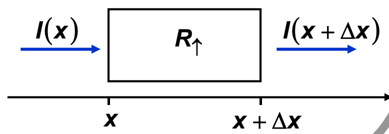
Because of transitions photons will be lost from the E&M wave traveling inside the solid. This loss will result in decay of the wave Intensity with distance travelled:

$$\Rightarrow I(x) = e^{-\alpha x} \, I(x=0)$$

$$\Rightarrow \frac{I(x)}{dx} = -\alpha \, I(x)$$

$$\Rightarrow I(x+\Delta x) - I(x) = -\alpha \;\; \Delta x \, I(x)$$

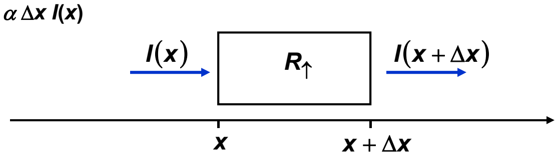
Loss coefficient or absorption coefficient

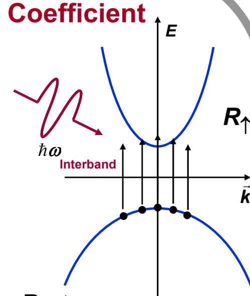


Transition Rates per Unit Volume and Loss Coefficient

$$\Rightarrow I(x+\Delta x) - I(x) = -\alpha \;\; \Delta x \, I(x)$$

The wave power loss (per unit area) in small distance Δx is $\alpha \, \Delta x \, I(x)$





The wave power loss in small distance Δx must also equal: $\hbar\omega \, R_\uparrow \, \Delta x$

Therefore:

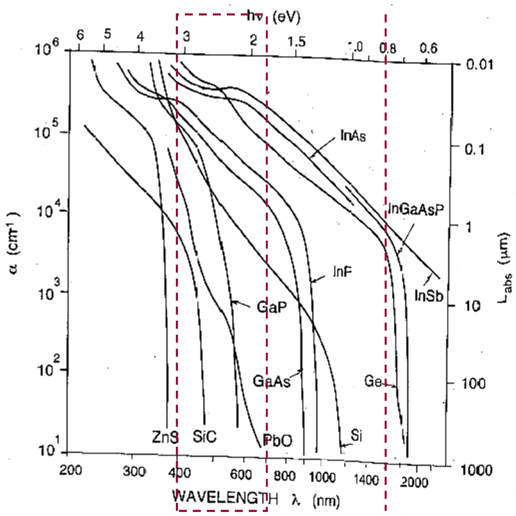
$$\hbar\omega \, R_\uparrow \, \Delta x = \alpha \, \Delta x \, I$$

$$\Rightarrow \alpha(\omega) = \frac{\hbar\omega \, R_\uparrow(\omega)}{I} = 2\pi \left(\frac{e}{2m}\right)^2 \left(\frac{2\eta}{\omega}\right) \left| \vec{P}_{cv} \cdot \hat{n} \right|^2 \frac{1}{2\pi^2} \left(\frac{2m_r}{\hbar^2}\right)^{3/2} \sqrt{\hbar\omega - E_g}$$

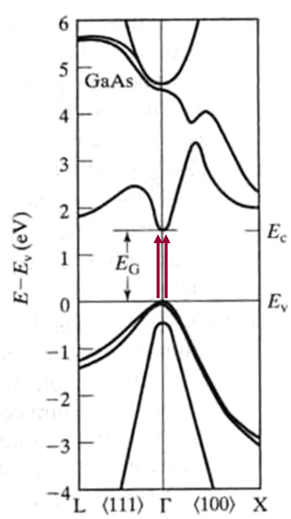
$$= \left(\frac{e}{m}\right)^2 \frac{\pi}{\varepsilon_o \, n\omega \, c} \left| \vec{P}_{cv} \cdot \hat{n} \right|^2 \frac{1}{2\pi^2} \left(\frac{2m_r}{\hbar^2}\right)^{3/2} \sqrt{\hbar\omega - E_g}$$

Values of $\alpha(\omega)$ for most semiconductors can range from a few hundred cm^{-1} to hundred thousand cm^{-1}

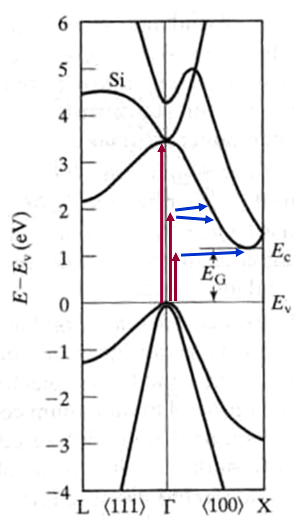
Loss Coefficient of Semiconductors



Direct Bandgap and Indirect Bandgap Semiconductors



Direct bandgap
(Direct optical transitions)



Direct bandgap
(Indirect phonon-assisted transitions)

$$\frac{1}{m_r} = \left(\frac{1}{m_h} + \frac{1}{m_e}\right)$$

If m_e approaches $-m_h$, then m_r becomes very large

Then, $\alpha \propto (m_r)^{3/2}$ also becomes very large

Stimulated Absorption and Stimulated Emission

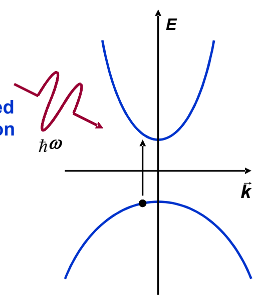
Throwing back in the occupation factors one can write more generally:

$$R_{\uparrow}(\omega)=\frac{2\pi}{\hbar}\left(\frac{eA_o}{2m}\right)^2\left\langle\left|\vec{P}_{cv}\cdot\hat{n}\right|^2\right\rangle\,2\times\int_{\text{FBZ}}\frac{d^3\vec{k}}{(2\pi)^3}\,f_v(\vec{k})\left[1-f_c(\vec{k})\right]\delta\left(E_c(\vec{k})-E_v(\vec{k})-\hbar\omega\right)$$

Stimulated Absorption:

The process of photon absorption is called stimulated absorption (because, quite obviously, the process is initiated by the incoming radiation or photons some of which eventually end up getting absorbed)

Stimulated absorption



An incoming photon can also cause the reverse process in which an electron makes a downward transition

This reverse process is initiated by the term in Hamiltonian that has the $e^{i\omega t}$ time dependence:

$$\hat{H}=\hat{H}_o+\hat{V}_{\uparrow}\,e^{-i\omega t}+\hat{V}_{\downarrow}\,e^{i\omega t}$$

$$\hat{V}_{\uparrow}=\frac{e\,A_o e^{i\vec{q}\cdot\hat{\vec{r}}}}{2m}\,\hat{\vec{P}}\cdot\hat{n}\qquad\qquad \hat{V}_{\downarrow}=\frac{e\,A_o e^{-i\vec{q}\cdot\hat{\vec{r}}}}{2m}\,\hat{\vec{P}}\cdot\hat{n}$$

Stimulated Emission and Spontaneous Emission

Following the same procedure as for stimulated absorption, one can write the rate per unit volume for the downward transitions as:

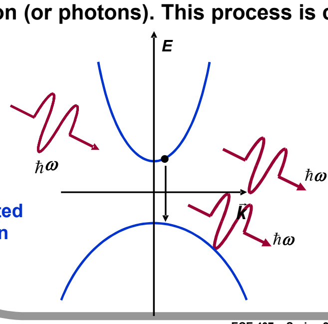
$$R_{\downarrow}(\omega)=\frac{2\pi}{\hbar}\left(\frac{eA_o}{2m}\right)^2\left\langle\left|\vec{P}_{cv}\cdot\hat{n}\right|^2\right\rangle\,2\times\int_{\text{FBZ}}\frac{d^3\vec{k}}{(2\pi)^3}\,f_c(\vec{k})\left[1-f_v(\vec{k})\right]\delta\left(E_c(\vec{k})-E_v(\vec{k})-\hbar\omega\right)$$

Stimulated Emission:

In the downward transition, the electron gives off its energy to the electromagnetic field, i.e. it emits a photon! The process of photon emission caused by incoming radiation (or by other photons) is called stimulated emission.

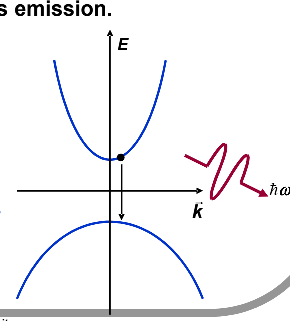
Spontaneous Emission:

Electrons can also make downward transitions even in the absence of any incoming radiation (or photons). This process is called spontaneous emission.



Stimulated emission

Spontaneous emission



Stimulated Absorption and Stimulated Emission

The net stimulated electronic transition rate is the difference between the stimulated emission and stmulated absorption rates:

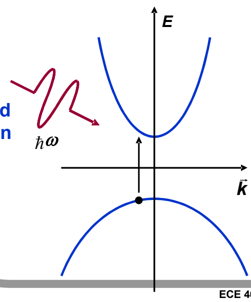
$$R_{\uparrow}(\omega)-R_{\downarrow}(\omega)=\frac{2\pi}{\hbar}\left(\frac{eA_o}{2m}\right)^2\left\langle\left|\vec{P}_{cv}\ .\ \hat{n}\right|^2\right\rangle\ 2\times\int_{\text{FBZ}}\frac{d^3\vec{k}}{(2\pi)^3}\left[f_v(\vec{k})-f_c(\vec{k})\right]\delta\left(E_c(\vec{k})-E_v(\vec{k})-\hbar\omega\right)$$

And the more accurate expression for the loss coefficient is then:

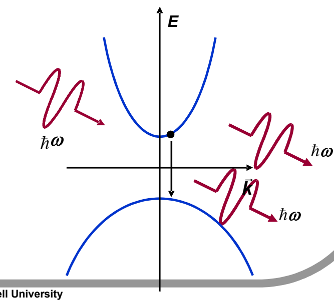
$$\alpha(\omega)=\frac{\hbar\omega\left(R_{\uparrow}(\omega)-R_{\downarrow}(\omega)\right)}{I}$$

$$=\left(\frac{e}{m}\right)^2\frac{\pi}{\varepsilon_o n\omega\, c}\left\langle\left|\vec{P}_{cv}\ .\ \hat{n}\right|^2\right\rangle\ 2\times\int_{\text{FBZ}}\frac{d^3\vec{k}}{(2\pi)^3}\left[f_v(\vec{k})-f_c(\vec{k})\right]\delta\left(E_c(\vec{k})-E_v(\vec{k})-\hbar\omega\right)$$

Stimulated absorption



Stimulated emission



Optical Gain in Semiconductors

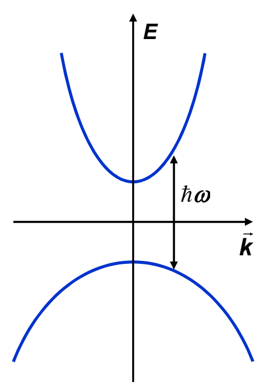
$$\alpha(\omega)=\left(\frac{e}{m}\right)^2\frac{\pi}{\varepsilon_o n\omega\, c}\left\langle\left|\vec{P}_{cv}\ .\ \hat{n}\right|^2\right\rangle\ 2\times\int_{\text{FBZ}}\frac{d^3\vec{k}}{(2\pi)^3}\left[f_v(\vec{k})-f_c(\vec{k})\right]\delta\left(E_c(\vec{k})-E_v(\vec{k})-\hbar\omega\right)$$

Note that the Intensity decays as: $I(x)=e^{-\alpha x}\,I(x=0)$

What if α were to become negative? Optical gain !!

$$\alpha(\omega)<0\quad\Rightarrow\quad R_{\downarrow}(\omega)>R_{\uparrow}(\omega)$$

A negative value of α implies optical gain (as opposed to optical loss) and means that stimulated emission rate exceeds stimulated absorption rate



Optical gain is possible if:

$$f_v(\vec{k})-f_c(\vec{k})<0\qquad\text{for}\qquad E_c(\vec{k})-E_v(\vec{k})=\hbar\omega$$
$$\Rightarrow f_c(\vec{k})-f_v(\vec{k})>0\qquad\text{for}\qquad E_c(\vec{k})-E_v(\vec{k})=\hbar\omega$$

Optical Gain in Semiconductors

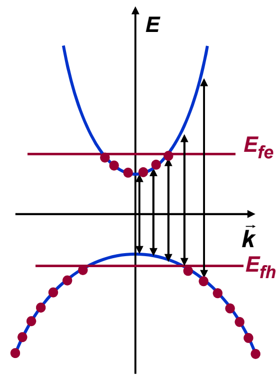
Optical gain is only possible in non-equilibrium situations when the electron and hole Fermi levels are not the same

Suppose:

$$f_c(\vec{k}) = f\left(E_c(\vec{k}) - E_{fe}\right) = \frac{1}{1 + e^{\left(E_c(\vec{k}) - E_{fe}\right)/KT}}$$

$$f_v(\vec{k}) = f\left(E_v(\vec{k}) - E_{fh}\right) = \frac{1}{1 + e^{\left(E_v(\vec{k}) - E_{fh}\right)/KT}}$$

$$E_c(\vec{k}) - E_v(\vec{k}) = \hbar\omega$$



Then the condition for optical gain at frequency ω is:

$$f\left(E_c(\vec{k}) - E_{fe}\right) - f\left(E_v(\vec{k}) - E_{fh}\right) > 0$$

$$\Rightarrow E_{fe} - E_{fh} > E_c(\vec{k}) - E_v(\vec{k})$$

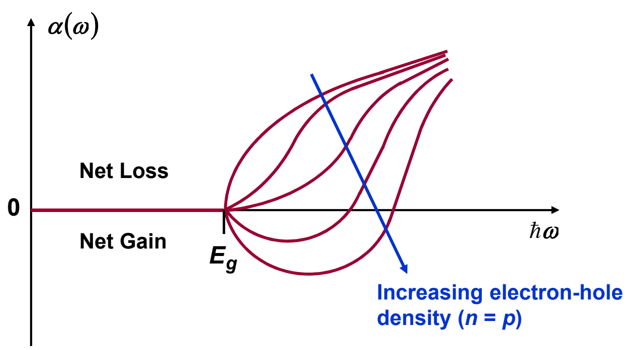
$$\Rightarrow E_{fe} - E_{fh} > \hbar\omega$$

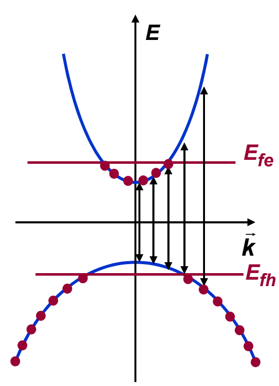
The above is the condition for population inversion (lots of electrons in the conduction band and lots of holes in the valence band)

Optical Gain in Semiconductors

The loss coefficient is a function of frequency:

$$\alpha(\omega) = \left(\frac{e}{m}\right)^2 \frac{\pi}{\varepsilon_o n \omega c} \left\langle \left|\vec{P}_{cv} \cdot \hat{n}\right|^2 \right\rangle \; 2 \times \int_{FBZ} \frac{d^3\vec{k}}{(2\pi)^3} \left[f_v(\vec{k}) - f_c(\vec{k})\right] \delta\left(E_c(\vec{k}) - E_v(\vec{k}) - \hbar\omega\right)$$

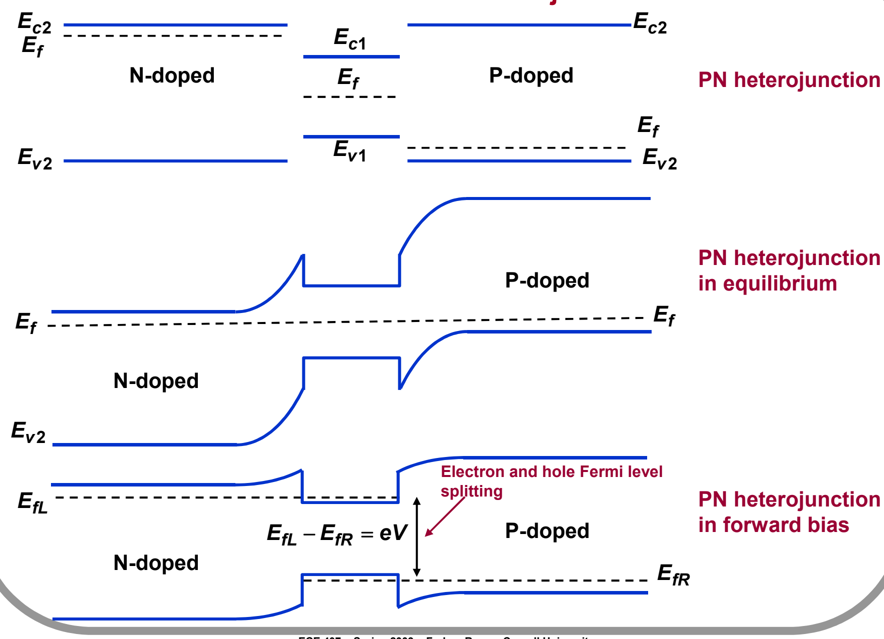




Optical gain for frequencies for which:

$$E_g < \hbar\omega < E_{fe} - E_{fh}$$

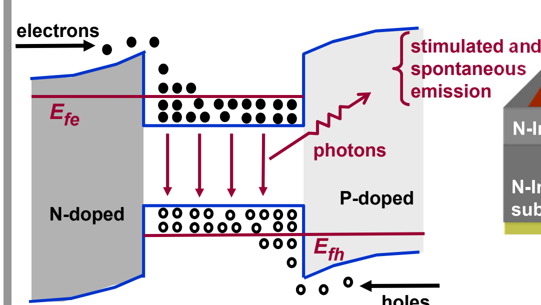
Semiconductor PN Heterojunctions



Semiconductor Heterostructure Lasers

A Heterostructure Laser (Band Diagram)

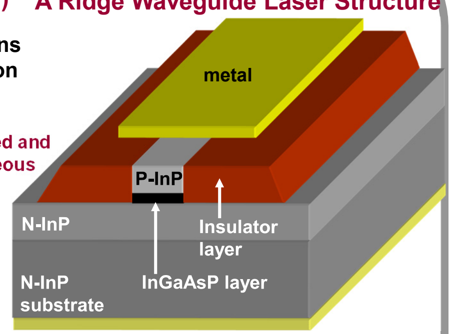
Non-equilibrium electron and hole populations can be sustained in a forward bias pn junction

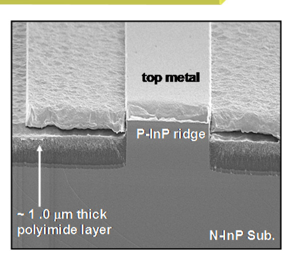


$$E_{fe} - E_{fh} > \hbar\omega$$
$$\Rightarrow \quad eV > \hbar\omega$$

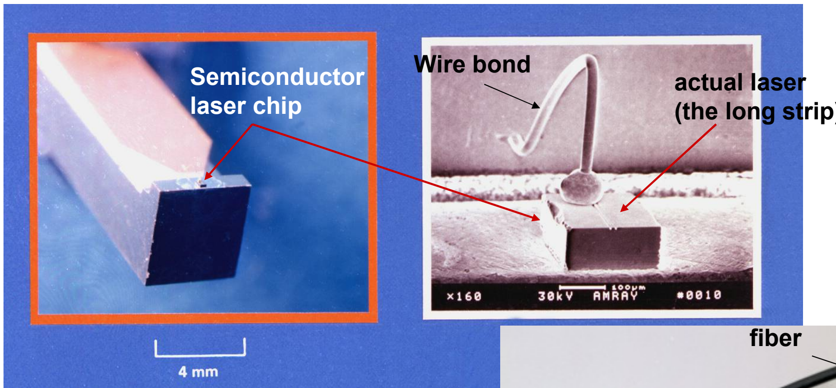
All lasers used in fiber optical communications are semiconductor lasers

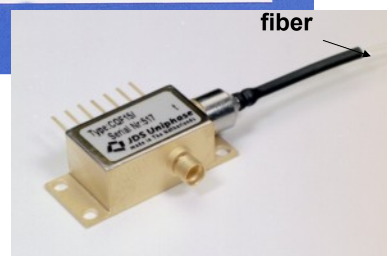
A Ridge Waveguide Laser Structure





Semiconductor Heterostructure Lasers





A commercial packaged semiconductor laser