Handout 28

Ballistic Quantum Transport in Semiconductor Nanostructures

In this lecture you will learn:

- Electron transport without scattering (ballistic transport)
- The quantum of conductance and the quantum of resistance
- Quantized conductance

Rolf Landauer (IBM)
(1927-1999)

Lester F. Eastman (Cornell)
(1928-)

Electron Transport Physics in Nanoscale Systems

Hydrodynamic and ballistic transport

Quantized conductance

Coulomb blockage of tunneling and single electron transistors

Coherent carrier transport

Universal conductance fluctuations

Integer and fractional quantum Hall effects

Charge density wave and spin density wave transport

Anderson localization and weak localization

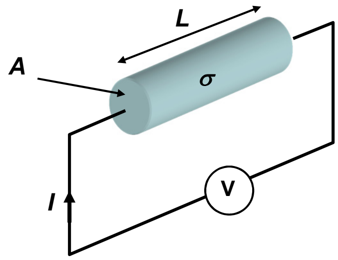
Metal-insulator transitions and Mott insulators

Molecular electronics and polarons

Strongly correlated electrons: Ferromagnets, aniterromagnets, and high-Tc superconductors, spin liquids, topological insulators

Conductors and Dissipation

Traditional View of Conductors:



$$E = \frac{V}{L}$$

$$J = \sigma E$$

$$I = AJ = A\sigma E = \frac{\sigma A}{L} V = GV = \frac{V}{R}$$

$$\Rightarrow G = \frac{\sigma A}{L} = \frac{1}{R}$$

Power Dissipation in Conductors:

E&M (energy continuity equation) tells us that the power dissipation per unit volume of a resistor is:

$$= J\,E = \sigma E^2 = \frac{J^2}{\sigma}$$

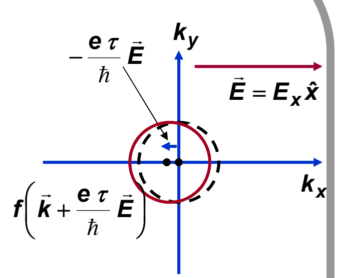
Power dissipation in the entire resistor is:

$$P = J\,E\,(AL) = J^2 \frac{AL}{\sigma} = J^2 A^2 \frac{L}{\sigma A} = I^2 R = \frac{I^2}{G}$$

Characteristic Velocity for Conduction

Consider a 3D solid in which the energy dispersion for conduction band near a band minimum is given by:

$$E_c(\vec{k}) = E_c + \frac{\hbar^2 k^2}{2m_e} \quad \Rightarrow \quad \vec{v}_c(\vec{k}) = \frac{1}{\hbar}\nabla_{\vec{k}} E_c(\vec{k}) = \frac{\hbar \vec{k}}{m_e}$$



Current Density:

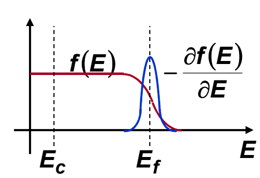
$$\vec{J} = -2\,e \times \int \frac{d^3\vec{k}}{(2\pi)^3} f\left(\vec{k} + \frac{e\tau}{\hbar}\vec{E}\right)\vec{v}_c(\vec{k})$$

$$= -2\,e \times \int \frac{d^3\vec{k}}{(2\pi)^3}\left[\cancelto{0}{f(\vec{k})} + \frac{e\tau}{\hbar}\vec{E}\,.\,\nabla_{\vec{k}} f(\vec{k})\right]\vec{v}_c(\vec{k})$$

$$= -2\,e \times \int \frac{d^3\vec{k}}{(2\pi)^3}\left[\frac{e\tau}{\hbar}\vec{E}\,.\,\nabla_{\vec{k}} E_c(\vec{k})\frac{\partial f(E)}{\partial E}\right]\vec{v}_c(\vec{k})$$

$$= 2\,e^2\tau \times \int \frac{d^3\vec{k}}{(2\pi)^3}\left(-\frac{\partial f(E)}{\partial E}\right)\left[\vec{E}\,.\,\vec{v}_c(\vec{k})\right]\vec{v}_c(\vec{k})$$

$$= \frac{ne^2\tau}{m_e}\vec{E}$$



Only electrons close to the Fermi energy contribute to the conductivity in metals or heavily doped semiconductors at low temperatures

ECE 407 – Spring 2009 – Farhan Rana – Cornell University

Characteristic Velocity for Conduction and Mean Free Path

Characteristic Velocity:

The characteristic velocity is the average velocity of those electrons that contribute to the conductivity:

$$\left\langle v^2 \right\rangle = \frac{\int_{FBZ} \frac{d^d \vec{k}}{(2\pi)^d} \left(-\frac{\partial f(E)}{\partial E} \right) \vec{v}(\vec{k}) . \vec{v}(\vec{k})}{\int_{FBZ} \frac{d^d \vec{k}}{(2\pi)^d} \left(-\frac{\partial f(E)}{\partial E} \right)}$$

For metals and heavily doped semiconductors at low temperatures: $\sqrt{\left\langle v^2 \right\rangle} \approx v_F$

For low doped semiconductors at high temperatures: $\sqrt{\left\langle v^2 \right\rangle} \approx \sqrt{\frac{d\, KT}{m_e}}$

Mean Free Path:

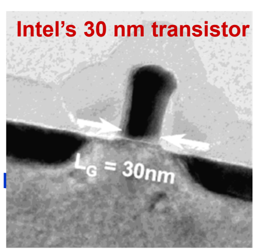
The mean free path ℓ is defined as the average distance an electron travels before it scatters. It is given by:

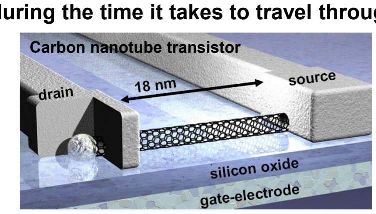
$$\ell = \sqrt{\left\langle v^2 \right\rangle}\, \tau$$

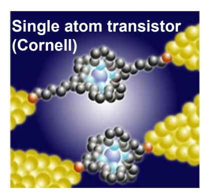
where τ is the scattering time.

Ballistic Electron Transport

The length scales involved in the smallest transistors and nanoscale devices, such as carbon nanotubes and molecular conductors, can be small enough so that the electrons do not scatter during the time it takes to travel through the device







When the length L of the conductor becomes much smaller than the mean free path ℓ the transport is termed "ballistic" meaning that the electrons do not scatter during the time it takes to travel through the conductor

Ballistic transport condition: $L \ll \ell$

Questions:

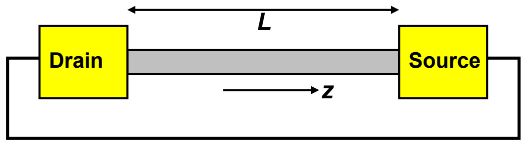
• What happens when $L \ll \ell$?

• The formulas for conductivity that have the scattering time τ in them are clearly no longer valid since there is no scattering: $\sigma = \frac{ne^2 \tau}{m_e}$ $\quad \{\tau \rightarrow \infty$

• What about dissipation?

Ballistic Electron Transport in a 1D Conductor

Consider a 1D conductor (example, a quantum nanowire) that is contacted at both ends by an external circuit.

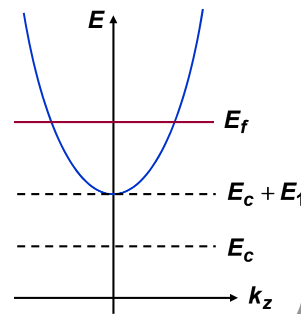


The dispersion relation for the electrons inside the quantum wire is:

$$E_c(k_z) = E_c + E_1 + \frac{\hbar^2 k_z^2}{2m_e}$$

$$v(k_z) = \frac{1}{\hbar}\frac{\partial E_c(k_z)}{\partial z} = \frac{\hbar k_z}{m_e}$$

Assume only one subband



The electron density (number per unit length) is:

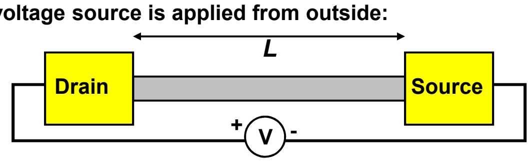
$$n = 2 \times \int_{-\infty}^{\infty} \frac{dk_z}{(2\pi)} f(E_c(k_z) - E_f)$$

The length L of the wire is short enough such that:

$$L << \ell$$

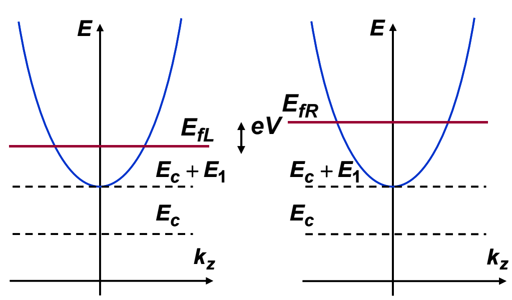
Electric Fields, Chemical Potentials, and Voltage Sources

Now suppose a voltage source is applied from outside:



In electronics, one never applies "electric fields" nor even "electrostatic potential differences" to circuits but only "chemical potential differences" by using voltage sources

The voltage source will raise the chemical potential (or the Fermi level) on one side of the conductor with respect to the other by an amount eV



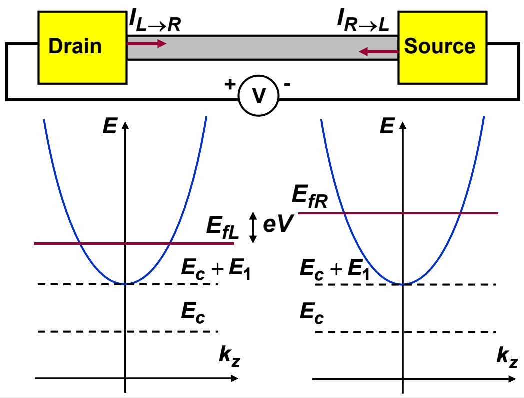
Electron Currents

At the left contact, the current due to electrons moving in the right direction is:

$$I_{L\to R} = (-e)2\times\int_0^{\infty}\frac{dk_z}{2\pi}\, v_c(k_z)\ f(E_c(k_z)-E_{fL})$$

At the right contact, the current due to electrons moving in the left direction is:

$$I_{R\to L} = (-e)2\times\int_{-\infty}^{0}\frac{dk_z}{2\pi}\, v_c(k_z)\ f(E_c(k_z)-E_{fR})$$

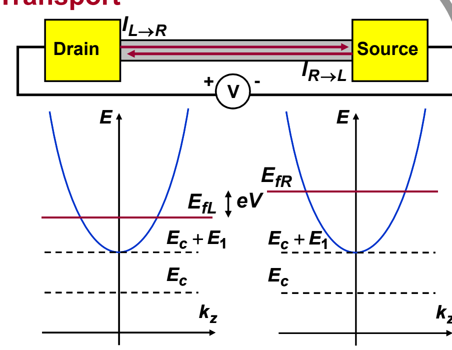


Arrows indicate the direction of electron flow (not the direction of current, which is opposite)

Ballistic Transport

Electrons do not scatter in the quantum wire. Therefore:

- All electrons that enter the wire from the left contact make it to the right contact
- All electrons that enter the wire from the right contact make it to the left contact



Total Current:

The net current is the sum of the currents due to the right-moving and left-moving electrons:

$$\begin{aligned} I &= I_{L\to R} + I_{R\to L} \\ &= (-e)2\times\int_0^{\infty}\frac{dk_z}{2\pi}\, v_c(k_z)\ f(E_c(k_z)-E_{fL}) + (-e)2\times\int_{-\infty}^{0}\frac{dk_z}{2\pi}\, v_c(k_z)\ f(E_c(k_z)-E_{fR}) \\ &= e\,2\times\int_0^{\infty}\frac{dk_z}{2\pi}\, v_c(k_z)\ \left[f(E_c(k_z)-E_{fR}) - f(E_c(k_z)-E_{fL})\right] \end{aligned}$$

Ballistic Transport Conductance

Total Current:

$$I = e\,2\times\int_0^{\infty}\frac{dk_z}{2\pi}\,v_c(k_z)\,\left[f\left(E_c(k_z)-E_{fR}\right)-f\left(E_c(k_z)-E_{fL}\right)\right]$$

$$= 2\times\frac{e}{2\pi}\int_{E_c+E_1}^{\infty}\left|\frac{dk_z}{dE}\right| dE\; v_c(k_z)\,\left[f\left(E-E_{fR}\right)-f\left(E-E_{fL}\right)\right]$$

$$= 2\times\frac{e}{2\pi\hbar}\int_{E_c+E_1}^{\infty} dE\;\left[f\left(E-E_{fR}\right)-f\left(E-E_{fL}\right)\right]$$

⟶ Very simple

Assume T ≈ 0K:

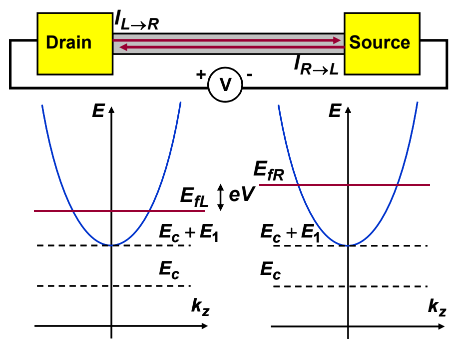
$$I = \frac{e}{\pi\hbar}(eV)$$

$$= \frac{e^2}{\pi\hbar}V$$

Conductance:

$$I = GV$$

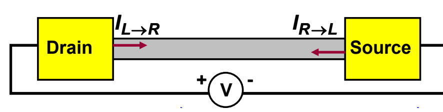
$$\Rightarrow\; G = \frac{e^2}{\pi\hbar}$$



Quantum of Conductance

The relation:

$$I = GV \quad\Rightarrow\quad G = \frac{e^2}{\pi\hbar}$$



defines the quantum of conductance as:

$$G_Q = \frac{e^2}{\pi\hbar} = 7.72\times10^{-5}\ \text{S}$$

The quantum of resistance is therefore: $R_Q = \dfrac{1}{G_Q} = \dfrac{\pi\hbar}{e^2} = 12.95\ \text{k}\Omega$

The Quantum of Conductance:

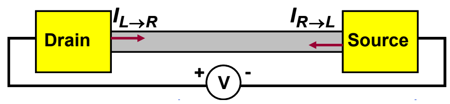
• The quantum of conductance is the smallest possible non-zero conductance of a completely ballistic conductor. Equivalently, the quantum of resistance is the highest possible resistance of a completely ballistic conductor.

• All completely ballistic conductors (whether in 1D, 2D, or 3D) will have conductance that is in multiples of the quantum conductance value (one can think of ballistic conductance in 2D and 3D as a number of 1D conductors in parallel)

The Question of Energy Dissipation

The relation:

$$I = G_Q V = \frac{V}{R_Q}$$



suggests that there should be power dissipation in the conductor given by:

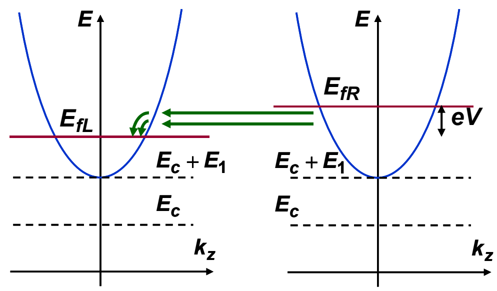
$$P = I^2 R_Q = \frac{V^2}{R_Q}$$

But, as we have seen, electrons do not loose any energy in the conductor – they do not scatter – they go ballistic. So where is the energy being dissipated?

Answer:

The energy is dissipated in the contact not in the conductor!

Electrons loose energy and thermalize when they reach the contact



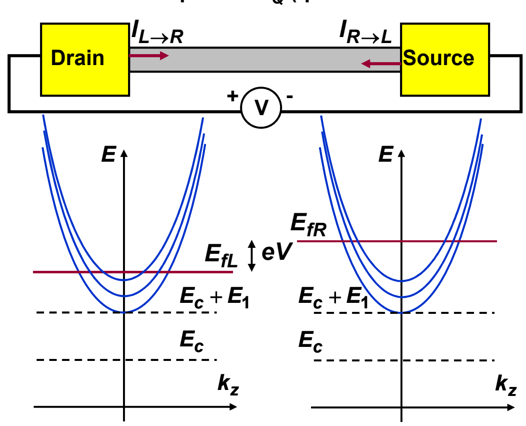
Multiple Subbands: Quantized Conductance

3 Subbands $\Rightarrow$ $E_c(p,k_z) = E_c + E_p + \dfrac{\hbar^2 k_z^2}{2m_e}$ $\quad \{ p = 1,2,3$

$$\Rightarrow I = \sum_{p=1}^{3} 2 \times \frac{e}{2\pi\hbar} \int_{E_c+E_p}^{\infty} dE \;\; [f(E-E_{fR}) - f(E-E_{fL})]$$

$$= 3\, G_Q\, V$$

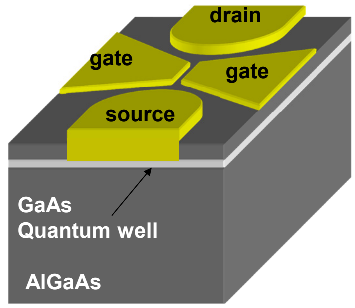
Conductance increases in multiples of G_Q (quantized conductance!)



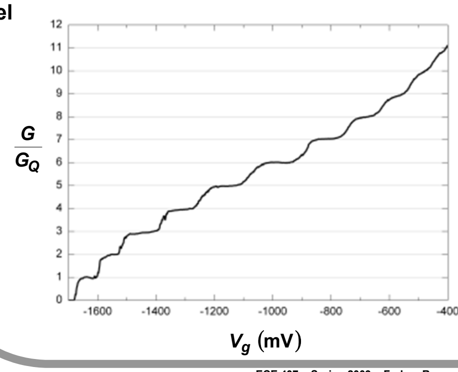
Quantized Conductance: Experiments with 1D Quantum Wires

Semiconductor Quantum Point Contacts:

- Electrons are confined in 2D in the quantum well
- Negative bias on metal gates repel electrons from underneath the gates creating a narrow 1D channel for electrons in the spacing between the gates
- The gate voltage can also control how many subbands of the 1D channel are below the Fermi level



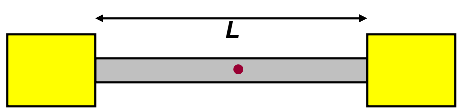
Phys. Rev. Lett., 60, 848 (1988)



The conductance (and resistance) is quantized so effectively in Quantum Hall Effect that it can give a value of the Plank's constant to one part in 10^8

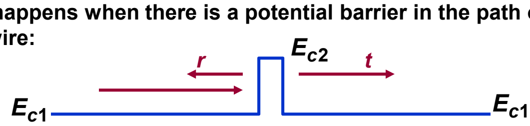
Scattering and Conductance in 1D

What if there is one scatterer (like an impurity atom) in the 1D channel?



Quantum Mechanical Reflection and Transmission from a Potential Barrier:

Consider what happens when there is a potential barrier in the path of an electron in a 1D quantum wire:



$$E_{c1}(k_z) = E_{c1} + E_1 + \frac{\hbar^2 k_z^2}{2m_e} \qquad E_{c2}(k_z) = E_{c2} + E_1 + \frac{\hbar^2 k_z^2}{2m_e}$$

Left Side Solution:

$$\phi_1(\vec{r}) = f(x,y)\, e^{ik_z z} + r\, f(x,y)\, e^{-ik_z z}$$

Right Side Solution:

$$\phi_1(\vec{r}) = t\, f(x,y)\, e^{ik_z z}$$

Transmission probability: $T_c = |t|^2$

Reflection probability: $R_c = |r|^2 = 1 - T$

Assume they are energy independent

Conductance as Transmission: Landauer's Formula

$$I=(-e)2\times\int_0^{\infty}\frac{dk_z}{2\pi}\,v_c(k_z)\;T_c\;f(E_c(k_z)-E_{fL})+(-e)2\times\int_{-\infty}^{0}\frac{dk_z}{2\pi}\,v_c(k_z)\;f(E_c(k_z)-E_{fR})$$

$$+(-e)2\times\int_0^{\infty}\frac{dk_z}{2\pi}\,v_c(k_z)\;R_c\;f(E_c(k_z)-E_{fR})$$

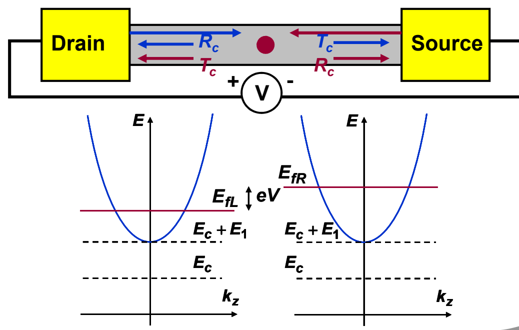
Use: $R_c+T_c=1$

$$=e\,2\times\int_0^{\infty}\frac{dk_z}{2\pi}\,v_c(k_z)\,T_c\,[f(E_c(k_z)-E_{fR})-f(E_c(k_z)-E_{fL})]$$

$$=2\times\frac{e\,T_c}{2\pi\,\hbar}\int_{E_c+E_1}^{\infty}dE\;\;[f(E-E_{fR})-f(E-E_{fL})]$$

$$=\left(\frac{e^2}{\pi\,\hbar}T_c\right)V$$

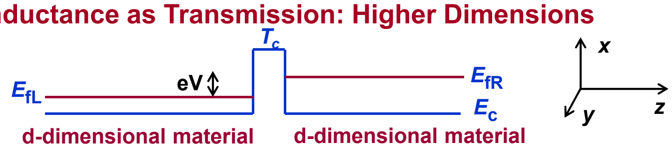
$$\Rightarrow G=\frac{e^2}{\pi\,\hbar}T_c\;<\;G_Q$$



Landauer's formula

$$\boxed{G=\frac{e^2}{\pi\,\hbar}T_c}$$

Conductance as Transmission: Higher Dimensions



$$E_c(\vec{k})=E_c+\frac{\hbar^2\left(k_x^2+k_y^2\right)}{2m_e}+\frac{\hbar^2k_z^2}{2m_e}=E_c+E_{xy}+E_z$$

$$J=J_{L\to R}+J_{R\to L}$$

$$=(-e)2\times\int\frac{dk_x dk_y}{(2\pi)^2}\int_0^{\infty}\frac{dk_z}{2\pi}\,v_{cz}(k_z)\;T_c\;f\left(E_c(\vec{k})-E_{fL}\right)\left[1-f\left(E_c(\vec{k})-E_{fR}\right)\right]$$

$$+(-e)2\times\int\frac{dk_x dk_y}{(2\pi)^2}\int_{-\infty}^{0}\frac{dk_z}{2\pi}\,v_{cz}(k_z)\;T_c\;f\left(E_c(\vec{k})-E_{fR}\right)\left[1-f\left(E_c(\vec{k})-E_{fL}\right)\right]$$

$$=(-e)2\times\int\frac{dk_x dk_y}{(2\pi)^2}\int_0^{\infty}\frac{dk_z}{2\pi}\,v_{cz}(k_z)\;T_c\;\left[f\left(E_c(\vec{k})-E_{fL}\right)-f\left(E_c(\vec{k})-E_{fR}\right)\right]$$

$$=-e\,\frac{m_eT_c}{2\pi^2\hbar^3}\int_0^{\infty}dE_{xy}\int_0^{\infty}dE_z\left[f\left(E_c+E_{xy}+E_z-E_{fL}\right)-f\left(E_c+E_{xy}+E_z-E_{fR}\right)\right]$$

$$=e^2\,\frac{m_eT_c}{2\pi^2\hbar^3}\left(\frac{E_{fL}+E_{fR}}{2}-E_c\right)V=GV$$

at T~0K