Handout 24

The Effective Mass Theorem and the Effective Mass Schrodinger Equation

In this lecture you will learn:

- Electron states in crystals with weak potential perturbations
- The effective mass theorem
- The effective mass Schrodinger equation
- The donor and acceptor impurity levels in crystals

G. H. Wannier, Phys. Rev., 52, 191 (1937).
J. C. Slater, Phys. Rev., 76, 1592 (1949).
J. M. Luttinger and W. Kohn, Phys. Rev., 97, 869 (1955).

Perturbed Electrons in Energy Bands

1) The quantum states of an electron in a crystal are given by Bloch functions that obey the Schrodinger equation:

$$\hat{H}\,\psi_{n,\vec{k}}(\vec{r}) = E_n(\vec{k})\,\psi_{n,\vec{k}}(\vec{r})$$

where the wavevector $\vec{k}$ is confined to the FBZ and "*n*" is the band index

2) Under a lattice translation, Bloch functions obey the relation:

$$\psi_{n,\vec{k}}(\vec{r}+\vec{R}) = e^{i\,\vec{k}\,.\,\vec{R}}\,\psi_{n,\vec{k}}(\vec{r})$$

Now we ask the following question: if an external potential is added to the crystal Hamiltonian,

$$\hat{H} + U(\vec{r})$$

then what happens? How do the electrons behave? How do we find the new energies and eigenstates?

$$\left[\,\hat{H} + U(\vec{r})\,\right]\psi(\vec{r}) = E\,\psi(\vec{r})$$

The external potential could represent, for example, an applied E-field or an applied B-field, or potentials due to impurity atoms, or inhomogeneous nanostructures

Some Preliminaries

Statement of problem: Need to solve,

$$\left[\hat{H} + U(\vec{r})\right]\psi(\vec{r}) = E\,\psi(\vec{r})$$

As always, we will start from a completely different point to solve the problem stated above

Recall that the energy bands are lattice-periodic in the reciprocal space,

$$E_n\left(\vec{k}+\vec{G}\right) = E_n\left(\vec{k}\right)$$

When a function in real space is lattice-periodic, we can expand it in a Fourier series,

$$V\left(\vec{r}+\vec{R}\right) = V(\vec{r}) \quad \Rightarrow \quad V(\vec{r}) = \sum_j V\left(\vec{G}_j\right) e^{i\,\vec{G}_j\,.\,\vec{r}}$$

⇒ When a function is lattice-periodic in reciprocal space, we can also expand it in Fourier series of the form,

$$E_n\left(\vec{k}+\vec{G}\right) = E_n\left(\vec{k}\right) \quad \Rightarrow \quad E_n\left(\vec{k}\right) = \sum_j E_n\left(\vec{R}_j\right) e^{i\,\vec{R}_j\,.\,\vec{k}}$$

A Brief Review

$$E_n\left(\vec{k}+\vec{G}\right) = E_n\left(\vec{k}\right) \quad \Rightarrow \quad E_n\left(\vec{k}\right) = \sum_j E_n\left(\vec{R}_j\right) e^{i\,\vec{R}_j\,.\,\vec{k}}$$

Recall the operator:

$$\hat{E}_n(-i\nabla) = \sum_j E_n\left(\vec{R}_j\right) e^{\vec{R}_j\,.\,\nabla}$$

When we apply this operator to a Bloch function from the same band (i.e. the *n*-th band) we got:

$$\begin{aligned}
\hat{E}_n(-i\nabla)\,\psi_{n,\vec{k}}(\vec{r}) &= \sum_j E_n\left(\vec{R}_j\right) e^{\vec{R}_j\,.\,\nabla}\;\psi_{n,\vec{k}}(\vec{r}) \\
&= \sum_j E_n\left(\vec{R}_j\right)\;\psi_{n,\vec{k}}\left(\vec{r}+\vec{R}_j\right) \\
&= \sum_j E_n\left(\vec{R}_j\right) e^{i\,\vec{k}\,.\,\vec{R}_j}\;\psi_{n,\vec{k}}(\vec{r}) \\
&= E_n\left(\vec{k}\right)\;\psi_{n,\vec{k}}(\vec{r})
\end{aligned}$$

The result above implies that the action of the operator $\hat{E}_n(-i\nabla)$ on a Bloch function belonging to the same band is that of the Hamiltonian!

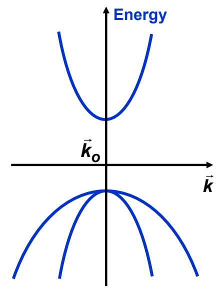
$$E_n(-i\nabla)\,\psi_{n,\vec{k}}(\vec{r}) = \hat{H}\,\psi_{n,\vec{k}}(\vec{r}) = E_n\left(\vec{k}\right)\psi_{n,\vec{k}}(\vec{r})$$

Solution Strategy

Now we come back to the problem:

$$\left[\hat{H} + U(\vec{r})\right]\psi(\vec{r}) = E\,\psi(\vec{r})$$

We want to see how the Bloch function $\psi_{n,\vec{k}_o}(\vec{r})$ is perturbed by the potential.



We write the solution as a superposition using Bloch functions from the same *n*-th band :

$$\psi(\vec{r}) = \sum_{\vec{k}\text{ near }\vec{k}_o} c(\vec{k})\,\psi_{n,\vec{k}}(\vec{r})$$

to get,

$$\left[\hat{H} + U(\vec{r})\right]\psi(\vec{r}) = E\,\psi(\vec{r})$$

$$\Rightarrow \left[\hat{E}_n(-i\nabla) + U(\vec{r})\right]\psi(\vec{r}) = E\,\psi(\vec{r})$$

where we have replaced the Hamiltonian operator by $\hat{E}_n(-i\nabla)$

We are seeking a solution near a particular point $\vec{k}_o$ in k-space. For example, near a band extremum. For $\vec{k}$ near $\vec{k}_o$ we can approximate all Bloch functions as,

$$\psi_{n,\vec{k}}(\vec{r}) = e^{i\,\vec{k}\,.\,\vec{r}}\,u_{n,\vec{k}}(\vec{r}) \approx e^{i\,\vec{k}\,.\,\vec{r}}\,u_{n,\vec{k}_o}(\vec{r}) = e^{i\,(\vec{k}-\vec{k}_o)\,.\,\vec{r}}\,\psi_{n,\vec{k}_o}(\vec{r})$$

The Envelope Function

$$\begin{aligned}\psi(\vec{r}) &= \sum_{\vec{k}\text{ near }\vec{k}_o} c(\vec{k})\,\psi_{n,\vec{k}}(\vec{r})\\ &= \sum_{\vec{k}\text{ near }\vec{k}_o} c(\vec{k})\,e^{i\,(\vec{k}-\vec{k}_o)\,.\,\vec{r}}\,\psi_{n,\vec{k}_o}(\vec{r})\\ &= \left[\sum_{\vec{k}\text{ near }\vec{k}_o} c(\vec{k})\,e^{i\,(\vec{k}-\vec{k}_o)\,.\,\vec{r}}\right]\psi_{n,\vec{k}_o}(\vec{r})\\ &= \phi(\vec{r})\;\psi_{n,\vec{k}_o}(\vec{r})\end{aligned}$$

The above expression shows that we are approximating the solution as a product of a Bloch function and another (unknown) function $\phi(\vec{r})$ which is called the envelope function. By construction the envelope function is slowly varying in space (on atomic scale).

We use the above form of the solution in the equation,

$$\left[\hat{E}_n(-i\nabla) + U(\vec{r})\right]\phi(\vec{r})\,\psi_{n,\vec{k}_o}(\vec{r}) = E\,\phi(\vec{r})\,\psi_{n,\vec{k}_o}(\vec{r})$$

First we look at:

$$\hat{E}_n(-i\nabla)\;\phi(\vec{r})\,\psi_{n,\vec{k}_o}(\vec{r})$$

The Effective Mass Schrodinger Equation

$$\hat{E}_n(-i\nabla)\,\phi(\vec{r})\,\psi_{n,\vec{k}_o}(\vec{r}) = \sum_j E_n(\vec{R}_j)\,e^{\vec{R}_j\cdot\nabla}\,\phi(\vec{r})\,\psi_{n,\vec{k}_o}(\vec{r})$$
$$= \sum_j E_n(\vec{R}_j)\,\phi(\vec{r}+\vec{R}_j)\,\psi_{n,\vec{k}_o}(\vec{r}+\vec{R}_j)$$
$$= \sum_j E_n(\vec{R}_j)\,e^{i\,\vec{k}_o\cdot\vec{R}_j}\,\phi(\vec{r}+\vec{R}_j)\,\psi_{n,\vec{k}_o}(\vec{r})$$
$$= \psi_{n,\vec{k}_o}(\vec{r})\sum_j E_n(\vec{R}_j)\,e^{i\,\vec{k}_o\cdot\vec{R}_j}\,\phi(\vec{r}+\vec{R}_j)$$
$$= \psi_{n,\vec{k}_o}(\vec{r})\sum_j E_n(\vec{R}_j)\,e^{i\,\vec{k}_o\cdot\vec{R}_j}\,e^{\vec{R}_j\cdot\nabla}\,\phi(\vec{r})$$
$$= \psi_{n,\vec{k}_o}(\vec{r})\sum_j E_n(\vec{R}_j)\,e^{i\,(\vec{k}_o-i\nabla)\cdot\vec{R}_j}\,\phi(\vec{r})$$
$$= \psi_{n,\vec{k}_o}(\vec{r})\,\hat{E}_n(\vec{k}_o-i\nabla)\,\phi(\vec{r})$$

This implies:

$$\left[\hat{E}_n(-i\nabla)+U(\vec{r})\right]\phi(\vec{r})\,\psi_{n,\vec{k}_o}(\vec{r}) = E\,\phi(\vec{r})\,\psi_{n,\vec{k}_o}(\vec{r})$$
$$\Rightarrow \psi_{n,\vec{k}_o}(\vec{r})\left[\hat{E}_n(\vec{k}_o-i\nabla)+U(\vec{r})\right]\phi(\vec{r}) = E\,\phi(\vec{r})\,\psi_{n,\vec{k}_o}(\vec{r})$$

The Effective Mass Theorem

Finally we have the following equation for the envelope function:

$$\left[\hat{E}_n(\vec{k}_o-i\nabla)+U(\vec{r})\right]\phi(\vec{r}) = E\,\phi(\vec{r})$$

The effective mass theorem states the following:

a) In the presence of a weak perturbing potential the solution for electron states near $\vec{k}_o$ in k-space can be represented as a product of a slowly varying envelope function and a Bloch function

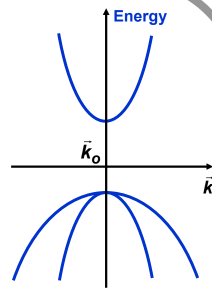
$$\psi(\vec{r}) = \phi(\vec{r})\;\psi_{n,\vec{k}_o}(\vec{r})$$

b) The slowly varying envelope function obeys the effective mass Schrodinger equation:

$$\left[\hat{E}_n(\vec{k}_o-i\nabla)+U(\vec{r})\right]\phi(\vec{r}) = E\,\phi(\vec{r})$$

The Envelope Function

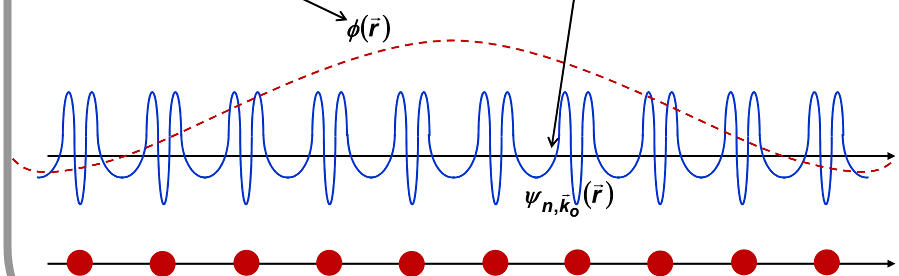
$$\left[\hat{E}_n\left(\vec{k}_o - i\nabla\right) + U(\vec{r})\right]\phi(\vec{r}) = E\,\phi(\vec{r})$$



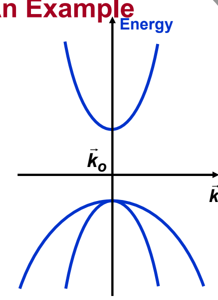
Electron wavefunction → $\psi(\vec{r}) = \phi(\vec{r})\;\psi_{n,\vec{k}_o}(\vec{r})$

Slowly varying envelope function

Bloch function



The Effective Mass Schrodinger Equation: An Example



Consider a conduction energy band with the dispersion:

$$E_c(\vec{k}) = E_c + \frac{\hbar^2(k_x - k_{ox})^2}{2m_{xx}} + \frac{\hbar^2(k_y - k_{oy})^2}{2m_{yy}} + \frac{\hbar^2(k_z - k_{oz})^2}{2m_{zz}}$$

Now suppose an external potential $U(\vec{r})$ is present. The electron states near the conduction band bottom in the presence of the external potential are described by the effective mass equation:

$$\left[\hat{E}_c\left(\vec{k}_o - i\nabla\right) + U(\vec{r})\right]\phi(\vec{r}) = E\,\phi(\vec{r})$$

Note that one has to make the following replacements in the energy dispersion relation:

$$E_c(\vec{k}) \rightarrow \hat{E}_c\left(\vec{k}_o - i\nabla\right) \quad \Rightarrow \quad \boxed{k_x \rightarrow k_{ox} - i\frac{\partial}{\partial x} \qquad k_y \rightarrow k_{oy} - i\frac{\partial}{\partial y} \qquad k_z \rightarrow k_{oz} - i\frac{\partial}{\partial z}}$$

The operator $\hat{E}_c\left(\vec{k}_o - i\nabla\right)$ is then:

$$E_c\left(\vec{k}_o - i\nabla\right) = E_c - \frac{\hbar^2}{2m_{xx}}\frac{\partial^2}{\partial x^2} - \frac{\hbar^2}{2m_{yy}}\frac{\partial^2}{\partial y^2} - \frac{\hbar^2}{2m_{zz}}\frac{\partial^2}{\partial z^2}$$

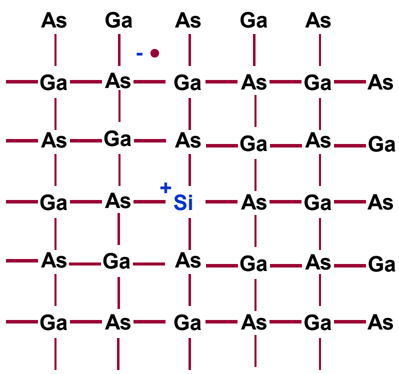
The effective mass Shrodinger equation becomes:

$$\left[-\frac{\hbar^2}{2m_{xx}}\frac{\partial^2}{\partial x^2} - \frac{\hbar^2}{2m_{yy}}\frac{\partial^2}{\partial y^2} - \frac{\hbar^2}{2m_{zz}}\frac{\partial^2}{\partial z^2} + E_c + U(\vec{r})\right]\phi(\vec{r}) = E\,\phi(\vec{r})$$

Donor Impurities in Semiconductors

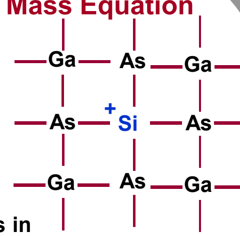
One of the earliest applications of the effective mass theorem was the donor and acceptor impurity states and energy levels in semiconductors

Consider a semiconductor (say GaAs) in which one Ga atom site is occupied by a Si atom, as shown:



- Silicon has one more electron in the outermost shell compared to Ga (4 in Si compared to 3 in Ga)
- Since only 3 electrons are needed to form co-valent bonds with the nearby As atoms, the extra electron does not participate in bonding and can drift away leaving behind a positively charged Si atom

Donor Impurities in Semiconductors: Effective Mass Equation

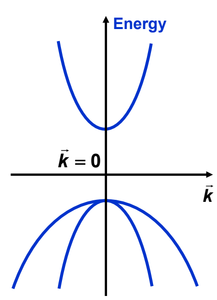


The positively charged Si atoms presents a Coulomb potential to the lattice. Therefore the potential energy is:

Attractive positive potential: $$U(\vec{r}) = -\frac{e^2}{4\pi\,\varepsilon_s\,|\vec{r}|}$$

We need to figure out how the electron states and energy levels in the conduction band are modified because of this Coulomb potential

$$\left[\hat{H} + U(\vec{r})\right]\psi(\vec{r}) = E\,\psi(\vec{r})$$



We are interested in how the states near the conduction band bottom get modified, so we assume

$$\psi(\vec{r}) = \phi(\vec{r})\;\psi_{c,\vec{k}_o=0}(\vec{r})$$

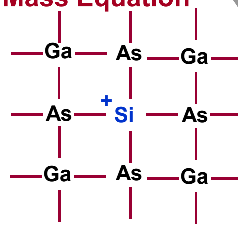
And we know that the envelope function satisfies the effective mass Schrodinger equation

$$\left[\hat{E}_c\left(\vec{k}_o - i\nabla\right) + U(\vec{r})\right]\phi(\vec{r}) = E\,\phi(\vec{r})$$

Donor Impurities in Semiconductors: Effective Mass Equation

We seek a solution near the conduction band bottom at $\vec{k}_o = 0$:

$$\Rightarrow \left[\hat{E}_c(-i\nabla) + U(\vec{r})\right]\phi(\vec{r}) = E\,\phi(\vec{r})$$



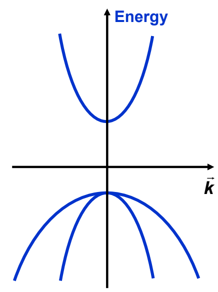
The conduction band dispersion in GaAs implies:

$$E_c(\vec{k}) = E_c + \frac{\hbar^2 k^2}{2m_e} \quad \Rightarrow \quad E_c(\vec{k}_o - i\nabla) = E_c(-i\nabla) = E_c - \frac{\hbar^2\nabla^2}{2m_e}$$

So we get the equation:

$$\left[E_c - \frac{\hbar^2\nabla^2}{2m_e} - \frac{e^2}{4\pi\,\varepsilon_s\,r}\right]\phi(\vec{r}) = E\,\phi(\vec{r})$$

$$\Rightarrow \left[-\frac{\hbar^2\nabla^2}{2m_e} - \frac{e^2}{4\pi\,\varepsilon_s\,r}\right]\phi(\vec{r}) = (E - E_c)\,\phi(\vec{r})$$



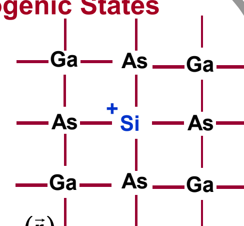
The above equation looks like the Schrodinger equation for an electron in a hydrogen atom with the exceptions that:

i)The mass is the effective mass m_e instead of the free-electron mass m

ii)The dielectric constant is ε_s instead of ε_o

Donor Impurities in Semiconductors: Hydrogenic States

$$\left[-\frac{\hbar^2\nabla^2}{2m_e} - \frac{e^2}{4\pi\,\varepsilon_s\,r}\right]\phi(\vec{r}) = (E - E_c)\,\phi(\vec{r})$$



Solutions are:

$$\phi(\vec{r}) = \phi_{n\ell m}(\vec{r}) = R_{n\ell}(r)\,Y_{\ell m}(\theta,\phi)$$

Remember that the actual wavefunction is: $\psi(\vec{r}) = \phi(\vec{r})\;\psi_{c,\vec{k}_o=0}(\vec{r})$

Where:

1) n is a positive integer ≥ 1 $\quad (n = 1,2,\ldots\ldots)$
2) ℓ is a postive integer $< n$ $\quad (\ell = 0,1,2,\ldots\ldots(n-1)$ for s, p, d, f, ...)
3) m is an integer such that $|m| \leq \ell$ $\quad (m = -\ell,\ldots -1,0,+1,\ldots. + \ell)$

The corresponding energy eigenvalues are:

$$E - E_c = -\frac{E_o}{n^2} \longrightarrow n = 1,2,3\ldots\ldots$$

$$\Rightarrow \quad E = E_c - \frac{E_o}{n^2}$$

$$E_o = \frac{m_e}{2\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_s}\right)^2$$

Donor Impurities in Semiconductors: Hydrogenic States

$$E = E_c - \frac{E_o}{n^2} \longrightarrow n = 1,2,3......$$

$$E_o = \frac{m_e}{2\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_s}\right)^2 = (13.6\text{ eV})\left(\frac{m_e}{m}\right)\left(\frac{\varepsilon_o}{\varepsilon_s}\right)^2$$

Ground state (lowest energy state):

$$\Rightarrow \quad n = 1 \quad \ell = 0 \quad m = 0$$

$$E = E_c - E_o$$

$$\phi_{1s}(\vec{r}) = \phi_{n=1\,\ell=0\,m=0}(\vec{r}) = \frac{1}{\sqrt{\pi a_o^3}}\, e^{-r/a_o}$$

$$a_o = \left(\frac{4\pi\varepsilon_s}{e^2}\right)\frac{\hbar^2}{m_e} = (0.53\text{ A})\left(\frac{\varepsilon_s}{\varepsilon_o}\right)\left(\frac{m}{m_e}\right)$$

Effective Bohr radius

In GaAs: $m_e = .067\, m$ and $\varepsilon_s = 12.4\, \varepsilon_o$

$$E = E_c - E_o = E_c - (13.6\text{ eV})\left(\frac{m_e}{m}\right)\left(\frac{\varepsilon_o}{\varepsilon_s}\right)^2 \approx E_c - 5.9\text{ meV}$$

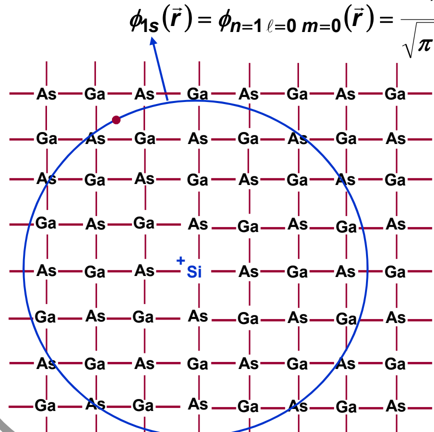
$$a_o = (0.53\text{ A})\left(\frac{\varepsilon_s}{\varepsilon_o}\right)\left(\frac{m}{m_e}\right) \approx 98\text{ A} \longrightarrow \textbf{Very large!}$$

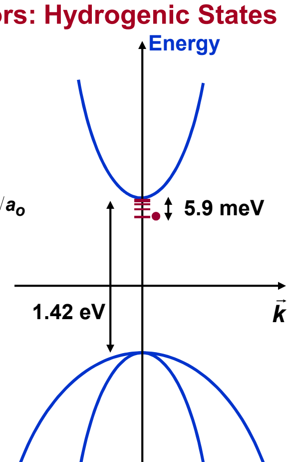
Donor Impurities in Semiconductors: Hydrogenic States

$$E = E_c - \frac{(13.6\text{ eV})}{n^2}\left(\frac{m_e}{m}\right)\left(\frac{\varepsilon_o}{\varepsilon_s}\right)^2 \approx E_c - \frac{5.9\text{ meV}}{n^2}$$

$$a_o = (0.53\text{ A})\left(\frac{\varepsilon_s}{\varepsilon_o}\right)\left(\frac{m}{m_e}\right) \approx 98\text{ A}$$

$$\phi_{1s}(\vec{r}) = \phi_{n=1\,\ell=0\,m=0}(\vec{r}) = \frac{1}{\sqrt{\pi a_o^3}}\, e^{-r/a_o}$$

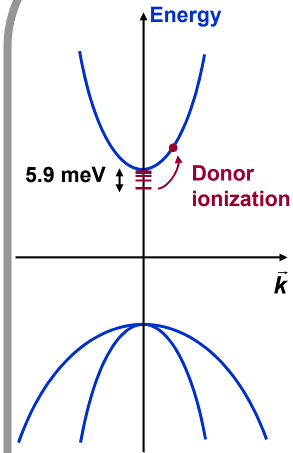




The positively charge donor atoms create new quantum states whose energies are slightly below the conduction band edge and whose wavefunctions are localized near the donor atom

$$\psi(\vec{r}) = \phi_{1s}(\vec{r})\ \psi_{c,\vec{k}_o=0}(\vec{r})$$

Donor Impurities in Semiconductors: N-Type Doping



• At very low temperatures the electron resides in the donor energy level and the donor atom is neutral

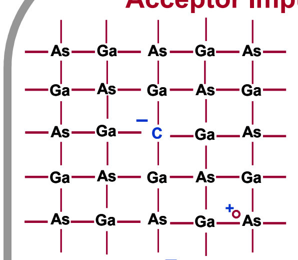
• At room temperature, the electron in the donor energy level can acquire enough energy to jump to the conduction band

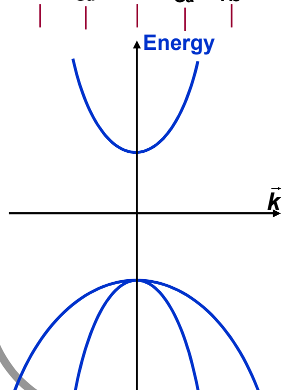
When this happens the donor is said to have ionized

• Once in the conduction band the electron can move around and is no longer localized at the donor atom

• Donor impurities can therefore be used to dope semiconductors n-type

Acceptor Impurities in Semiconductors: P-Type Doping





Consider a semiconductor (say GaAs) in which one As atom site is occupied by a carbon atom, as shown:

• C has one less electron in the outermost shell compared to As (4 in C compared to 5 in As)

• Since 4 electrons are needed to form covalent bonds with the nearby Ga atoms, the required electron is taken from the valence band resulting in a negatively charged C atom and a hole in the valence band

Solution: $\psi(\vec{r}) = \phi(\vec{r})\ \psi_{hh,\vec{k}_o=0}(\vec{r})$

$$\Rightarrow \left[\hat{E}_{hh}(-i\nabla) + U(\vec{r})\right]\phi(\vec{r}) = E\ \phi(\vec{r})$$

Negative repulsive potential: $U(\vec{r}) = +\dfrac{e^2}{4\pi\,\varepsilon_s\,|\vec{r}|}$

hh-band dispersion: $E_{hh}(\vec{k}) = E_v - \dfrac{\hbar^2 k^2}{2m_{hh}}$

$$\Rightarrow \quad E_{hh}(-i\nabla) = E_v + \frac{\hbar^2\nabla^2}{2m_{hh}}$$

Acceptor Impurities in Semiconductors: P-Type Doping

The effective mass Schrodinger equation becomes:

$$\left[E_v + \frac{\hbar^2 \nabla^2}{2 m_{hh}} + \frac{e^2}{4\pi\, \varepsilon_s |\vec{r}|} \right] \phi(\vec{r}) = E\, \phi(\vec{r})$$

Rearrange:

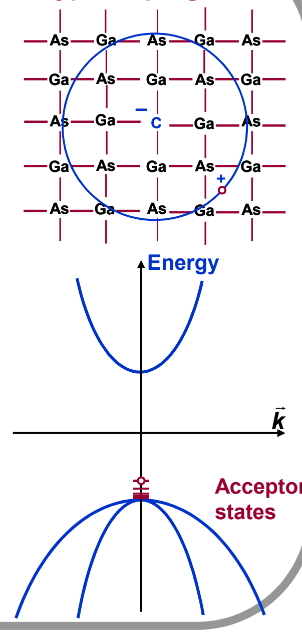
$$\left[-\frac{\hbar^2 \nabla^2}{2 m_{hh}} - \frac{e^2}{4\pi\, \varepsilon_s |\vec{r}|} \right] \phi(\vec{r}) = \left(-E + E_v\right) \phi(\vec{r})$$

Again we end up with a Schrodinger-like equation for a Hydrogen atom which has the solution:

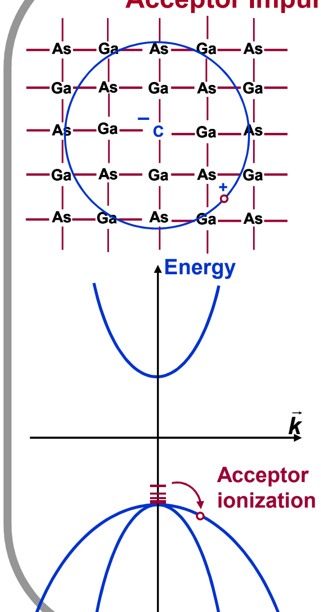
$$\phi(\vec{r}) = \phi_{n\ell m}(\vec{r}) = R_{n\ell}(r) Y_{\ell m}(\theta, \phi)$$

$$-E + E_v = -\frac{E_o}{n^2} \quad\Rightarrow\quad E = E_v + \frac{E_o}{n^2}$$

$$E_o = \frac{m_{hh}}{2\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_s}\right)^2 = \left(13.6\ \text{eV}\right)\left(\frac{m_{hh}}{m}\right)\left(\frac{\varepsilon_o}{\varepsilon_s}\right)^2$$

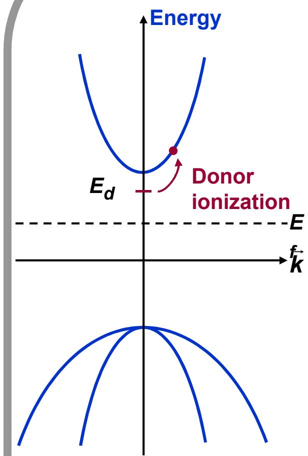


Acceptor Impurities in Semiconductors: P-Type Doping



- Acceptor atom gives rise to hydrogenic energy levels near the valence band maximum
- At very low temperatures the hole resides in the acceptor energy level and the acceptor atom location is overall neutral
- At room temperature, the hole in the acceptor energy level can acquire enough energy to jump to the valence band
  When this happens the acceptor is said to have ionized
- Once in the valence band the hole can move around and is no longer localized at the acceptor atom
- Acceptor impurities can therefore be used to dope semiconductors p-type

Donor Ionization Statistics



In the grand canonical ensemble the probability of a system to have total particles N and total energy E is:

$$P(N,E) = A\, e^{-(E-E_f N)/KT}$$

The donor level can have the following possible states:

1) No electrons present

$$P(N=0, E=0) = A$$

2) One spin-up electron present

$$P(N=1, E=E_d) = A\, e^{-(E_d-E_f)/KT}$$

3) One spin-down electron present

$$P(N=1, E=E_d) = A\, e^{-(E_d-E_f)/KT}$$

4) Two or more electrons present

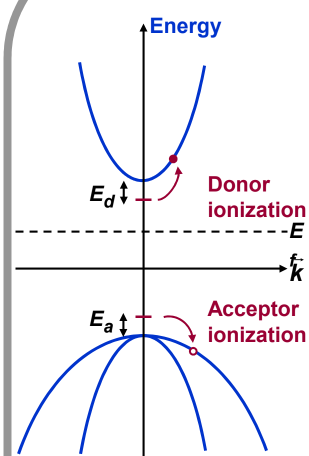
$$P(N>1, E) = 0$$

→ Coulomb repulsion does not allow it

Sum of all probabilities should equal unity:

$$\Rightarrow A\left[1+2\,e^{-(E_d-E_f)/KT}\right] = 1 \quad \Rightarrow \quad A = \frac{1}{1+2\,e^{-(E_d-E_f)/KT}}$$

Donor Ionization Statistics



Probability that the donor level is ionized = Probability that the donor level has no electrons

$$= P(N=0, E=0)$$

$$= A$$

$$= \frac{1}{1+2\,e^{-(E_d-E_f)/KT}}$$

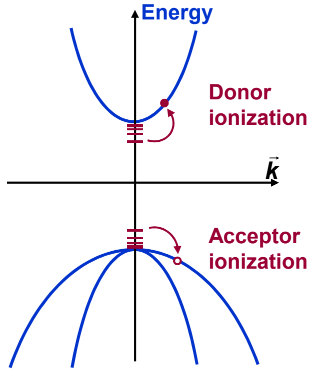
If the total donor impurity concentration is N_d then the concentration of ionized donors N^+_d is equal to:

$$N_d^+ = \frac{N_d}{1+2\,e^{-(E_d-E_f)/KT}}$$

For acceptors we have a similar relation:

$$N_a^- = \frac{N_a}{1+2\,e^{(E_a-E_f)/KT}}$$

Carrier Statistics in Doped Semiconductors



Consider a semiconductor that is doped with both donor and acceptor impurity atoms

- The total charge must be zero:

$$N_d^+ - N_a^- + p - n = 0$$

The above equation can be used to find the position of the equilibrium Fermi level since every term depends on the Fermi level position (one equation in one unknown)

$$N_d^+ = \frac{N_d}{1 + 2\, e^{-(E_d - E_f)/KT}}$$

$$N_a^- = \frac{N_a}{1 + 2\, e^{(E_a - E_f)/KT}}$$