ECE 4070/MSE 5470
Physics of Semiconductors and Nanostructures
Final Exam, May 18, 2015

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Instructions:

- There are **FOUR** problems in this exam
- Every problem must be done in the blue booklet
- Only work done on the blue exam booklets will be graded. Do not attach your own sheets to the exam booklets under any circumstances
- To get partial credit you must show all the relevant work
- Correct answers with wrong reasoning will not get points
- All questions do not carry equal points
- All questions do not have the same level of difficulty, use your time judiciously

DO NOT WRITE IN THIS SPACE

1 Miscellaneous [30 points]

Answer the following questions **briefly**.

(a) Why is a typical metal reflective and opaque?

(b) Electrons in a quantum wire move in 1-dimension. Argue why they can be more resistant to *elastic* scattering than if the motion was in higher dimensions.

(c) How does sound velocity change from Germanium to Silicon to Diamond - all of the same crystal structure?

(d) We saw that even in the absence of scattering in a ballistic transistor, the current saturates. Why?

(e) Sketch the typical optical absorption processes in $E(k) - k$ diagrams for a direct-bandgap and an indirect-bandgap semiconductor. Also sketch the corresponding absorption spectra.

(f) Though both photons and phonons are modeled as bosons, it has proven much more difficult to make phonon lasers than photon lasers. Why?

2 Bandstructure and Ballistic Transport [30 points]

A fictitious 2D semiconductor has an energy dispersion $E(\mathbf{k}) = \pm\hbar v_F\sqrt{|\mathbf{k}|^2 + k_0^2}$, where $\mathbf{k} = (k_x, k_y)$, $\hbar$ is the reduced Planck's constant, v_F a characteristic velocity, and k_0 is a constant. Assume a spin degeneracy g_s and valley degeneracy g_v.

(a) Find the energy bandgap E_g, and show that the effective mass m_{xx} at the band edge is related to the bandgap by $2m_{xx}v_F^2 = E_g$.

(b) Find and sketch the density of states.

(c) Find the group velocity $\mathbf{v}_g(\mathbf{k})$ of the state $\mathbf{k}$. Sketch the magnitude and direction of the group velocities in the $\mathbf{k}-$space.

(d) Now the conduction band of this 2D semiconductor sheet is connected by ohmic contacts to a source and a drain. A voltage V is applied across these two terminals. Set up the expressions that will give the ballistic current in response to the applied voltage.

(e) The temperature is held very low, $T \to 0$ K. Estimate the ballistic current as a function of voltage if $qV >> \hbar v_F k_0$.

3 Acoustic phonon scattering in different dimensions [25 points]

We have investigated the scattering of electrons by acoustic phonons though the deformation potential mechanisms. The end results on the effect of mobility appear as $\mu = \frac{e\langle\tau\rangle}{m^\star}$, where $\langle\tau\rangle$ is the ensemble-averaged scattering time.

In this problem, we will investigate the effect of dimensionality on the temperature dependence of acoustic phonon scattering. The acoustic phonon scattering potential is $W(\mathbf{r}, t) = D_c \nabla \cdot \mathbf{u}(\mathbf{r}, t)$, where D_c is the deformation potential, $\mathbf{u}(\mathbf{r}, t) = u_0 e^{i(\mathbf{q}\cdot\mathbf{r} - \omega_q t)}\hat{\mathbf{n}}$ is the vibration of an atom from its equilibrium position at $\mathbf{r}$ at time t in the direction $\hat{\mathbf{n}}$.

(a) Fermi's golden rule states the scattering rate $\frac{1}{\tau(\mathbf{k}\to\mathbf{k}')} = \frac{2\pi}{\hbar}|\langle\mathbf{k}'|W(\mathbf{r})|\mathbf{k}\rangle|^2\delta(E_k - E_{k'} \pm \hbar\omega_q)$. Write out the matrix elements for electron transport in various dimensions $d = 3, 2, 1$. Show that it is proportional to u_0^2.

(b) Argue why $u_0^2 \propto \frac{k_B T}{\hbar\omega}$ for $\hbar\omega << k_B T$.

(c) Show that for low energy acoustic phonons, summing the scattering over all allowed states $\sum_{\mathbf{k}'} \frac{1}{\tau(\mathbf{k}\to\mathbf{k}')}$ yields that the scattering rate is proportional to the band density of states $\langle\frac{1}{\tau(k)}\rangle \propto g_d(E_k)$ for d-dimensions.

(d) For non-degenerate electron distributions, the kinetic energy $E_k \approx k_B T$ to an excellent approximation. Using (b) and (c), and assuming $\langle\frac{1}{\tau}\rangle \approx \frac{1}{\langle\tau\rangle}$, show that we get the dependence $\mu \propto T^{-\frac{3}{2}}$ for $d = 3$ dimensions. You did this in the assignments by a more detailed technique.

(e) Now find the temperature dependence of acoustic phonon scattering for lower dimensions: for $d = 2$ and $d = 1$.

4 Photonic Properties of Semiconductor Nanostructures [15 points]

Consider a direct bandgap 1D semiconductor with a parabolic conduction band structure $E_c(k) = E_c + \frac{\hbar^2 k^2}{2m_c^\star}$, but a very heavy valence band with no dispersion $E_v(k) = E_v$, and an energy bandgap $E_c - E_v = E_g$. At equilibrium, the valence band states are filled and the conduction band states are empty.

(a) Sketch an $E(k) - k$ diagram for the electron energies in the 1D semiconductor.

(b) Photons of energy $\hbar\omega > E_g$ are incident on the 1D semiconductor. Sketch on your answer to part (a) which states will be involved in optical transitions.

(c) Explain the microscopic processes occurring in steady state - when the photon source is kept on.

(d) If the source of photons is suddenly turned off, describe the processes that occur thereafter.