These pages include virtually all Quiz, Midterm, and Final Examination questions I have used in M&AE 5070 over the years. Note that some of them involve concepts in Control that we are only beginning to study in class (and for which you will not be responsible on Quiz B).

The final page is a Summary of the equations of longitudinal and lateral/directional motions that I will provide for your reference in Quiz B.

For the following questions circle either **T** or **F**, depending upon whether the statement is true or false.

T **F** **1.** Static stability is a *necessary* condition for a vehicle eventually to return to an equilibrium state following an arbitrary infinitesimal perturbation.

T **F** **2.** Static stability is a *sufficient* condition for a vehicle eventually to return to an equilibrium state following an arbitrary infinitesimal perturbation.

T **F** **3.** An airfoil section having conventional subsonic camber (i.e., concave down) will have a positive pitching moment at zero lift.

T **F** **4.** A flight vehicle having positive $\partial \mathbf{C}_{mcg}/\partial\alpha$ will be statically stable to longitudinal perturbations (with controls fixed).

T **F** **5.** For a given center-of-gravity location, an aircraft with a forward tail (canard) usually has less static longitudinal stability when the controls are freed than when they are fixed.

T **F** **6.** The lift-curve slopes of aircraft are reduced when the controls are free, relative to when the controls are fixed, regardless of whether the tail is forward or aft of the wing.

T **F** **7.** A large-aspect-ratio, forward-swept wing having symmetrical sections must be twisted such that the angle of attack at the tips is less than that at the root in order for it to have a positive pitching moment at zero lift.

T **F** **8.** All other geometric factors being equal, the contribution of the horizontal tail to the total lift-curve slope C_{L_α} for a vehicle will be larger for a forward tail (canard) than for an aft tail.

T **F** **9.** *Forward* wing sweep contributes to unstable dihedral effect (i.e., to positive $\mathbf{C}_{l\beta}$).

T **F** **10.** Wing sweepback contributes to stable weathercock stability.

T **F** **11.** The control-free neutral point is always *ahead* of the basic neutral point.

T **F** **12.** The unforced response of a flight vehicle to a purely longitudinal perturbation from a state of longitudinal trim usually consists of two oscillatory modes.

T **F** **13.** The unforced response of a flight vehicle to a purely lateral/directional perturbation from a state of longitudinal trim usually consists of two oscillatory modes.

T **F** **14.** The long-period (phugoid) longitudinal mode is more heavily damped near the drag-divergence Mach number (where compressibility effects are important), than at low speeds.

T **F** **15.** Changes in drag associated with changes in airspeed cause the short-period longitudinal mode to be relatively heavily damped.

T **F** **16.** Dynamic stability for a linear, time-invariant system is guaranteed if all the roots of the characteristic equation of the plant matrix have negative real parts.

T **F** **17.** The response of a linear, time-invariant system to an *impulsive* forcing at time $t = 0$ is equivalent to the response to an appropriately perturbed initial condition.

Circle the letters corresponding to *as many* answers as are correct for the following question:

18. When a vehicle is trimmed (longitudinally) with the controls free, which of the following statements is/are always true?

a. The net moment about the control-free neutral point is zero;

b. The net hinge-moment about the elevator hinge line is zero;

c. The center-of-gravity is ahead of the control-free neutral point;

d. None of the above.

Identify each stability derivative in the first column with its corresponding symbol from the second column.

19. _____	Pitch stiffness	$\mathbf{C}_{np}$
20. _____	Dihedral effect	$\mathbf{C}_{nr}$
21. _____	Yawing moment due to roll rate	$\mathbf{C}_{l\beta}$
22. _____	Pitch damping	$\mathbf{C}_{lr}$
23. _____	Yaw damping	$\mathbf{C}_{mq}$
24. _____	Rolling moment due to yaw rate	$\mathbf{C}_{n\beta}$
25. _____	Weathercock stability	$\mathbf{C}_{m\alpha}$

26. The tail efficiency factor η can be either less than, or greater than, unity. Explain what factor(s) might cause η to be greater than unity. Explain what factor(s) might cause η to be less than unity.

27. Explain, briefly, why the floating tendency of a horizontal tail plane, pivoted at its aerodynamic center, is zero.

28. Of the effects of the fuselage on static longitudinal stability listed below, the *least* important is

a. The forward shift in the basic neutral point;

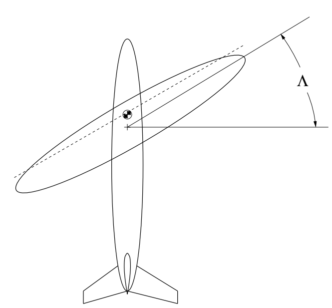
b. The positive shift in pitching moment coefficient at zero lift;

c. The increase in the lift-curve slope C_{L_α} of the configuration.

29. How would you expect the value of the angle-of-attack damping stability derivative $C_{m_{\dot\alpha}}$ to be different (if at all) for a canard configuration than for a conventional aft-tailed configuration?

30. How would you expect the value of the *pitch damping* stability derivative to be different (if at all) for a canard configuration than for a conventional aft-tailed configuration? Discuss both the sign and the magnitude of the derivative, assuming the same magnitude of tail volume coefficient $|V_H|$ for both cases.

31. One proposal for an efficient transonic airliner uses a *yawed wing*, swept uniformly through the angle Λ so that the right wing is swept forward while the left wing is swept back, as illustrated in the sketch. One disadvantage of such a vehicle is that it is no longer possible to decouple the longitudinal and lateral/directional modes, even when analyzing small perturbations from equilibrium. In discussing the following, you may assume the vehicle c.g. is located at the quarter-chord station of the intersection of the wing with the plane $y = 0$.



a. Suggest one relevant aerodynamic stability derivative that describes a *lateral or directional moment* caused by a *longitudinal perturbation* for such a vehicle. Explain how it arises.

b. Suggest one relevant aerodynamic stability derivative that describes a *longitudinal force or moment* caused by a *lateral or directional perturbation* for such a vehicle. Explain how it arises.

32. The contribution of the tail to lift due to pitch rate and angle of attack rate are usually described by

$$\frac{\partial \mathbf{C}_L}{\partial q} = -2V_H a_H \tag{1}$$

$$\frac{\partial \mathbf{C}_L}{\partial \hat{\dot{\alpha}}} = -2V_H a_t \frac{\partial \epsilon}{\partial \alpha} \tag{2}$$

Discuss whether any error introduced in either Eq. (1) or (2) by the use of ℓ_t in the definition of V_H, rather than the distance ℓ_t' from the *center-of-gravity* to the tail aerodynamic center.

33. A university professor (who specializes in theoretical aerodynamics) decides to increase the yaw damping of a vehicle by adding a vertical fin *ahead* of the vehicle center of gravity. If he plans to use the fin also try to compensate for adverse yaw by having the fin contribute to positive C_{n_p}, should he place the fin above or below the vehicle center of gravity? Why?

34. Winglets are small (nearly vertical) fins added to the wing tips to improve the aerodynamic efficiency of the wing, but they also can affect stability and control. For our purposes, assume winglets are vertical fins added completely *above* the tips of the wings.

a. What two (lateral/directional) stability derivatives would you expect to be affected most by the addition of winglets? Why?

b. How will the contributions of the winglets to the two stability derivatives in part **a.**, be affected by wing sweep and/or dihedral?

35. A *spoiler* is a small split flap on the upper surface of the wing that can be deflected upward to decrease (or "spoil") the wing lift; deflection of the spoiler also increases drag. Spoilers can be deflected symmetrically on both wings to increase the descent rate (without increasing airspeed), or asymmetrically, on just one wing, to induce roll.

a. List an advantage of using spoilers in place of ailerons for roll control.

b. List a disadvantage of using spoilers in place of ailerons for roll control.

c. Would you expect the use of spoilers (rather than ailerons) to couple lateral/directional control to longitudinal motion? Why?

36. A variable-incidence canard is to be used for longitudinal control. The entire canard is free to rotate about a hinge line at the 20 per cent chord station (i.e., $0.05c_t$ ahead of the canard aerodynamic center).

a. Estimate the floating tendency $\partial C_{h_e}/\partial \alpha_t$ for this type of control.

b. Estimate the restoring tendency $\partial C_{h_e}/\partial \delta_e$ for this type of control.

[**Note:** Do *not* try to use *data* (experimental or otherwise) for part **d**; instead, use what you know from theory about moments and aerodynamic centers.]

37. Consider a lightweight hang glider constructed with a geometrically similar wing and canard, with the canard (forward tail) an upside-down, 1/2 scale model of the wing. The vehicle parameters are

Wing:	**Tail (Canard)**:	**Vehicle**:
$S = 8.0\ \text{m}^2$	$S_t = 2.0\ \text{m}^2$	$W/S = 125.\ \text{N/m}^2$
$b = 6.93$ m	$b_t = 3.465$ m	
$\bar{c} = 1.15$ m	$\bar{c}_t = 0.575$ m	
$C_{m_{0_w}} = -.025$	$C_{m_{0_t}} = +.025$	
	$x_{ac_w} - x_{ac_t} = 2.4$ m	

Both wing and tail are unswept; you may assume the spanwise efficiency factors for both wing and tail are $e = 1.0$, corresponding to elliptic loadings. The fuselage structure is tubular aluminum, which produces negligible aerodynamic forces and moments.

a. Assuming the two-dimensional lift-curve slopes for the wing and tail airfoil sections are $a_0 = 2\pi$, what are the wing and tail lift curve slopes? [**Note:** since the wing and tail are geometrically similar, their lift-curve slopes will be the same.]

b. For this configuration it is reasonable to neglect the downwash correction – i.e., to set $d\epsilon/d\alpha = 0$ – and to assume the tail efficiency factor $\eta = 1$.

i. Why is setting $d\epsilon/d\alpha = 0$ and $\eta = 1$ reasonable for this configuration?

ii. What is the total (vehicle) lift curve slope?

iii. Where is the basic neutral point for this configuration (measured from the leading edge of the wing mean aerodynamic chord in per cent mean aerodynamic chord, positive aft)?

c. The pilot intends to achieve longitudinal control simply by shifting his/her weight (i.e., by moving the vehicle center of gravity).

i. Assuming the vehicle structural weight is negligible in comparison to the pilot's weight, and that the static margin is 0.25 for a trimmed lift coefficient of 0.5, how far must the pilot move (in meters) to change the trim trimmed lift coefficient from 0.5 to 1.0?

ii. In what direction (fore or aft, relative to the wing aerodynamic center) must the pilot move to increase the lift coefficient (i.e., to slow down)?

iii. Will the vehicle stability increase or decrease at lower speeds (i.e., as the pilot trims the vehicle at larger lift coefficients).

d. The designer now decides to incorporate a variable-incidence canard for control, hinging the entire canard at the 20 per cent chord station (i.e., $0.05c_t$ ahead of the canard aerodynamic center).

i. Estimate the floating tendency $\partial C_{h_e}/\partial \alpha_t$ for this type of control.

ii. Estimate the restoring tendency $\partial C_{h_e}/\partial \delta_e$ for this type of control.

iii. The location of the canard hinge was chosen so that the canard would float with zero hinge moment for a specific canard lift coefficient. What is the value of this canard lift coefficient?

[**Note:** Do *not* try to use *data* (experimental or otherwise) for part **d**; instead, use what you know from theory about moments and aerodynamic centers.]

38. Consider an aircraft for which the horizontal tail is an exact, full-size replica of the wing. The wing has lift-curve slope $\mathbf{C}_{L\alpha} = a$, and is symmetrical, so $C_{m_{\text{a.c.}}} = 0$. The tail is a "flying tail" – i.e., the entire tail pivots (for control) about its aerodynamic center. The aerodynamic center of the tail is $5\bar{c}$ aft of the wing aerodynamic center, where $\bar{c}$ is the wing mean aerodynamic chord. Vehicle coefficients are based on the wing area S and mean aerodynamic chord $\bar{c}$. Angle of attack is considered positive nose up, and tail deflection δ is considered positive trailing edge down.

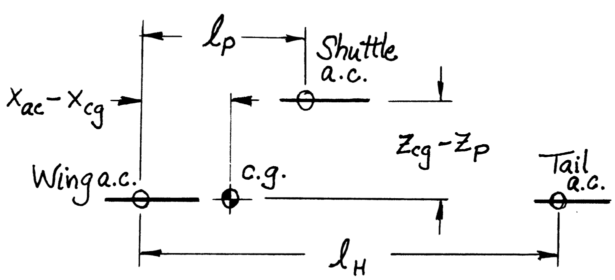
a. If the influence of the wing on the tail is ignored (i.e., no downwash effect), for what value of control deflection δ is the vehicle trimmed if the vehicle center of gravity is $2\bar{c}$ aft of the wing aerodynamic center?

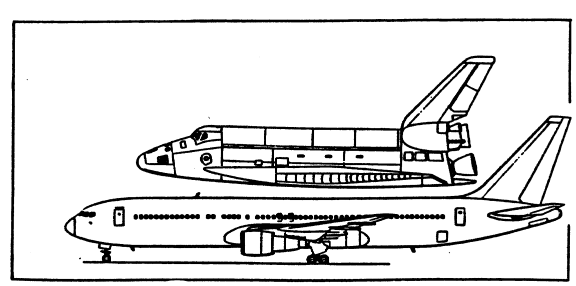
b. Is the configuration of part **a** statically stable? Where is the basic neutral point?

c. Where is the control-free neutral point (under the same assumptions as for part **a**)?

d. If the downwash at the tail due to the wing is $\epsilon = \alpha/2$, so that the tail sees only half the angle of attack, where is the basic neutral point?

39. We wish to consider using the Boeing 767-300 to transport the Space Shuttle Orbiter between the East and West coasts of the U.S. The Shuttle will be carried piggy-back (just as has been done in the past using the Boeing 747 aircraft).





The relevant dimensions are identified in the sketch above, and the needed vehicle parameters are

767-300 Wing:	**767-300 Tail**:	**Shuttle Orbiter**:
$S = 283.\ \text{m}^2$	$S_H = 76.4\ \text{m}^2$	$S_P = 250.\ \text{m}^2$
$b = 47.6\ \text{m}$	$b_H = 18.6\ \text{m}$	$b_P = 23.8\ \text{m}$
$\bar{c} = 6.69\ \text{m}$	$\bar{c}_H = 4.67\ \text{m}$	$\bar{c}_P = 12.0\ \text{m}$
	$\partial\epsilon/\partial\alpha = 0.38$	
$(C_{L_\alpha})_w = 4.34$	$(C_{L_\alpha})_H = 3.69$	$(C_{L_\alpha})_P = 2.60$
	$\ell_H = 24.0\ \text{m}$	$\ell_P = 3.3\ \text{m}$
		$z_{\text{cg}} - z_P = 6.0\ \text{m}$

Due to the relatively small aspect ratio of the Shuttle Orbiter wing, the variation of the shuttle *induced drag* with angle of attack will be an important factor in the stability of the vehicle. [**Note:** For this analysis, it may be useful to think of the shuttle as simply another tail, but one whose aerodynamic center is displaced vertically and for which variation in drag is significant.]

In order to see the effect of this (externally-carried) *payload* (that's why we use the subscript 'P'), we need to develop an expression for the pitch stiffness C_{m_α} of the complete vehicle, including the shuttle, taking into account the lift forces on the wing, tail, and shuttle, and the *drag* forces on the shuttle. To simplify matters, we will neglect the effects of the fuselage on pitch stability, and also assume that the effects of the wing downwash at the shuttle and the shuttle downwash at the wing and tail are both negligible.

a. Develop an expression for the lift coefficient of the entire vehicle. What is the value of the lift-curve slope $a = C_{L_\alpha}$ for the entire vehicle?

b. Develop an expression for the pitch stiffness of the entire vehicle by summing moments about the center of gravity, then differentiating with respect to angle of attack. Include only the *induced* component of the shuttle drag, and assume it is given by

$$(C_{D_P})_i = \frac{C_{L_P}^2}{\pi \mathcal{R}_P}$$

c. Write an expression for the location of the basic neutral point of the vehicle: $(x_{\text{np}} - x_{\text{ac}})/\bar{c}$. You should find that it depends upon the *magnitude* of the lift generated by the shuttle. Why is this, and what are its implications for stability? [No more than 50 words, please].

40. We wish to gain insight into the *damping* of the phugoid mode by considering the simple *first-order* system that describes perturbations to the longitudinal state in which the vehicle center of gravity is constrained to move in a straight line, and the vehicle is also restrained from any pitching motions. In this situation only the axial force equation is relevant, and it can be interpreted as a drag equation.

a. Write the equation for speed perturbations u resulting from this motion in *dimensional* form.

b. What is the condition that must be satisfied for the motion to be *damped* (i.e., for speed perturbations to decay exponentially)?

c. For the Boeing 747 in its powered descent condition (Nelson's $\mathbf{M} = 0.25$ condition) the derivative $X_u = -.0188\ \text{sec}^{-1}$. What is the time (in seconds) for a perturbation in speed to damp to half its initial amplitude in this case?

d. For the Boeing 747 at the high-speed transonic condition (Heffley's $\mathbf{M} = 0.90$ condition) the derivative $X_u = -.0219\ \text{sec}^{-1}$. What is the time (in seconds) for a perturbation in speed to damp to half its initial amplitude in this case?

e. Convert the equation you developed in part **a** to *non-dimensional* form for the variable u/u_0; use standard notation so that the coefficients appearing in this equation are functions only of dimensionless parameters, such as the aircraft mass parameter, the Mach number, and relevant aerodynamic coefficients and derivatives.

41. The roots of the cubic equation for the non-dimensional frequency λ

$$\lambda^3 + 5.20\lambda^2 + 1.26\lambda + 1.30 = 0,$$

consist of one large (negative) real root and a small, lightly damped, complex pair.

a. Estimate the large (negative) root.

b. Use Bairstow's approximation to estimate the complex pair of roots.

c. What are the non-dimensional times to damp to half amplitude $\hat{t}_{1/2}$ of each of the modes, and the non-dimensional period $\hat{T}$ of the oscillatory mode?

42. Analysis of the lateral oscillation (Dutch roll mode) is sometimes simplified by assuming that the vehicle *does not roll*, so that the resulting motion consists of a flat, side-slipping and yawing motion. This is an attractive approximation to the true Dutch roll mode because the characteristic equation corresponding to this constrained mode is simply a quadratic.

a. What important destabilizing effect does this approximation neglect?

b. Write the (dimensional) equations of motion for this approximation in state-space form. You may neglect the control terms, as we are interested only in the unforced response of the vehicle, and assume that the side force due to yaw rate Y_r and the product of inertia I_{xz} are negligibly small.

c. Write the characteristic equation for this motion, and express the undamped natural frequency ω_{n_0} and the damping ratio ζ in terms of the usual (dimensional) stability derivatives and vehicle parameters.

d. Would you expect this *approximation* of the Dutch roll mode ever to predict an unstable (i.e., exponentially growing) oscillation? Why?

e. For the Douglas DC-8 at cruise (Schmidt's Condition 3) use the relevant *(dimensional)* stability derivatives and vehicle parameters from Appendix B to determine the undamped natural frequency and damping ratio of the Dutch roll mode using this approximation.

f. Recall that, according to the exact linear analysis, the undamped natural frequency and damping ratio for this case are $\omega_n = 1.498$ sec^{-1} and 0.079, respectively. Why do you think the motion is more highly damped *in this approximation* than for the exact linear result?

g. What two stability derivatives must be small in order for this motion to be a good approximation to the true Dutch roll mode for the vehicle?

43. Consider a vehicle designed with a simplified control system, consisting of only an elevator and ailerons – i.e., with no rudder. You may also assume that

$$Y_{\delta_a} = N_{\delta_a} = 0 \tag{3}$$

i.e., that aileron deflection generates negligible side force and no adverse yaw. We wish to investigate the properties of a steady turn for such a vehicle. A steady turn corresponds to having

$$\dot{\beta} = \dot{p} = \dot{r} = 0 \tag{4}$$

and

$$\phi = \text{ const.} \tag{5}$$

a. Simplify the linearized equations for lateral-directional motions according to the above information to give a system of equations for the unknowns β, r, and δ_a for a given bank angle ϕ.

b. Solve your system of equations for the yaw rate r corresponding to a given bank angle ϕ.

c. The turn will be "coordinated" if the sideslip angle $\beta = 0$. Use the side force equation to determine the yaw rate r as a function of bank angle ϕ for a coordinated turn.

d. Compare your answers to **b** and **c** to develop a criterion for when the aileron-only system can approximate a coordinated turn.

44. As was shown in class, equations approximating the long-period longitudinal response of an aircraft from a level flight equilibrium state can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} u/u_0 \\ \theta \end{pmatrix} = \begin{pmatrix} X_u & -g_0/u_0 \\ -Z_u & 0 \end{pmatrix}\begin{pmatrix} u/u_0 \\ \theta \end{pmatrix} \tag{6}$$

We can also add the effect of throttle input by adding the terms

$$\begin{pmatrix} X_{\delta_T}/u_0 \\ M_{\delta_T} \end{pmatrix}\delta_T \tag{7}$$

to the right hand side of these equations, where δ_T represents a perturbation from equilibrium in the throttle (thrust) setting.

a. Is this system *controllable* when $M_{\delta_T} = 0$? Explain.

b. For a modified version of the Boeing 747 in its high-speed configuration at $\mathbf{M} = 0.90$ at 40,000 feet in the standard atmosphere, the dimensional stability derivatives relevant to this problem are

$$\begin{aligned} u_0 &= 871.3 \text{ ft/sec} & g_0 &= 32.174 \text{ ft/sec}^2 \\ X_u &= -.0219 \text{ sec}^{-1} & X_{\delta_T} &= 1.0 \text{ ft/sec}^2 \\ Z_u &= -.0836 \text{ sec}^{-1} & Z_{\delta_T} &= 0.0 \\ & & M_{\delta_T} &= 0.0 \end{aligned}$$

Determine the steady-state response to a step input of $\delta_T = 0.1$.

c. Interpret the result of part **b** physically – i.e., explain why only the pitch angle θ changed (and not the velocity u).

d. What is the physical origin of the derivative M_{δ_T}? When would setting its value to zero be a good approximation?

45. Analysis of the lateral oscillation (Dutch roll mode) can be simplified by assuming that the vehicle *does not yaw*, so that the resulting motion consists of a coupled side-slipping and rolling motion. This approximation to the Dutch roll mode results in a cubic characteristic equation.

a. Write the (dimensional) equations of motion for this approximation in state-space form. You may neglect the control terms, as we are interested only in the unforced response of the vehicle, and assume that the initial equilibrium corresponds to level flight ($\Theta_0 = 0$), and that the side force due to roll rate Y_p and the product of inertia I_{xz} are negligibly small.

b. Write the characteristic equation for the plant matrix of part **a**.

c. Use the Bairstow approximation to reduce the characteristic equation to a *quadratic*.

d. Would you expect *this approximation* of the Dutch roll mode ever to predict an unstable (i.e., exponentially growing) response? Why?

e. For a particular aircraft at its standard cruise condition, the relevant *(dimensional)* stability derivatives are

$$\begin{aligned} Y_v &= -.0040 \text{ sec}^{-1} & Y_p &= 0.0 \text{ ft/sec} \\ L_v &= -.0400 \text{ (ft sec)}^{-1} & L_p &= -.800 \text{ sec}^{-1} \end{aligned}$$

For these values, what is the nature of the unforced response? Is the motion stable or unstable?

f. What important *stabilizing* effects does this approximation neglect?

46. If the stability derivative $M_{\dot{w}}$ is negligible, the equations of motion for an aircraft mounted in a wind tunnel and free to rotate in pitch about its center of mass can be written

$$\dot{\mathbf{x}} = \begin{pmatrix} M_q & u_0 M_w \\ 1 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} M_{\delta_e} \\ 0 \end{pmatrix} \delta_e \tag{8}$$

where

$$\mathbf{x} = \begin{pmatrix} q \\ \theta \end{pmatrix} \tag{9}$$

is a reduced longitudinal state variable.

The dimensional stability derivatives corresponding to the tunnel operating conditions are given by

$$M_w = 0\ , \quad M_q = -.50 \text{ sec}^{-1} \quad \text{and} \quad M_{\delta_e} = -.10 \text{ sec}^{-2}$$

a. Is this system statically stable?

b. What will be the response $\mathbf{x}(t)$ of the system to the initial perturbation $\mathbf{x}(0) = [0 \quad 0.1]^T$?

c. For the same values of aerodynamic stability derivatives given above, we wish to stabilize the system using state-variable feedback in the form

$$\eta = -\mathbf{k}^T \mathbf{x} \tag{10}$$

so that the response of the system is a damped oscillation with period $T = 4$ sec and time to damp to half amplitude $t_{1/2} = 0.4$ sec. Determine the gain vector required to achieve this response, assuming you can measure both θ and q.

d. For the same values of aerodynamic stability derivatives given above, we wish to stabilize the system using linear-quadratic optimal control. Determine the gain vector for the special case when the state variable and control input are equally weighted – i.e., when the performance index to be minimized is given by

$$J_\infty = \int_t^\infty \left(\mathbf{x}^T \mathbf{x} + {\delta_e}^2 \right) \,\mathrm{d}\tau \tag{11}$$

47. Consider a vehicle designed with a simplified control system, consisting of only an elevator and ailerons – i.e., with *no rudder*. You may also assume that

$$Y_{\delta_a} = N_{\delta_a} = 0\,, \tag{12}$$

that Y_r is negligible relative to u_0, and that $I_{xz} = 0$. We wish to investigate the properties of a steady turn for such a vehicle as a perturbation from level-flight equilibrium $\Theta_0 = 0$. A *steady* turn corresponds to having

$$\dot{v} = \dot{p} = \dot{r} = 0 \tag{13}$$

and

$$\phi = \text{ const.} \tag{14}$$

a. Simplify the linearized equations for lateral-directional motions according to the above information to give a *system of algebraic* equations for the unknowns v and r as functions of δ_a and the bank angle ϕ.

b. Solve your system of equations for the yaw rate r corresponding to a given bank angle ϕ.

c. The turn will be *coordinated* if the sideslip velocity $v = 0$. Use the side force equation to determine the yaw rate r as a function of bank angle ϕ for a *coordinated* turn.

d. Compare your answers to **b** and **c** to develop a criterion for when the aileron-only system can approximate a coordinated turn. Express your criterion in terms of *dimensionless* aerodynamic derivatives.

1 Longitudinal Motions

For the longitudinal equations, the state and control vectors are

$$\mathbf{x} = [u \quad w \quad q \quad \theta]^T \qquad \text{and} \qquad \eta = [\delta_e \quad \delta_T]^T \tag{15}$$

and the plant and control matrices are given by

$$\mathbf{A} = \begin{pmatrix} X_u & X_w & 0 & -g\cos\Theta_0 \\ \frac{Z_u}{1-Z_{\dot{w}}} & \frac{Z_w}{1-Z_{\dot{w}}} & \frac{u_0+Z_q}{1-Z_{\dot{w}}} & -\frac{g\sin\Theta_0}{1-Z_{\dot{w}}} \\ M_u + \frac{Z_u M_{\dot{w}}}{1-Z_{\dot{w}}} & M_w + \frac{Z_w M_{\dot{w}}}{1-Z_{\dot{w}}} & M_q + M_{\dot{w}}\frac{u_0+Z_q}{1-Z_{\dot{w}}} & -M_{\dot{w}}\frac{g\sin\Theta_0}{1-Z_{\dot{w}}} \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{16}$$

and

$$\mathbf{B} = \begin{pmatrix} X_{\delta_e} & X_{\delta_T} \\ \frac{Z_{\delta_e}}{1-Z_{\dot{w}}} & \frac{Z_{\delta_T}}{1-Z_{\dot{w}}} \\ M_{\delta_e} + \frac{Z_{\delta_e} M_{\dot{w}}}{1-Z_{\dot{w}}} & M_{\delta_T} + \frac{Z_{\delta_T} M_{\dot{w}}}{1-Z_{\dot{w}}} \\ 0 & 0 \end{pmatrix} \tag{17}$$

respectively. The dimensionless mass and moment-of-inertia variables are defined for the longitudinal problem as

$$\mu = \frac{2m}{\rho S\bar{c}} \qquad \text{and} \qquad i_y = \frac{8I_y}{\rho S\bar{c}^3} = \frac{4\mu I_y}{m\bar{c}^2} \tag{18}$$

2 Lateral-Directional Motions

For lateral-directional motions, the state and control vectors are

$$\mathbf{x} = [v \quad p \quad \phi \quad r]^T \qquad \text{and} \qquad \eta = [\delta_r \quad \delta_a]^T \tag{19}$$

and the plant and control matrices are given by

$$\mathbf{A} = \begin{pmatrix} Y_v & Y_p & g\cos\Theta_0 & Y_r - u_0 \\ \frac{L_v+i_x N_v}{1-i_x i_z} & \frac{L_p+i_x N_p}{1-i_x i_z} & 0 & \frac{L_r+i_x N_r}{1-i_x i_z} \\ 0 & 1 & 0 & 0 \\ \frac{N_v+i_z L_v}{1-i_x i_z} & \frac{N_p+i_z L_p}{1-i_x i_z} & 0 & \frac{N_r+i_z L_r}{1-i_x i_z} \end{pmatrix} \tag{20}$$

and

$$\mathbf{B} = \begin{pmatrix} Y_{\delta_r} & 0 \\ \frac{L_{\delta_r}+i_x N_{\delta_r}}{1-i_x i_z} & \frac{L_{\delta_a}+i_x N_{\delta_a}}{1-i_x i_z} \\ 0 & 0 \\ \frac{N_{\delta_r}+i_z L_{\delta_r}}{1-i_x i_z} & \frac{N_{\delta_a}+i_z L_{\delta_a}}{1-i_x i_z} \end{pmatrix} \tag{21}$$

respectively. The dimensionless mass and moment-of-inertia variables are defined for the lateral-directional problem as

$$\mu = \frac{2m}{\rho S b}\,, \qquad i_x = \frac{I_{xz}}{I_x}\,, \qquad \text{and} \qquad i_z = \frac{I_{xz}}{I_z}\,. \tag{22}$$

3 Matrix Help

The inverse of the two-by-two matrix

$$\mathbf{A} = \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix}$$

is given by

$$\mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})} \begin{pmatrix} a_{2,2} & -a_{1,2} \\ -a_{2,1} & a_{1,1} \end{pmatrix}$$