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We will consider using state-variable feedback to improve the lateral-directional response of the Boeing 747 aircraft, particularly the Dutch Roll mode. Assume initial trim conditions corresponding to the cruise condition at $\mathbf{M} = 0.80$ and 40,000 feet in the Standard Atmosphere. For this equilibrium condition, the dimensionless stability and control derivatives are given by:

$$\begin{array}{lllll} \mathbf{C}_{y_\beta} = -0.88\,, & \mathbf{C}_{y_p} = 0.0\,, & \mathbf{C}_{y_r} = 0.0\,, & \mathbf{C}_{y_{\delta_r}} = +0.1157\,, & \mathbf{C}_{y_{\delta_a}} = 0.0 \\ \mathbf{C}_{l_\beta} = -0.277\,, & \mathbf{C}_{l_p} = -0.334\,, & \mathbf{C}_{l_r} = +0.300\,, & \mathbf{C}_{l_{\delta_r}} = +0.0070\,, & \mathbf{C}_{l_{\delta_a}} = +0.0137 \\ \mathbf{C}_{n_\beta} = +0.195\,, & \mathbf{C}_{n_p} = -0.0415\,, & \mathbf{C}_{n_r} = -0.327\,, & \mathbf{C}_{n_{\delta_r}} = -0.1256\,, & \mathbf{C}_{n_{\delta_a}} = +0.0002 \end{array} \quad (1)$$

and the dimensional properties are given by:

$$\begin{array}{lll} W = 636,600.\ \text{lbf}\,, & S = 5,500.\ \text{ft}^2\,, & b = 195.7\ \text{ft}\,, \\ I_x = 18.2 \times 10^6\ \text{slug-ft}^2\,, & I_z = 49.7 \times 10^6\ \text{slug-ft}^2\,, & I_{xz} = -1.56 \times 10^6\ \text{slug-ft}^2 \end{array}$$

Extend the MATLAB code you wrote for Exercise Set VI to include single-variable state-variable feedback for lateral-directional motions. Compute the dimensional stability derivatives from the dimensionless aerodynamic coefficients and the dimensional properties of the airframe given above.

1. Determine the characteristic response times for the rolling, spiral, and Dutch Roll modes. You should find that the oscillatory (Dutch Roll) mode is very lightly damped, with a damping ratio of only about $\zeta_{\text{DR}} = 0.047$. Plot the time history of the state variables for the first 20 seconds of response to a 5 *degree* perturbation in sideslip angle β.

2. Determine the gains required, using state-variable feedback with *rudder control only*, to increase the damping ratio of the Dutch Roll mode to $\zeta = 0.30$, while keeping the undamped natural frequency of the mode, and the times to damp to half amplitude of the rolling and spiral modes, unchanged.

- **a.** Plot the time history of the state variables for the first 20 seconds of response to a 5 *degree* perturbation in sideslip angle β, and compare with the result from Problem **1**.
- **b.** Determine the most significant changes in the augmented plant matrix (i.e., $\mathbf{A}^* - \mathbf{A}$), and relate them to *effective* changes in the vehicle stability derivatives.

3. Determine the gains required, using state-variable feedback with *aileron control only*, to increase the damping ratio of the Dutch Roll mode to $\zeta = 0.30$, while keeping the undamped natural frequency of the mode, and the times to damp to half amplitude of the rolling and spiral modes, unchanged.

- **a.** Plot the time history of the state variables for the first 20 seconds of response to a 5 *degree* perturbation in sideslip angle β, and compare with the results from Problems **1** and **2**.
- **b.** Determine the most significant changes in the augmented plant matrix (i.e., $\mathbf{A}^* - \mathbf{A}$), and relate them to *effective* changes in the vehicle stability derivatives.

4. For the modified systems of both, Questions **2** and **3**, you should have found a very slow return of the roll angle to its equilibrium value after the specified initial perturbation. Comment on why this is so.

Note: There are quite simple ways to do this using more advanced functions in the MATLAB Control System Toolkit, but *I expect you to use only basic matrix operations* (such as **poly**, **roots**, **damp**, **eig**, etc.) to determine the augmented plant matrix $\mathbf{A}^*$ (and, of course, the state-space functions **ss** and **initial** to determine the responses in the time domain).