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We will consider the response of the Boeing 747 aircraft to longitudinal perturbations.

1. Write a simple MATLAB code to reproduce the results of the sample calculations presented in class. That is, assume the initial trim condition corresponds to powered approach at Mach 0.25 at standard sea level conditions, using the aerodynamic derivative and vehicle property values provided in Section 5.4.1 of the course notes. Use as input variables the *dimensionless* stability derivatives and any needed dimensional quantities (such as vehicle mass, wing area, and mean aerodynamic chord) given in the same table of data.

Compute the corresponding dimensional stability derivatives, then compute the elements of the plant matrix for linear response to longitudinal perturbations.

Use the MATLAB function **eig** to determine the undamped natural frequencies and damping ratios of the short-period and phugoid modes, and verify that they are equal to the values given in class; i.e.,

$$\omega_{n_{\mathrm{sp}}} = 0.882 \text{ sec}^{-1} \quad \omega_{n_{\mathrm{ph}}} = 0.134 \text{ sec}^{-1}$$
$$\zeta_{\mathrm{sp}} = 0.6255 \qquad \zeta_{\mathrm{ph}} = 0.0133$$

a. Define a state-space model using the MATLAB function **ss**, and use it to study the response of the vehicle to a unit perturbation in pitch angle θ. What is the principal (qualitative) difference between this response and the response to a pure angle-of-attack perturbation (as shown in class)? Explain the difference.

b. Also study the response of the vehicle to a unit perturbation in pitch *rate* q, and answer the same questions as above.

2. Study the changes in properties of the short-period and phugoid modes when the aircraft is performing a steady climb. Assume all flight conditions and dimensionless stability derivatives from Problem 1 remain unchanged, except now

$$C_L = 1.108 \cos \Theta_0$$

where Θ_0 is the equilibrium flight-path angle, taken to be $\Theta_0 = 5°$ for this exercise. Summarize the values of undamped natural frequencies and damping ratios for both modes, noting any differences from those for level-flight. Also, plot the response to a unit change in pitch angle θ, focusing on the long-term (phugoid) mode.

Important Notes:

In Chapter 5 of the class notes, entitled **Dynamic Stability**, the responses are plotted in terms of *non-dimensional* velocity u/u_0 and angle of attack w/u_0. One way to do this using the MATLAB **ss** utility is to define the plant matrix as earlier (in terms of the state variables u and w), then

1. Multiply the elements in the first two columns of the plant matrix by u_0; and
2. Premultiply the plant matrix by the diagonal matrix having elements $diag(1/u_0\,,\, 1/u_0\,,\, 1\,,\, 1)$

before defining the state-space data object. Alternatively, you can simply divide the corresponding elements of the state vector by u_0 before plotting them to enable direct comparison with the results in the class notes.

Also, be sure you have the latest (correct) data for the Boeing 747 aircraft; the correct gross weight for this flight condition is $W = 564,032.$ lbf.