Chapter 1

Introduction to Flight Dynamics

Flight dynamics deals principally with the response of aerospace vehicles to perturbations in their flight environments and to control inputs. In order to understand this response, it is necessary to characterize the aerodynamic and propulsive forces and moments acting on the vehicle, and the dependence of these forces and moments on the flight variables, including airspeed and vehicle orientation. These notes provide an introduction to the engineering science of flight dynamics, focusing primarily of aspects of stability and control. The notes contain a simplified summary of important results from aerodynamics that can be used to characterize the forcing functions, a description of static stability for the longitudinal problem, and an introduction to the dynamics and control of both, longitudinal and lateral/directional problems, including some aspects of feedback control.

1.1 Introduction

Flight dynamics characterizes the motion of a flight vehicle in the atmosphere. As such, it can be considered a branch of systems dynamics in which the system studies is a flight vehicle. The response of the vehicle to aerodynamic, propulsive, and gravitational forces, and to control inputs from the pilot determine the attitude of the vehicle and its resulting flight path. The field of flight dynamics can be further subdivided into aspects concerned with

- **Performance**: in which the short time scales of response are ignored, and the forces are assumed to be in quasi-static equilibrium. Here the issues are maximum and minimum flight speeds, rate of climb, maximum range, and time aloft (endurance).
- **Stability and Control**: in which the short- and intermediate-time response of the attitude and velocity of the vehicle is considered. Stability considers the response of the vehicle to perturbations in flight conditions from some dynamic equilibrium, while control considers the response of the vehicle to control inputs.
- **Navigation and Guidance**: in which the control inputs required to achieve a particular trajectory are considered.

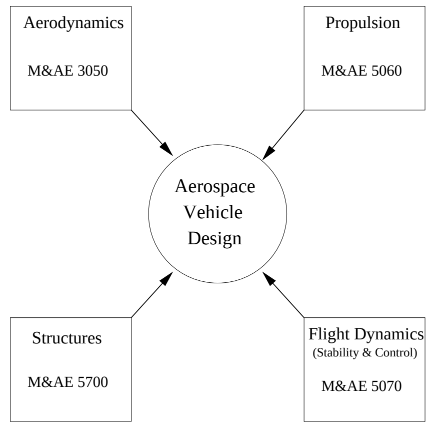

Figure 1.1: The four engineering sciences required to design a flight vehicle.

In these notes we will focus on the issues of stability and control. These two aspects of the dynamics can be treated somewhat independently, at least in the case when the equations of motion are linearized, so the two types of responses can be added using the principle of superposition, and the two types of responses are related, respectively, to the *stability* of the vehicle and to the ability of the pilot to *control* its motion.

Flight dynamics forms one of the four basic engineering sciences needed to understand the design of flight vehicles, as illustrated in Fig. 1.1 (with Cornell M&AE course numbers associated with introductory courses in these areas). A typical aerospace engineering curriculum with have courses in all four of these areas.

The aspects of stability can be further subdivided into (a) static stability and (b) dynamic stability. Static stability refers to whether the initial tendency of the vehicle response to a perturbation is toward a restoration of equilibrium. For example, if the response to an infinitesimal increase in angle of attack of the vehicle generates a pitching moment that reduces the angle of attack, the configuration is said to be statically stable to such perturbations. Dynamic stability refers to whether the vehicle ultimately returns to the initial equilibrium state after some infinitesimal perturbation. Consideration of dynamic stability makes sense only for vehicles that are statically stable. But a vehicle can be statically stable and dynamically unstable (for example, if the initial tendency to return toward equilibrium leads to an overshoot, it is possible to have an oscillatory divergence of continuously increasing amplitude).

Control deals with the issue of whether the aerodynamic and propulsive controls are adequate to trim the vehicle (i.e., produce an equilibrium state) for all required states in the flight envelope. In addition, the issue of "flying qualities" is intimately connected to control issues; i.e., the controls must be such that the maintenance of desired equilibrium states does not overly tire the pilot or require excessive attention to control inputs.

Several classical texts that deal with aspects of aerodynamic performance [1, 5] and stability and control [2, 3, 4] are listed at the end of this chapter.

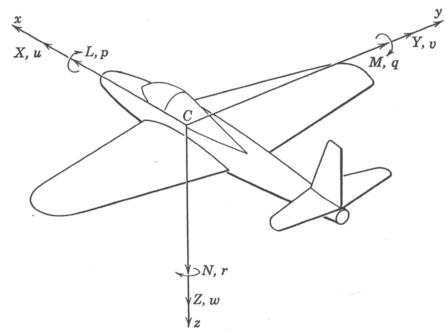

Figure 1.2: Standard notation for aerodynamic forces and moments, and linear and rotational velocities in body-axis system; origin of coordinates is at center of mass of the vehicle.

1.2 Nomenclature

The standard notation for describing the motion of, and the aerodynamic forces and moments acting upon, a flight vehicle are indicated in Fig. 1.2.

Virtually all the notation consists of consecutive alphabetic triads:

- The variables x, y, z represent coordinates, with origin at the center of mass of the vehicle. The x-axis lies in the symmetry plane of the vehicle[1] and points toward the nose of the vehicle. (The precise direction will be discussed later.) The z-axis also is taken to lie in the plane of symmetry, perpendicular to the x-axis, and pointing approximately down. The y axis completes a right-handed orthogonal system, pointing approximately out the right wing.
- The variables u, v, w represent the instantaneous components of linear velocity in the directions of the x, y, and z axes, respectively.
- The variables X, Y, Z represent the components of aerodynamic force in the directions of the x, y, and z axes, respectively.
- The variables p, q, r represent the instantaneous components of rotational velocity about the x, y, and z axes, respectively.
- The variables L, M, N represent the components of aerodynamic moments about the x, y, and z axes, respectively.
- Although not indicated in the figure, the variables ϕ, θ, ψ represent the angular rotations, relative to the equilibrium state, about the x, y, and z axes, respectively. Thus, $p = \dot{\phi}$, $q = \dot{\theta}$, and $r = \dot{\psi}$, where the dots represent time derivatives.

The velocity components of the vehicle often are represented as angles, as indicated in Fig. 1.3. The velocity component w can be interpreted as the angle of attack

$$\alpha \equiv \tan^{-1} \frac{w}{u} \tag{1.1}$$

[1] Virtually all flight vehicles have bi-lateral symmetry, and this fact is used to simplify the analysis of motions.

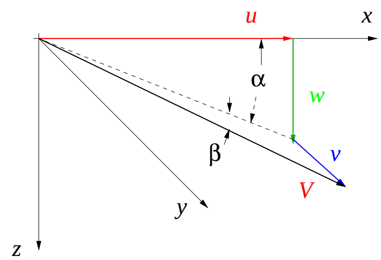

Figure 1.3: Standard notation for aerodynamic forces and moments, and linear and rotational velocities in body-axis system; origin of coordinates is at center of mass of the vehicle.

while the velocity component v can be interpreted as the sideslip angle

$$\beta \equiv \sin^{-1} \frac{v}{V} \tag{1.2}$$

1.2.1 Implications of Vehicle Symmetry

The analysis of flight motions is simplified, at least for small perturbations from certain equilibrium states, by the bi-lateral symmetry of most flight vehicles. This symmetry allows us to decompose motions into those involving *longitudinal* perturbations and those involving *lateral/directional* perturbations. Longitudinal motions are described by the velocities u and v and rotations about the y-axis, described by q (or θ). Lateral/directional motions are described by the velocity v and rotations about the x and/or z axes, described by p and/or r (or ϕ and/or ψ). A longitudinal equilibrium state is one in which the lateral/directional variables v, p, r are all zero. As a result, the side force Y and the rolling moment p and yawing moment r also are identically zero. A longitudinal equilibrium state can exist only when the gravity vector lies in the x-z plane, so such states correspond to wings-level flight (which may be climbing, descending, or level).

The important results of vehicle symmetry are the following. If a vehicle in a longitudinal equilibrium state is subjected to a perturbation in one of the longitudinal variables, the resulting motion will continue to be a longitudinal one – i.e., the velocity vector will remain in the x-z plane and the resulting motion can induce changes only in u, w, and q (or θ). This result follows from the symmetry of the vehicle because changes in flight speed ($V = \sqrt{u^2 + v^2}$ in this case), angle of attack ($\alpha = \tan^{-1} w/u$), or pitch angle θ cannot induce a side force Y, a rolling moment L, or a yawing moment N. Also, if a vehicle in a longitudinal equilibrium state is subjected to a perturbation in one of the lateral/directional variables, the resulting motion will *to first order* result in changes only to the lateral/directional variables. For example, a positive yaw rate will result in increased lift on the left wing, and decreased lift on the right wing; but these will approximately cancel, leaving the lift unchanged. These results allow us to gain insight into the nature of the response of the vehicle to perturbations by considering longitudinal motions completely uncoupled from lateral/directional ones, and vice versa.

1.2.2 Aerodynamic Controls

An aircraft typically has three aerodynamic controls, each capable of producing moments about one of the three basic axes. The *elevator* consists of a trailing-edge flap on the horizontal tail (or the ability to change the incidence of the entire tail). Elevator deflection is characterized by the deflection angle δ_e. Elevator deflection is defined as positive when the trailing edge rotates downward, so, for a configuration in which the tail is aft of the vehicle center of mass, the control derivative

$$\frac{\partial M_{cg}}{\partial \delta_e} < 0$$

The *rudder* consists of a trailing-edge flap on the vertical tail. Rudder deflection is characterized by the deflection angle δ_r. Rudder deflection is defined as positive when the trailing edge rotates to the left, so the control derivative

$$\frac{\partial N_{cg}}{\partial \delta_r} < 0$$

The *ailerons* consist of a pair of trailing-edge flaps, one on each wing, designed to deflect differentially; i.e., when the left aileron is rotated up, the right aileron will be rotated down, and vice versa. Aileron deflection is characterized by the deflection angle δ_a. Aileron deflection is defined as positive when the trailing edge of the aileron on the right wing rotates up (and, correspondingly, the trailing edge of the aileron on the left wing rotates down), so the control derivative

$$\frac{\partial L_{cg}}{\partial \delta_a} > 0$$

By vehicle symmetry, the elevator produces only pitching moments, but there invariably is some cross-coupling of the rudder and aileron controls; i.e., rudder deflection usually produces some rolling moment and aileron deflection usually produces some yawing moment.

1.2.3 Force and Moment Coefficients

Modern computer-based flight dynamics simulation is usually done in dimensional form, but the basic aerodynamic inputs are best defined in terms of the classical non-dimensional aerodynamic forms. These are defined using the dynamic pressure

$$Q = \frac{1}{2}\rho V^2 = \frac{1}{2}\rho_{\mathrm{SL}} V_{\mathrm{eq}}^2$$

where ρ is the ambient density at the flight altitude and V_{eq} is the *equivalent airspeed*, which is defined by the above equation in which ρ_{SL} is the standard sea-level value of the density. In addition, the vehicle reference area S, usually the wing planform area, wing mean aerodynamic chord $\bar{c}$, and wing span b are used to non-dimensionalize forces and moments. The force coefficients are defined as

$$\begin{aligned} \mathbf{C}_X &= \frac{X}{QS} \\ \mathbf{C}_Y &= \frac{Y}{QS} \\ \mathbf{C}_Z &= \frac{Z}{QS} \end{aligned} \tag{1.3}$$

while the aerodynamic moment coefficients are defined as

$$\begin{aligned} \mathbf{C}_l &= \frac{L}{QSb} \\ \mathbf{C}_m &= \frac{M}{QS\bar{c}} \\ \mathbf{C}_n &= \frac{N}{QSb} \end{aligned} \tag{1.4}$$

Note that the wing *span* is used as the reference moment arm for the rolling and yawing moments, while the mean aerodynamic chord is used for the pitching moment.

Finally, we often express the longitudinal forces in terms of the lift L and drag D, and define the corresponding lift and drag coefficients as

$$\begin{aligned} \mathbf{C}_L &\equiv \frac{L}{QS} = -\mathbf{C}_Z \cos\alpha + \mathbf{C}_X \sin\alpha \\ \mathbf{C}_D &\equiv \frac{D}{QS} = -\mathbf{C}_Z \sin\alpha - \mathbf{C}_X \cos\alpha \end{aligned} \tag{1.5}$$

Note that in this set of equations, L represents the *lift force*, not the rolling moment. It generally will be clear from the context here, and in later sections, whether the variable L refers to the lift force or the rolling moment.

1.2.4 Atmospheric Properties

Aerodynamic forces and moments are strongly dependent upon the ambient density of the air at the altitude of flight. In order to standardize performance calculations, standard values of atmospheric properties have been developed, under the assumptions that the atmosphere is static (i.e., no winds), that atmospheric properties are a function only of altitude h, that the temperature is given by a specified piecewise linear function of altitude, and that the acceleration of gravity is constant (technically requiring that properties be defined as functions of *geopotential* altitude. Tables for the properties of the Standard Atmosphere, in both SI and British Gravitational units, are given on the following pages.

h (m)	T (K)	p (N/m^2)	ρ (kg/m^3)	a (m/s)
0	288.15	101325.00	1.225000	340.29
500	284.90	95460.78	1.167268	338.37
1000	281.65	89874.46	1.111641	336.43
1500	278.40	84555.84	1.058065	334.49
2000	275.15	79495.01	1.006488	332.53
2500	271.90	74682.29	0.956856	330.56
3000	268.65	70108.27	0.909119	328.58
3500	265.40	65763.78	0.863225	326.58
4000	262.15	61639.91	0.819125	324.58
4500	258.90	57727.98	0.776770	322.56
5000	255.65	54019.55	0.736111	320.53
5500	252.40	50506.43	0.697100	318.48
6000	249.15	47180.64	0.659692	316.43
6500	245.90	44034.45	0.623839	314.36
7000	242.65	41060.35	0.589495	312.27
7500	239.40	38251.03	0.556618	310.17
8000	236.15	35599.41	0.525162	308.06
8500	232.90	33098.64	0.495084	305.93
9000	229.65	30742.07	0.466342	303.79
9500	226.40	28523.23	0.438895	301.63
10000	223.15	26435.89	0.412701	299.46
10500	219.90	24474.00	0.387720	297.27
11000	216.65	22631.70	0.363912	295.07
11500	216.65	20915.84	0.336322	295.07
12000	216.65	19330.06	0.310823	295.07
12500	216.65	17864.52	0.287257	295.07
13000	216.65	16510.09	0.265478	295.07
13500	216.65	15258.34	0.245350	295.07
14000	216.65	14101.50	0.226749	295.07
14500	216.65	13032.37	0.209557	295.07
15000	216.65	12044.30	0.193669	295.07
15500	216.65	11131.14	0.178986	295.07
16000	216.65	10287.21	0.165416	295.07
16500	216.65	9507.26	0.152874	295.07
17000	216.65	8786.45	0.141284	295.07
17500	216.65	8120.29	0.130572	295.07
18000	216.65	7504.64	0.120673	295.07
18500	216.65	6935.66	0.111524	295.07
19000	216.65	6409.82	0.103068	295.07
19500	216.65	5923.85	0.095254	295.07
20000	216.65	5474.72	0.088032	295.07

Table 1.1: Properties of the International Standard Atmosphere; SI units.

h (ft)	T (°R)	p (lbf/ft^2)	ρ (slug/ft^3)	a (ft/s)
0	518.67	2116.20	0.002377	1116.44
1000	515.10	2040.84	0.002308	1112.60
2000	511.54	1967.66	0.002241	1108.74
3000	507.97	1896.62	0.002175	1104.87
4000	504.41	1827.68	0.002111	1100.99
5000	500.84	1760.78	0.002048	1097.09
6000	497.27	1695.87	0.001987	1093.17
7000	493.71	1632.92	0.001927	1089.25
8000	490.14	1571.87	0.001868	1085.31
9000	486.57	1512.68	0.001811	1081.35
10000	483.01	1455.31	0.001755	1077.38
11000	479.44	1399.72	0.001701	1073.40
12000	475.88	1345.86	0.001648	1069.40
13000	472.31	1293.69	0.001596	1065.38
14000	468.74	1243.17	0.001545	1061.35
15000	465.18	1194.25	0.001496	1057.31
16000	461.61	1146.91	0.001447	1053.25
17000	458.05	1101.10	0.001400	1049.17
18000	454.48	1056.78	0.001355	1045.08
19000	450.91	1013.92	0.001310	1040.97
20000	447.35	972.48	0.001266	1036.84
21000	443.78	932.42	0.001224	1032.70
22000	440.21	893.70	0.001183	1028.55
23000	436.65	856.30	0.001143	1024.37
24000	433.08	820.18	0.001103	1020.18
25000	429.52	785.30	0.001065	1015.97
26000	425.95	751.63	0.001028	1011.74
27000	422.38	719.14	0.000992	1007.50
28000	418.82	687.79	0.000957	1003.24
29000	415.25	657.56	0.000923	998.96
30000	411.69	628.42	0.000889	994.66
31000	408.12	600.33	0.000857	990.34
32000	404.55	573.27	0.000826	986.01
33000	400.99	547.20	0.000795	981.65
34000	397.42	522.10	0.000765	977.27
35000	393.85	497.95	0.000737	972.88
36000	390.29	474.70	0.000709	968.47
37000	389.97	452.42	0.000676	968.07
38000	389.97	431.19	0.000644	968.07
39000	389.97	410.96	0.000614	968.07
40000	389.97	391.67	0.000585	968.07
41000	389.97	373.29	0.000558	968.07
42000	389.97	355.78	0.000532	968.07
43000	389.97	339.08	0.000507	968.07
44000	389.97	323.17	0.000483	968.07
45000	389.97	308.00	0.000460	968.07

Table 1.2: Properties of the International Standard Atmosphere; British Gravitational units.

Bibliography

[1] John Anderson, **Introduction to Flight**, McGraw-Hill, New York, Fourth Edition, 2000.

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[5] Richard Shevell, **Fundamentals of Flight**, Prentice Hall, Englewood Cliffs, New Jersey, Second Edition, 1989.