Having a Part Twice Over¹
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Abstract: I argue that it is intuitive and useful to think about composition in light of the familiar functionalist distinction between role and occupant. This involves factoring the standard notion of parthood into two related notions: being a parthood slot and occupying a parthood slot. One thing is part of another just in case it fills one of that thing’s parthood slots. This move opens room to rethink mereology in various ways, and, in particular, to see the mereological structure of a composite as potentially outreaching the individual entities that are its parts. I sketch one formal system that allows things to have individual entities as parts multiple times over. This is particularly useful to David Armstrong, given Lewis’ charge that his structural universals must do exactly that. I close by reflecting upon the nature and point of formal mereology.

Keywords: mereology, parthood, composition, extensionality, strong supplementation, causal role, structural universals

1. Introduction

A thing can stand in the very same relation to many other things. I have five cousins; I stand in the cousin of relation to 5 different people. I am three feet from all kinds of stuff; I stand in the being three feet from relation to lots of things. And I am also part of many things—the fusion of the contents of this room, my family, the Cornell philosophy department.² I stand in the parthood relation to all of them and more.

Nothing controversial yet. I shall try harder.

The same relation can hold multiple times between the very same entities at the very same time. The cousin of relation, for example, can hold twice between the same two individuals. Two people are cousins twice over, or “double cousins,” as they are called, just in case they are the children of pairs of siblings.³ Similarly, the being three feet from relation

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¹ Acknowledgments deleted for blind review.
² Perhaps I am not a part of my family and the department, but only a member. Allow poetic license so that I can maintain a single example throughout the paragraph. It is clear that in general, things can be part of multiple wholes.
³ A bit more slowly: x is the cousin of y =df one of x’s parents is the sibling of one of y’s parents. So if each of x’s parents is a sibling of one of y’s parents, x and y satisfy the definition twice. Consider: two siblings, Allen and Bob, marry two siblings, Catherine and Deborah. Allen and Catherine have a child, Elise. Bob and Deborah have a child, Frank. Elise and Frank are cousins because they are the children, respectively, of Allen and his brother Bob. But they are also cousins because they are the children, respectively, of Catherine and her sister Deborah. Elise and Frank are double cousins, cousins twice over.
can hold multiple times between the same two entities: consider two antipodal points on a sphere, such that the shortest distance between them along the surface is three feet. Since there are many (infinitely many) three foot long arcs between them on the surface, the points are three feet from each other many times over. But… parthood? Surely my examples grind to a halt here. One thing cannot be part of the same whole many times over. I have two hands, yes, but each is part of me once and once only.

David Lewis, certainly, wants nothing to do with the idea that the parthood relation can obtain multiple times between the same entities. His primary objection to what he calls the ‘pictorial’ conception of David Armstrong’s structural universals is that

Each methane molecule has not one hydrogen atom but four. So if the structural universal methane is to be an isomorph of the molecules that are its instances, it must have the universal hydrogen as a part not just once, but four times over. Likewise for bonded, since each molecule has four bonded pairs of atoms. But what can it mean for something to have a part four times over? What are there four of? There are not four of the universal hydrogen, or of the universal bonded; there is only one (1986, 34).

I will argue that Lewis is wrong here. We can make sense of the idea of an entity’s having a part twice—or four times—over. My primary point is to explain in general terms how this could be, and sketch a system of formal mereology that allows it. It is not to defend Armstrong’s views about structural universals, though the account is indeed helpful to him, and I will return to it at the end.

2. An Odd Case

To get the intuitive idea going, (try to) consider the following case. Let w be a world containing two simples, a and b, and no other simples. Let w also contain an object c that is the fusion of those two simples. And let it also contain a distinct object d: the fusion of a, b, and c.

“Double cousinhood” is a legally recognized concept. In the U.S. state of North Carolina, it is legal to marry your cousin, but not your double cousin, presumably because double cousins are more genetically similar. Antipodal points are points on the surface of the sphere that are directly opposite each other: the shortest straight distance between them is the diameter of the sphere. Terminology: any two points (antipodal or not) on the surface of the sphere lie along a great circle, i.e. a circle whose center is the center of the sphere. They are also connected by at least one great circle arc (a segment of a great circle). Antipodal points lie on infinitely many great circles and are connected by infinitely many great circle arcs of equal length. The length of these great circle arcs is ½ the circumference of the great circles (2πr). So the antipodal points are connected by infinitely many “surface paths” of πr. (I admit a debt to Wikipedia here.)
Note that d is, as described, a rather odd bird; it is the fusion of a composite object with its own parts. Indeed, the classical extensional mereologist will deny that the case is possible at all. She will claim that there can be no such d, distinct from c; there are only three objects in w, not four.

Her basic objection here is that there simply isn’t enough to d. How can d have c as a proper part, when it has no parts that are disjoint from c? And how can c fail to have d as a part, given that d has no parts disjoint from it? In fact, doesn’t d have the same parts as c? So aren’t they identical? They are, after all, both built from a and b. Surely one cannot get two distinct composites out of the same two parts. More formally: the case violates weak supplementation and thus strong supplementation as well. It also appears to violate the extensionality of proper parthood as well as the extensionality of overlap. So the classical extensional mereologist will have no truck with it.

All of that is correct as far as it goes. The case is clearly ruled out by classical extensional mereology. But I will offer a different mereological system, one that permits the case above—and which does not simply deny the axioms that cause the trouble, but replaces them with analogues that recapture some of the intuitive force behind the originals. The key move? Distinguish entities that are parts from parthood slots.

If you doubt that the case violates the extensionality of proper parthood, you are likely to be sympathetic to the overall moral of this paper (though perhaps not every detail). Your doubt presumably stems from the intuition that c and d do not share all their proper parts—after all, isn’t c a proper part of d, but not of c? This intuition is much easier to make sense of in the context of the new system I propose, and I shall return to it toward the end of section 6.
3. Roles and Occupants

To see the idea here, set parthood aside and recall the familiar distinction between roles and occupants. For example, Barack Obama currently occupies the “President of the United States” role. Role and occupant are not identical. The Presidency is an office, essentially characterized by the United States Constitution; Obama is a man, essentially characterized (if at all) by, say, his DNA and origin. Or consider what a certain sort of functionalist would say about pain. Pain itself is a second order functional property, essentially characterized by its typical causes and effects. Any particular pain in a particular creature, however, is some physical state or other that ‘plays the pain role’.6

What is it for something to be a role or an occupant? Particular roles are defined causally and/or nomologically, where the latter is understood quite broadly. Sometimes the relevant laws are physical ones, but sometimes they are not. The laws that characterize the Presidency, for example, are of a rather different sort; that role is characterized by the powers and responsibilities laid out in the Constitution. What it is for a person to occupy the role—to be the President—is for her to actually have those powers and responsibilities. Or, in the functionalism example, pain is said to be a second-order property, the property of having some property or other whose instantiation is caused in certain ways and causes certain effects. For a person to be in pain—to instantiate the property—is to instantiate some physical property that actually is caused by and causes the right sorts of things. Notice, however, that all of that is about particular roles, and occupants thereof. Something rather vaguer must be said about what it is to be a causal role full stop. To be a causal role is to be a location in a causal nexus, to be defined by various forward- and backward-looking causal powers. To be an occupant of a causal role is to have the relevant forward and backward looking causal powers.7

Here are three platitudes about occupants and roles. First platitude: the same role can have different occupants at different times (and the same role-type can have different occupants in different systems). Although Obama is the 2011 occupant of the “President of

6 This way of putting the point assumes role rather than realizer functionalism, but both views turn on the same distinction between role and occupant. Role functionalists identify the mental state-type pain with the role itself, while realizer functionalists identify it with whatever occupies the relevant causal role in a certain species or individual. That is, role functionalists take mental predicates like ‘pain’ to rigidly designate second-order functional properties, while realizer functionalists take them to nonrigidly designate first-order physical properties.

7 This is a dispositional characterization; one could alternatively insist that the occupant actually cause and be caused in the relevant ways.
the United States” role, other people have occupied it in the past and will occupy it in the future. Similarly, although my pain right here right now is realized by a particular pattern of C–fiber firing, later pains of mine—as well as pains of yours, or of an octopus, or of a Martian—may well be realized by something else. Second platitude: however, no particular role can have multiple occupants at a time. That seems partially definitive of being a single role. Third platitude: a single entity can play different roles at different times, and, indeed, at the same time. Obama used to be an Illinois senator, now he is President—and, as President, is simultaneously both Chief Executive and Commander-in-Chief of the Armed Forces. Or, if you prefer, the treasurer of a local running club might also be the president of the school board.

4. Mereological Roles

I claim that the role-occupant distinction can be usefully exploited in mereology. The central idea is to cleave the standard notion of parthood in two, into being a parthood slot and occupying (or filling) a parthood slot. Those two notions together define the being a part of relation: for one thing \( x \) to be part of \( y \) is for \( x \) to occupy one of \( y \)’s parthood slots.

In the formal system I sketch below, I will take those two notions as primitive. But notice that we have exactly as much grasp—if imperfect grasp—on them as we do of the analogue notions above. There, both rolehood and occupancy were characterized causal-nomologically; here, they are to be understood mereologically. To be a parthood slot is to be a location in a mereological nexus, to be an aspect of the mereological structure of a whole, to be defined by what it is part of and what is part of it. To occupy a parthood slot is to actually stand in the relevant parthood relations, to actually be part of some things and have other things as parts.

To help illustrate what I have in mind, consider the uncontroversial phenomenon of mereological overlap. Two things overlap just in case they share a part, as adjoining offices overlap when they share a wall. In light of the suggestion on the table, it is quite natural to think of overlap as occurring when one entity occupies parthood slots—plays mereological roles—in more than one whole. Indeed, looked at in this light, mereological overlap is directly parallel to the familiar if unnamed phenomenon in which one entity plays causal

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8 Perhaps this should be: no particular instance of a role has more than one occupant at a single time. The Presidency only ever has one instance, but there are many pains, and two senators from each state.
roles in the functioning of more than one system: the person who is both treasurer of the running club and president of the school board, a gear that does two jobs.

Distinguishing between parthood slots and the entities that fill them opens up room to rethink the relations between parts and wholes. Importantly, we need not rethink them; classical extensional mereology can in fact be reformulated in these terms. But the distinction allows the formulation of a variety of other formal systems that cannot be formulated without it. I will offer one such system shortly. First, however, I will spell out the mereological analogues of the above three platitudes about causal roles. Doing so will further illuminate the distinction between parthood slots and the entities that fill them.

5. The Three ‘Platitudes’

Admittedly, calling these mereological analogues ‘platitudes’ seems a bit of a stretch. So I shall simply call them claims:

Claim 1: Parthood slots can have different occupants at different times.
Claim 2: No particular parthood slot can have multiple occupants at the same time.
Claim 3: A single entity can occupy multiple parthood slots at the same or different times.

Now, only the third claim really matters for my purposes in this paper; it amounts to the claim that things can have parts twice over. Still, if the distinction between parthood slots and occupants is to be pursued, all three claims should make sense, and indeed seem reasonably natural. And they are—exactly as natural as the three platitudes about causal roles. In order to continue warming you up to the notion of a parthood slot, let me say a bit about the first two before turning to the third.

The second claim, like the second platitude, seems partially definitive of how parthood slots are individuated. It is both as prima facie reasonable and as hard to defend as the second platitude.

The first claim says that things can change their parts over time. This is not only a coherent thought, but a very familiar one; it is the denial of mereological essentialism. Or is it? In fact, matters are a bit more complicated than this, in interesting ways. One effect of
the proposed approach is that it allows the formulation of multiple kinds of mereological essentialism, notably the following two:

(ME₁) Objects have both their parthood slots and the occupants thereof essentially. (If \( z \) is a parthood slot of \( y \), and if \( x \) fills \( z \), then it is necessary both that \( z \) is a parthood slot of \( y \) and that \( x \) fills \( z \).

(ME₂) Objects have their parthood slots essentially, but not the occupants thereof. (If \( z \) is a parthood slot of \( y \), and if \( x \) fills \( z \), then it is necessary that \( z \) is a parthood slot of \( y \).

The ordinary version of mereological essentialism is closest to (ME₁). That is because the simple claim that ‘objects have their parts essentially’ translates, in the current terminology, to the claim that if there is a \( z \) such that \( z \) is a parthood slot of \( y \), and \( x \) fills \( z \), then it is necessary that there is a \( z \) such that \( z \) is a parthood slot of \( y \), and \( x \) fills \( z \). And (ME₁) is, as expected, inconsistent with claim 1.

(ME₂), in contrast, states a position that is consistent with claim 1, and that cannot be articulated on the standard way of talking about parthood. But it is worth articulating. Consider the view that, for example, tables must have the number of legs they actually have, although they need not have the particular legs they actually have. This is roughly the kind of thing that (ME₂) says. It states a version of mereological essentialism according to which objects can survive the replacement of parts as long as their mereological structure stays fixed, but can survive neither the gain of genuinely additional parts—new filled parthood slots—nor the unreplaced loss of parts, where that involves the loss of the parthood slot as well as the entity that fills it. (The thought that the unreplaced loss of the occupant entails the loss of the slot itself is captured in the axiom of Single Occupancy below.) On such a view, a four-legged table can survive the replacement of a leg, but it cannot become five-legged or three-legged.

Is it plausible to think that things have their parthood slots essentially? That is a topic for future inquiry; I will not pursue it here. Whether or not (ME₂)-style mereological essentialism is defensible or desirable, the mere fact that it is easily understandable lends support to the current approach which factors parthood into slots and occupants. I count this illumination of the varieties of mereological essentialism among the virtues of my system that

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9 A third is (ME₃): Occupants do not have their parthood slots essentially, but do have the entities which actually occupy their parthood slots essentially. However, this is an odd view that does not obviously map on to any intuitive claims about the relation between objects and their parts.

10 This particular characterization of (ME₂) assumes that there are no reified absences, or at least that they cannot fill slots. If there were such things, a four-legged table could lose a leg while retaining exactly the same parthood slots, all of which are filled—one by an absence. Thanks to an anonymous referee here.
I sketch in section 8. For the moment, just notice that the first claim—the analogue of the first platitude—denies (ME1) and remains silent on (ME2); it is plausible to the extent that the ordinary version of mereological essentialism is not.

Again, the first and second claims are not strictly speaking required for my purposes here, and I do not claim to have established that they are true. The primary point of introducing them is to pursue the analogy with causal roles. Even mereological essentialists (i.e., those who deny claim 1) should find it interesting that their view winds up structurally parallel to the denial of the idea that causal roles can have different occupants at different times. Because I do find both plausible, the formal system I introduce in section 6 reflects them. The first claim is permitted by the system; the second claim is entailed by an axiom (A7).

The all-important third claim, too, is permitted by the system. Let me finally say a bit to render it plausible. Just as in the causal-nomological case, there is strong reason to think that a single entity can occupy multiple parthood slots. At any rate, most people already accept it. First, most people accept that a single entity can occupy multiple parthood slots at different times. After all, most people not only deny mereological essentialism, but also its converse; most people deny that things are essentially parts of the wholes that they are currently part of. And to say that a thing can be a part of x at t1 and part of a distinct y at t2 is precisely to say that it can occupy different parthood slots at different times. Second, everyone also accepts that a single entity can occupy multiple parthood slots at the same time. This is the familiar phenomenon of mereological overlap that I have already discussed. Mereological overlap is precisely an instance of the third claim: the shared part occupies two parthood slots, one in each whole, at a single time.

So the third claim looks quite plausible. Nonetheless, perhaps it is too strong. As stated, it has a quite controversial consequence: namely, that a single entity can simultaneously occupy two distinct parthood slots in a single whole. And to do that is to be a part twice over. We have arrived at the issue from the beginning of the paper. Should we, or should we not, believe such a thing? After all, the third claim can be weakened to accommodate the above phenomena while ruling this one out:

Claim 3*: a single entity can occupy different parthood slots at different times, or at the same time within different wholes.
Both claims 3 and 3* allow entities to be parts of different entities at different times, and allow distinct objects to overlap. The difference is that 3 does, and 3* does not, allow the controversial case in which an entity serves as a part twice over. Our question becomes: should we believe 3 or 3*?

I don’t intend to provide a knockdown argument for 3 over 3*. But I would like to point out two reasons to prefer 3, or at least to take the weakening to be insufficiently motivated.

First, there is no pressure to similarly weaken the third platitude about causal-nomological roles. The relevant weakening would yield something like:

Platitude 3*: a single entity can play different causal-nomological roles at different times, or at the same time within different systems.

I don’t know how to individuate ‘systems’. But whatever exactly they are supposed to come to, Obama occupies two roles—Chief Executive and Commander in Chief—within the same system.

Second, in the mereological case, it is actually rather odd to deny claim 3 while endorsing 3*. That is, it is rather odd to say that an entity can occupy two parthood slots in two distinct wholes, but not within a single whole. To see why, consider the (a?) fusion of a pair of overlapping entities. Let composite object o₁ be composed of simples a and b, and let composite object o₂ be composed of simples b and c. Let o₃ be the (a?) fusion of o₁ and o₂.

How many simple proper parts does o₁ have? Two. How many simple proper parts does o₂ have? Two. How many simple proper parts does o₃ have? Well, o₃ is the result of combining two simple proper parts with two simple proper parts, so—four. Wait, no, three!

More carefully, utilizing the distinction between slot and occupants: o₁ has two filled parthood slots, as does o₂. Yet there are only three entities doing the filling. Between them, that is, there are four slots and three occupants. And here is the oddity: claim 3* allows b to do double duty across wholes, and occupy two parthood slots. But claim 3* does not allow b to do double duty within the fusion. It only allows the fusion o₃ to have three filled parthood slots. So where did the fourth one go? According to the view that denies claim 3 but endorses claim 3*, fusions are not only “nothing over and above” their parts, but are in fact “something less and below” their parts! On this view, mereological structure is lost by fusion. 3* counts four parthood slots across o₁ and o₂, but only three in their sum.
One lesson to be learned here is simply that the very fact that we can easily understand both answers to the question, “how many parts does o3 have?” helps show, again, that factoring parthood into slots and occupants is not so very foreign. But the other lesson is that, having thus factored parthood into slots and occupants, 3 starts looking better than 3*. We start to see reason to allow objects to have parts twice over.

But if we want to agree to that, we need to rethink mereology. In the next section, I codify this nascent picture into a formal system. It will be consistent with all three claims from this section, though henceforth the focus is on claim 3.

6. A Formal System

To get us going, recall that formal mereologies typically start with the predicate calculus with identity, and add one primitive predicate, usually parthood (though in principle it could instead be proper parthood or overlap). It is at this point—before the addition of any axioms—that the role-approach first diverges.

Instead of introducing one primitive predicate P, we introduce two. Let P_s be a primitive predicate intuitively understood as being a parthood slot of something. Let F be a primitive predicate intuitively understood as filling or occupying a parthood slot.\(^{11}\) That is, P_sxy means that x is a parthood slot of y, and Fxy means that x fills y. Now we introduce the parthood predicate by definition, rather than taking it as a primitive:

\[
P_{\text{Parthood}} = \text{df} \exists y \exists z y \in z \land F_{xz}
\]

Intuitively: x is part of y just in case x fills one of y’s parthood slots.

Other predicates can also be introduced by definition. Here are four, two familiar and two less so:

- **Proper Parthood**
  \[
  \text{PP}_{xy} = \text{df} P_{xy} \land \neg P_{yx}
  \]
  Intuitively: x is a proper part of y just in case x is part of y but y is not part of x.

- **Overlap**
  \[
  \text{O}_{xy} = \text{df} \exists z (P_{zx} \land P_{zy})
  \]
  Intuitively: x and y overlap just in case they share a part (just in case something fills a parthood slot in both).

- **Slot-Overlap**
  \[
  \text{O}_{sxy} = \text{df} \exists z (P_{szx} \land P_{szy})
  \]
  Intuitively: x and y slot-overlap just in case they share a parthood slot z.

- **Proper Parthood Slot**
  \[
  \text{PP}_{sxy} = \text{df} P_{sxy} \land \neg F_{yx}
  \]

\(^{11}\) ‘F’ for ‘filling’ rather than ‘O’ for ‘occupying’ in order to leave ‘O’ available for ‘overlap’.

Intuitively: $x$ is a proper parthood slot of $y$ just in case $x$ is a parthood slot of $y$ that $y$ does not fill.

On this approach, note, we quantify over both roles and occupants.\(^{12}\)

Where do we go from here? Wherever we want. The only difference thus far is that parthood is not a primitive, but defined in terms of two primitives: for $x$ to be part of $y$ is for $x$ to fill one of $y$’s parthood slots. This defined notion of parthood can figure in the same axioms and theorems as the primitive notion does. Further, the above definitions of proper parthood and overlap are exactly the usual ones. Consequently, the nonstandard starting point is compatible with any formal system, including classical extensional mereology (see Simons 1987 and Varzi 2010 for excellent overviews)—a system that does not allow objects to have a part multiple times over. The unusual choice of primitives allows, but does not force, a nonstandard system. It all depends on what axioms are chosen. As an illustration, I shall build up a system that does allow objects to have parts multiple times over. Again, it is but one possibility among many. I will gesture towards some other options once my own suggested system is on the table.

How shall our primitive relations, slothood and occupancy, behave? I hereby stipulate that both relations only hold between entities of different sorts, which I shall call ‘slots’ and ‘fillers’ for lack of better terms. (To fix ideas, you can think of ‘fillers’ as material objects, though the picture I am sketching is intended to be neutral about whether things other than material objects can stand in parthood relations.) To see the thought here, compare the fact that causal roles are occupied by entities, or properties, or property-instances… not causal roles. And just as causal roles neither themselves have nor fill causal roles, parthood slots neither themselves have nor fill parthood slots. Instead, both the occupancy relation and the slothood relation hold only between fillers and slots: only fillers fill slots, only fillers have slots, and only slots are filled. These ideas are captured by the following three axioms:

(A1) Only Slots are Filled
\[
F_{xy} \rightarrow \exists z P_{sz} z
\]
Intuitively: if $x$ fills $y$, then $y$ is a parthood slot of something.

(A2) Slots Cannot Fill
\[
F_{xy} \rightarrow \neg \exists z P_{xz} z
\]
Intuitively: if $x$ fills $y$, then $x$ is not a parthood slot of anything.

\(^{12}\) Is this a cost? Is this ontologically profligate? No more so than functionalism is; functionalists quantify over causal roles without qualm. The notion of a mereological role, or parthood slot, can be understood in second-order terms just as functional roles can be. I suspect that the reason parthood slots sound creepier than causal roles do is simply that I have been discussing them in much more general terms than functionalists do. Functionalists do not, on the whole, say much about what it is to be a causal role, how they are individuated, and so forth. They instead take some particular property, like *pain*, and try to characterize the particular causal role that is definitive of it.
(A3) Slots Don’t Have Slots \[ P_{xy} \rightarrow \neg \exists z P_{xz} \]

Intuitively: if \( x \) is a slot of \( y \), \( x \) does not itself have any parthood slots.

It follows from axioms (A1)-(A3) that both occupancy and slothood are irreflexive, asymmetric, and (trivially) transitive.

(T1) Filler-Irreflexivity \[ \neg F_{xx}^{13} \]
(T2) Filler-Asymmetry \[ F_{xy} \rightarrow \neg F_{yx}^{14} \]
(T3) Filler-Transitivity \[ (F_{xy} \& F_{yz}) \rightarrow F_{xz}^{15} \]
(T4) Slot-Irreflexivity \[ \neg P_{sx}^{16} \]
(T5) Slot-Asymmetry \[ P_{xy} \rightarrow \neg P_{yx}^{17} \]
(T6) Slot-Transitivity \[ (P_{xy} \& P_{yz}) \rightarrow P_{xz}^{18} \]

I have relegated the proofs of these theorems to footnotes.

Here are four further axioms characterizing slothood and occupancy. First, I stipulate that there are improper parthood slots: everything that has parthood slots at all has a parthood slot that it itself fills.

(A4) Improper Parthood Slots \[ \exists y P_{yz}x \rightarrow \exists z P_{xz}y \& F_{xz} \]

Second, things ‘inherit’ the slots of the things that fill their slots—the slots of a thing’s slot occupants are slots of that thing too. (If you like: the slots of the fillers are slots of the filled.)

(A5) Slot Inheritance \[ [(P_{z,y}x \& F_{xz}) \& P_{z,x}y] \rightarrow P_{z,y}x \]

Third, distinct entities cannot mutually fill each other’s parthood slots.

(A6) Mutual Occupancy is Identity \[ (P_{z,y}x \& F_{xz}) \& (P_{z,x}y \& F_{yz}) \rightarrow x=y \]

And fourth, every parthood slot is filled exactly once.

(A7) Single Occupancy \[ P_{xy} \rightarrow \exists ! z F_{xz} \]

(A7) combines two very plausible ideas. One is that there is no bare mereological structure; there are no unfilled parthood slots. The other, deriving from the second platitude from section 5, is that no parthood slots have more than one occupant simultaneously.

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13 Proof that (T1) follows from (A1) and (A2): suppose \( F_{xx} \) for **reductio**. By (A1) \( x \) is a parthood slot of something, but by (A2) \( x \) is not a parthood slot of anything. Contradiction.
14 Proof that (T2) follows from (A1) and (A2): if \( F_{xy} \), then by (A1) \( y \) is a parthood slot, and so by (A2) cannot fill anything. In particular, it does not fill \( x \): \( \neg F_{yx} \).
15 Proof that (T3) follows from (A1) and (A2): if \( F_{xy} \), then by (A1) \( y \) is a parthood slot. By (A2), \( y \) does not fill any parthood slot. So (\( F_{xy} \& F_{yz} \) → \( F_{xz} \) is vacuously true.
16 Proof that (T4) follows from (A3): suppose \( P_{sx} \) for **reductio**. Then \( x \) has a parthood slot (namely \( x \)), but by (A3) \( x \) does not have any parthood slots. Contradiction.
17 Proof that (T5) follows from (A3): if \( P_{xy} \), then \( x \) is a parthood slot. By (A3), it cannot have any parthood slots. So, **a fortiori**, it cannot have \( y \) as a parthood slot.
18 Proof that (T6) follows from (A3): if \( P_{xy} \), then \( y \) has a parthood slot. By (A3), \( y \) can’t be a parthood slot of anything. So (\( P_{xy} \& P_{yz} \) → \( P_{xz} \) is vacuously true.
All of these axioms govern our primitive predicates, slothood and occupancy. But it should be clear that various theorems follow about parthood. In particular, we can recover the transitivity, anti-symmetry, and almost the reflexivity of parthood. (Again, I relegate the proofs to footnotes.)

(T7) Transitivity  \( (P_{xy} \& P_{yz}) \rightarrow P_{xz} \)
(T8) Anti-Symmetry  \( (P_{xy} \& P_{yx}) \rightarrow x = y \)
(T9) Conditional Reflexivity  \( \exists z P_{szx} \rightarrow P_{xx} \)

In the current system, the reflexivity of parthood is restricted to things that have parthood slots. That’s because (A3) and the definition of parthood entail that parthood slots cannot have parts at all. They therefore are not parts of themselves. This might seem odd, but it is not in fact much of a departure from standard mereology. It simply makes explicit a commitment to the idea that not everything is even up for having parts. That idea can be, and often is, combined with standard mereology. For example, some people think that the parthood relation only obtains between material objects. But anyone who both thinks that and believes in abstract entities will need to similarly conditionalize reflexivity. On such a view, not everything is part of itself. The fact that my system makes explicit something that is typically left implicit should not be a mark against it.

Let us pause to draw two interesting lessons. First, although the system as I have thus far characterized it requires that every parthood slot be uniquely occupied—that is (A7)—it does not require that each occupier occupies exactly one parthood slot. The current system allows an entity to fill more than one slot. Indeed, that was the third of our three platitudes about roles and occupants, and it is reflected in the formal system by the fact that

\[ \forall x (\exists y P_{xy} \leftrightarrow \exists z P_{szx})^{22} \]

19 Proof that (T7) follows from (A5), (A7), and the definition of parthood: suppose for conditional proof that \( x \) is part of \( y \) and \( y \) is part of \( z \). That is, \( y \) has a parthood slot \( s_y \) that \( x \) fills, and \( z \) has a parthood slot \( s_z \) that \( y \) fills. (A5) entails that \( z \) inherits \( y \)'s parthood slots, which include \( s_y \). So \( z \) has a parthood slot \( s_z \) that \( x \) fills (and, by (A7), only \( x \) fills). So \( x \) is part of \( z \). Thus \( (P_{xy} \& P_{yz}) \rightarrow P_{xz} \).

20 Given the definition of parthood, this is equivalent to (A6).

21 Proof that (T9) follows from (A4) and the definition of parthood: (A4) says that anything that has parthood slots at all has a parthood slot that it itself fills. By the definition of parthood, it follows that anything that has parthood slots is a part of itself. That is (T9).
But it is not a theorem of the system that everything has the same number of parts as parthood slots, nor the same number of proper parts as proper parthood slots. On the current picture, to be a part is to be the occupant of a parthood slot. Thus the number of parts a composite has is the number of entities that fill its parthood slots. Consequently, composites can have fewer parts than parthood slots. They do so when at least one filler occupies more than one slot—which is, of course, exactly when they have one or more parts twice over.

Notice too that the notion of “having a part twice over” here exactly matches the examples from the beginning of the paper, when I first suggested that a relation can hold multiple times between the same relata. A relation holds multiple times between the same relata just in case those relata more than once satisfy a sufficient condition on the relation’s obtaining. Consider, again, the example of the cousinhood relation. Two people are cousins when one of the first person’s parents is sibling-related to one of the second person’s parents. That is a sufficient condition on the obtaining of the cousinhood relation. And it is a sufficient condition that the very same two people can satisfy twice: namely, when both of the first person’s parents are sibling-related to the second person’s parents. Two people are cousins twice over when they twice over satisfy the sufficient condition on cousinhood. Similarly here. Something is twice over a part of something else when the two twice over satisfy a sufficient condition on the obtaining of the parthood relation. That sufficient condition is simply that the first fills a parthood slot of the second. And like the condition on cousinhood, this is a sufficient condition that the very same things can satisfy twice: namely, when the first object fills two distinct parthood slots in the second. (We can now see a new way of understanding why classical extensional mereology cannot make sense of parthood-twice-over. In taking parthood as primitive, it provides no sufficient conditions on parthood, let alone one that can be met twice.)

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22 Proof of (T11), right-to-left: Suppose a has a parthood slot b. By (A7), b is uniquely filled by some c. By the definition of parthood, c is a part of a. Proof of (T11), left-to-right. Suppose a has a part, b. By the definition of parthood, b fills a parthood slot of a. So a has a parthood slot.

23 Proof of (T12), right-to-left: Suppose a is a proper parthood slot of b. By the definition of proper parthood slot, a is a parthood slot of b, not filled by b. By (A7), a is uniquely filled, so it is filled by some c≠b. Because c fills a parthood slot of b, c is a part of b. Because c is part of b but c≠b, (T8) then entails that b is not a part of c. By the definition of proper parthood, b is a proper part of c. So the fact that b has a proper parthood slot entails that it has a proper part. Proof of left-to-right: Suppose a is a proper part of b. By the definition of proper parthood, a is part of b and b is not part of a. By the definition of parthood, a fills a parthood slot of b—call it c. By (T8), b≠a. So c is filled by something distinct from b, and is therefore a proper parthood slot of b. Thus the fact that b has a proper part entails that it has a proper parthood slot.
So far, so good. But it might be wondered whether I have been too permissive. The system as thus far characterized not only allows things to be parts of the same whole twice over, but seemingly allows distinct wholes to have all of the same parts. This is a result of the fact that I have not taken the following as axioms or theorems:

**X Strong Supplementation**
\[ \neg P_{yx} \rightarrow \exists z (P_{zy} \& \neg O_{zx}) \]
Intuitively: if \( y \) is not a part of \( x \), then \( y \) has a part \( z \) that is neither part of \( x \) nor such that \( x \) is part of it.

**X Weak Supplementation**
\[ P_{xy} \rightarrow \exists z (P_{zy} \& \neg O_{zx}) \]
Intuitively: if \( x \) is a proper part of \( y \), then \( y \) has another part \( z \) that is neither part of \( x \) nor such that \( x \) is part of it.

**X Extensionality**
\[ (\exists z P_{zx} \lor \exists z P_{zy}) \rightarrow (x=y \leftrightarrow \forall z (P_{zx} \leftrightarrow P_{zy})) \]
Intuitively: composite objects are identical just in case they have exactly the same proper parts: iff the very same entities occupy their proper parthood slots.

Yet I did promise to do more than just deny the axioms that cause the trouble. And there is clearly something right about these further claims, especially weak supplementation and extensionality. As Simons says (1987, 26), weak supplementation seems definitive of the very concept of a proper part. And extensionality captures the seeming truism that two distinct entities cannot be exactly alike mereologically, that difference must somehow be mereologically reflected.

I now have the resources to capture these thoughts in a different way. This is easiest to see with respect to the extensionality intuition. The crucial question is, how must distinctness be ‘mereologically reflected’? In the entities that are proper parts, or in mereological structure? Instead of requiring that any two distinct entities have at least one difference in the entities that occupy their parthood slots, perhaps we can simply require that distinct entities have at least one difference in their parthood slots themselves.

Return to the early example of the putative composite \( d \). \( d \), recall, is the fusion of \( c \) with \( c \)’s proper parts, the simples \( a \) and \( b \). Consider again the question of whether \( c \) and \( d \) “have all the same parts”—a phrase which now seems rather crude. From the current perspective, there is a sense in which the answer is yes, and a sense in which the answer is no. In a sense, \( c \) and \( d \) each have two parts. It is the occupants that are the parts, after all, and exactly the same entities ultimately occupy \( c \)’s parthood slots as occupy \( d \)’s parthood slots—namely, \( a \) and \( b \). They do all the parthood work, as it were. But \( d \), unlike \( c \), has some
of those parts twice. c and d therefore differ in mereological structure: c has two proper parthood slots, and d has five. Those five slots are occupied by a, b, c, and, by transitivity, c’s proper parts... a and b. So there is a perfectly good sense in which c and d obey the basic extensionality intuition. Their distinctness does come with mereological difference.

Indeed, given the new understanding of parthood, c and d in fact obey the standard formulation of the extensionality of proper parthood, namely Extensionality just above. But they do not obey either Strong or Weak Supplementation, and there is a more unified way to accommodate such intuitions into the new system. Since the key thought has been that mereological difference can be measured in terms of parthood slots rather than just in terms of what ultimately occupies them, the original principles can be reformulated in terms of slothood rather than parthood proper. Indeed, all that needs to be added as an axiom is the rather untidy looking:

\[(A8) \text{Slot Strong Supplementation} \quad \exists z \exists \! P \! s \! z \! x \& \exists \! P \! s \! z \! y \rightarrow \neg (\exists \! z \! P \! s \! z \! x \& F \! y \! z \! y) \rightarrow \exists \! z (P \! s \! z \! y \& \neg P \! s \! z \! x)\]

Intuitively: if x and y have parthood slots, and y is not a part of x, then y has a parthood slot that isn’t a parthood slot of x.

Hopefully the intuitive gloss is clear enough, and makes up for all the variables. The extra complexity simply stems from the fact it is conditional on x and y both having parthood slots. It must be conditional for almost the same reason as the reflexivity of parthood must be. Just as it follows from (A3) and the definition of parthood that parthood slots cannot have parts, it follows from (A2) and the definition of parthood that parthood slots cannot be parts. Thus one easy way for some y to fail to be a part of x is by being a parthood slot. But if it is, then an unconditionalized Slot Strong Supplementation principle would entail that y—a slot—has to have a slot that isn’t a slot of x, contradicting Slots Don’t Have Slots (A3). (Note that a classical extensional mereologist who thinks that not everything can have or be a part needs to similarly restrict her own strong supplementation principle, just as she also needs to restrict reflexivity.)

24 Slot Strong Supplementation also differs from classical strong supplementation in that the final predicate is not an overlap predicate. I.e. one might think that the slot version would be \(\exists z (P \! s \! z \! x \& \exists \! z \! P \! s \! z \! y \& F \! y \! z \! y) \rightarrow \exists \! z (P \! s \! z \! y \& \neg P \! s \! z \! x)\). But this is trivial. Since parthood slots do not themselves have slots, the parthood slot z cannot slot overlap anything, let alone x. So the consequent would be satisfied as long as y has any parthood slot at all—which the antecedent guarantees.

25 Suppose one thinks that numbers neither have parts nor are parts of anything. Then the number 4 is not part of my toaster. By the standard Strong Supplementation principle, the number 4 has to have a part which is not part of my toaster, and such that my toaster is not part of it. But the number 4 can’t have a part which is not part of my toaster, ex hypothesi, the number 4 does not have parts at all.
everything has parts or can be a part. In classical extensional mereology, that would be an additional metaphysical commitment, outside the formal system.)

The system that results from adding strong supplementation to (A1)-(A7) entails the following reformulated versions of weak supplementation and extensionality:

\[(T13) \text{ Slot Weak Supplementation } \quad PPxy \rightarrow \exists z(Pszy \& \sim Pszx)\]

Intuitively: if \(x\) is a proper part of \(y\), then \(y\) has a parthood slot \(z\) that isn’t a parthood slot of \(x\).

\[(T14) \text{ Slot Extensionality } \quad (\exists PPzx \& \exists PPzy) \rightarrow [x=y \leftrightarrow \forall z (PPszx \leftrightarrow PPszy)]\]

Intuitively: composite objects are identical just in case they have exactly the same proper parthood slots.

These three principles capture at least some of what drives the original ones. They do require that there be a mereological difference between distinct entities, and that there be a remainder between a whole and each of its proper parts. They simply allow that such differences and remainders be structural rather than material. The principles therefore permit \(c\) and \(d\) to be distinct, and indeed permit \(c\) to be a proper part of \(d\). More generally, they allow two distinct objects to differ in mereological structure despite having the same entities as parts. The system that results from adding (A8) to the previous axioms still allows a single thing to occupy multiple parthood slots in a single whole, and therefore still allows the parthood relation to obtain multiple times between the very same entities. Having a part twice over is consistent with these supplementation principles.

Perhaps it is time for another picture, both to visually illustrate the system, and to provide a concrete model demonstrating its consistency. Consider again the trusty, if weird, composite \(d\). Here is what it looks like with slothood and occupancy explicitly represented.

(At-home exercise: draw the picture for the more straightforward case of an object with only two simple proper parts.)

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26 Proof of (T13). Assume for reductio that (T13) fails: let \(a\) be a proper part of \(b\), and yet \(b\) has no parthood slot that are not parthood slots of \(a\)—i.e., all of \(b\)’s parthood slots are also slots of \(a\). The fact that \(a\) is a proper part of \(b\), together with the definition of proper part, entails that \(b\) is not part of \(a\). By (A8), then, \(b\) has a parthood slot that is not a parthood slot of \(a\). Contradiction.

27 Proof of (T14). Assume for reductio that (T14) fails: let \(a\) and \(b\) be distinct composites that have exactly the same parthood slots. (The other direction of the consequent is guaranteed by Leibniz’s Law: no composite can have different proper parthood slots from itself.) By (T8), either \(a\) is not a part of \(b\) or \(b\) is not a part of \(a\). If \(a\) is not a part of \(b\), (A8) entails that \(a\) has a parthood slot that is not a parthood slot of \(b\). If \(b\) is not a part of \(a\), (A8) entails that \(b\) has a parthood slot that is not a slot of \(a\). Neither consequent obtains, because \(a\) and \(b\) have exactly the same parthood slots. Contradiction.
Letters represent the entities that occupy slots. Boxes represent slots, named s₁–s₆. Inclusion within a box represents occupancy—i.e., the letter inside a box occupies that slot. Dotted lines between a box and a letter represent slothood—i.e., that the box is a parthood slot of the letter, either directly or by Slot Inheritance.

Here is what the picture tells us. a and b are simples and therefore have only one (improper) parthood slot apiece—s₁ and s₂, respectively. a occupies two parthood slots, s₁ and s₅; similarly for b, which occupies slots s₂ and s₆. c only occupies one parthood slot (s₃), but has three—s₁, s₂, and s₃—two of which are proper, and one improper. d has six parthood slots: s₄ (improper), s₁, s₅, and s₆ (directly), and s₁ and s₂ (via Slot Inheritance). To zoom in on the particularly interesting part: a is an improper part of a, a proper part of c, and—twice over—a proper part of d. Mutatis mutandis for b. I leave it to the reader to ascertain that all of the axioms are satisfied by this model; the system is consistent.

7. Expanding or Otherwise Modifying the System

I have already emphasized that this formal system is but one way to develop the basic insight about slothood and occupancy. There are other ways to go: more axioms can be added, and perhaps some should be dropped. I will make some brief remarks about possible additions and subtractions in turn.
Possible additions. Notably missing are any axioms that take us from parts to wholes, rather than wholes to parts—that is, any axioms governing composition rather than decomposition (see Varzi 2010, §§3 and 4). This is only because, having said so many controversial things already, it seems wise to quit while I am ahead. Here are three ways in which my new machinery impacts decisions about which composition principles to adopt.

First, a fully unrestricted axiom of unrestricted composition is off the table. The reason should by now be familiar. (A2) and the definition of parthood together entail that parthood slots cannot be parts of anything. It follows that there are entities that have no fusion. Still, it remains an open option to say that any things that are up for being parts have a fusion—to say that there are trout-turkeys and all the rest. (It should be no surprise that the restriction that yields this almost-unrestricted composition principle is a cousin of the restrictions embedded in Conditional Reflexivity and Slot Strong Supplementation.)

Second, unrestricted composition becomes a little tricky even after we have set the slots aside. Just as the axioms allow (non-slot) things to have parts twice over, they also allow them to have multiple fusions. We cannot point at some (non-slot) entities, and speak of their unique fusion. For that has been said, two things a and b might have multiple fusions: the fusion of a and b, the fusion of a and a and b, and so forth.

Third, and relatedly, adding an almost-unrestricted composition axiom to the system thus far entails that the world is junky—i.e., that everything is a proper part of something else. This is quite different from classical extensional mereology, which is taken to be incompatible with junk largely because of unrestricted composition (see Bohn 2009, Schaffer 2010, 64). To see this, imagine a world with two simple fillers, a and b. Almost-unrestricted composition entails that they have a fusion; call it c. But if every two or more things have a fusion, then every composite must fuse with each and all of its own proper parts. So c and a must compose something, as must c and b, and c and a and b (also known as d). Now, unrestricted composition in a classical extensional setting strictly speaking also entails this—though I must say I’ve never seen anyone mention it. The difference is that, in such a setting, Strong Supplementation entails that the ‘various’ composites built out of a and b are all identical. Not so here.

28 Proof: Let c be composed of simples a and b, and e be composed of c and a. Strong Supplementation entails that if e is not part of c, e must have a part which does not overlap c. But it doesn’t. So e must be a part of c. But c is a part of e, so anti-symmetry entails that e=c.
All of that is merely to gesture at some of the interesting issues that arise when adding compositional principles to the basic system I have laid out. The formal system in section 6 is officially silent on these matters.

What about alternatives to that formal system that delete or modify axioms rather than adding more? There are many possibilities; here are some notable ones. First, some might prefer to leave out Slot Inheritance (A5). The only reason to include it, after all, is to derive the transitivity of parthood (T7) from the behavior of slothood and occupancy, and not everyone wants parthood to be transitive. Second, some might prefer to leave out Single Occupancy (A7), namely those who found the second platitude/claim from section 5 less than compelling. To reject (A7) is to claim that not every parthood slot is uniquely occupied. It could be rejected altogether—permitting unoccupied parthood slots—or it could be replaced with a weaker axiom that requires all slots to be occupied, but not to be uniquely occupied. Third, some might wish to allow different kinds of things to obey different axioms. For example, perhaps parthood is transitive for regions of space-time but not for material objects. Presumably the way to pursue this option is to isolate whatever core set of axioms are constitutive of slothood and occupancy, and treat the rest as substantive metaphysical hypotheses about the mereological behavior of certain kinds of thing. Fourth, some might wish to explicitly forbid or allow things to change either occupants of parthood slots or parthood slots themselves. Either would amount to axiomatizing a kind of mereological anti-essentialism.

8. Potential Applications

We have already seen two interesting consequences of thinking about mereology this way. First, it enables the formulation of different versions of mereological essentialism. Someone might well wish to claim that objects have their mereological structure—their parthood slots—essentially while denying that they have their particular entity-parts essentially. Second, it enables a richer way to count the parts of a fusion of overlapping objects; it makes sense of the inclination to count the overlapping parts both once and twice. I will now add two other potential applications to this list, bringing us to a total of four.

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29 Thanks to an anonymous referee for some ideas here.
Third, the system might offer a new tool for those who wish to countenance spatio-temporally coincident entities. Consider again c and d: they are obviously spatio-temporally coincident, and in a certain sense share all their parts, just as stock examples like Lumpl and Goliath are usually taken to. Yet there is another sense in which they do differ mereologically—they have the same entity-parts, but differ in how those entities occupy their parthood slots. Thus we have a kind of coincidence that does not violate extensionality, or at least a version of extensionality. Yet it also does not involve anything other than material parts, unlike Kathrin Koslicki’s view that spatio-temporally coincident entities do not violate extensionality because they have different formal parts (2008). So, if the basic idea could be extended beyond the case of c and d to the stock examples, perhaps there is a new route for defenders of coincident entities to maintain extensionality. Admittedly, I am not at all sure that it can be so extended. Would the claim be that the same material entities occupy different slots in Lumpl and Goliath? I also do not think that preserving extensionality obviates all of the hard questions about coincidents. (It dissolves the question, “how can they differ while being mereologically indiscernible?” But it does nothing to dissolve the question, “how can they differ mereologically while being otherwise indiscernible?”.) Nonetheless, I think it is important to see the new possibility here.

Fourth and finally, it provides exactly what David Armstrong needs in order to vindicate his views about structural universals and states of affairs. He believes that both are, as I put it elsewhere (blind citation 1), built—structural universals are built of other universals, and states of affairs are built of universals and individuals. Yet he also recognizes that neither can be composed of their parts in the ways licensed by classical extensional mereology. For one thing, some structural universals, such as methane, have other universals as parts multiple times over. For another thing, distinct states of affairs can be built out of the same constituents. (Consider a non-symmetric relation like loves; the state of affairs of John’s loving Mary is, tragically if familiarly, different from the state of affairs of Mary’s loving John.) So, “under the benign prodding of David Lewis” (1988, 312), who convinced him that “it’s no good thinking that a structural universal is composed of simpler universals which are literally parts of it” (Lewis 1986, 40-41), Armstrong came to claim that neither structural universals nor states of affairs are composed of parts at all. Instead, he claims that

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30 Admittedly, role-mereology doesn’t by itself to anything to block the regress worry (e.g. 1997, 117-118), but it can be supplemented with the same remarks Armstrong in fact makes about that.

This is unnecessary, even aside from any of the unorthodox machinery I have outlined here. What Armstrong needs to do—and knows he needs to do—is deny extensionality, at least as standardly construed. But that does not require postulating a new, mysterious, and oddly named\(^\text{31}\) relation; it just requires saying that ordinary composition is not extensional. In other words, Armstrong and Lewis should not be having a disagreement about what relation holds in these cases, but rather a disagreement about the nature of composition. The problem here is that Lewis assumes, and Armstrong follows him, that no relation counts as parthood unless extensionality holds. But is that part of the meaning of ‘parthood’? Unless it is analytic that two entities cannot be composed of the same parts—unless ‘non-extensional mereology’ is a contradiction in terms—Lewis is not automatically entitled to the terms ‘parthood’ and ‘composition’, and Armstrong need not have ceded them so quickly. He could speak of ‘non-extensional mereology’ instead of ‘non-mereological composition’.

At any rate, the system sketched above will do what Armstrong needs. We have already seen that it permits a single entity to occupy multiple parthood slots in one whole. It therefore allows the universal *hydrogen* to be a part of the universal *methane* four times over, even though there only exists one of them to occupy all four parthood slots. And it also allows two distinct states of affairs to “have the same parts” in the sense that the same entities occupy the (distinct) parthood slots in both. Finally, it does this without ignoring the extensionality intuition entirely. On the system in question, the universal *methane* (CH\(_4\)) differs mereologically from the simpler hydrocarbon universal *CH*, and the state of affairs of John’s loving Mary differs mereologically from the state of affairs of Mary’s loving John. They differ in mereological structure, rather than in the entities that do the parthood work. Armstrong can now answer Lewis’ questions: “what can it mean for something to have a part four times over? What are there four of?” There are four parthood slots. There need not be four distinct entities that fill them.

8. Conclusion

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\(^{31}\) Since ‘mereological’ simply means ‘of or pertaining to parts and wholes’, presumably all composition is mereological.
I have offered a novel way to formalize parthood relations. The central idea is the splitting of the parthood relation in two. Somewhat secondary is the particular system whose axioms I have sketched; it is largely an example to illustrate the possibilities. But the question nonetheless arises: should we really believe this theory?

Answering that question requires reflecting upon what formal mereology is all about. It is in part simply a formal codification of our ordinary notions of parthood and composition—just conceptual analysis in fancy, symbolic garb. But that is certainly not all it is. Conceptual analysis might secure the antisymmetry of parthood, or that it obeys weak supplementation. But it will not get us the hotly disputed claims of unrestricted composition or extensionality. Mereological systems do not merely make analytic claims about the nature of parthood, but also substantive and controversial philosophical claims about what exists. They are tools that serve philosophical purposes and reflect antecedent commitments. People with different purposes and different antecedent commitments will endorse different mereological systems. And there is value in knowing what the options are. Mereologists need not actually endorse every system they explore any more than modal logicians do.

So, facing the question head on, do I honestly think that objects can have a part twice over? No. But—and this is the crucial point—I do not think that I am entitled to that conclusion on the basis of the nature of parthood. That is, I do not think it is either obvious or analytic that objects cannot have parts twice over. It is a substantive claim in need of argument. I will neither provide one here, nor pretend that I have a decisive one. I will simply say that I am not inclined to believe that objects can have parts twice over for the same basic reason that I am not inclined to believe in spatio-temporally coincident objects more generally: the grounding problem. I do not understand how objects like c and d can differ mereologically, but in no other way; I do not like bare mereological difference any more than I like bare modal difference. Again, that is intended more as a sketch of a mindset than as an argument; I am somewhat conflicted about how serious I really think the grounding problem is (blind citations 2, 3). The point here is not to decide the matter, but simply to gesture at the kinds of consideration that might be relevant. Notice that they do not involve any claims about the nature of parthood or the meaning of the terms ‘part’ or ‘compose’.

Finally, recall that the distinction between being a parthood slot and filling a parthood slot is independent of the particular system I sketched in section 6. That distinction is
intuitive and useful in its own right. The basic overall thought is that there is more to mereology than matter. The mereological structure of an object might reach beyond the particular entities that are its parts.