More Detailed Black-Scholes Problem

A bank with $150,000,000 market value in assets, all invested in fairly high-risk corporate loans. The bank is obliged to repay $140,000,000 in deposit amounts with interest to depositors in three years. There are no other interest payments due to depositors; all deposits mature and are repaid with interest, included in the $140 million amount given above in three years. The current riskless rate of return is 2% per annum and the standard deviation of annual returns on the bank's assets (again, all in corporate loans) is .4. Assume that all Black-Scholes assumptions hold for this bank.

a. What is the initial value of the bank's equity?
b. Suppose that the bank is insured by a government insurer. What is the initial value of the deposit insurance policy to the bank?
c. What is the initial value of the bank's deposits?
d. Now, suppose that this bank promises charges its corporate customers, on average, 3.18% on its loans. Further assume that the bank maintains deposit insurance. Suppose that in three years, none of the bank's corporate borrowers default, so that the value of the bank's assets is $165,000,000 (which happens to be $150,000,000$\times^{.0318\times 3}$). What will be the value of deposits at that time? What will be the value of bank equity at that time? What will be the value of the deposit insurer claim on the bank at that time?
e. Suppose that instead, in three years, many corporate borrowers default, and the value of the bank's assets declines to $105,000,000. What will be the value of deposits at that time? What will be the value of bank equity at that time? What will be the value of the deposit insurer claim on the bank at that time?

Solution

a. At first glance, the book value of equity might appear to be $10,000,000 = $150,000,000 - $140,000,000, or perhaps a somewhat different amount if interest were properly accounted for. However, assets are risky. We know that the bank has an option to default on its deposit obligations and that Black-Scholes assumptions apply. Thus, we will treat equity value as a call option in a Black-Scholes context. More specifically, we seek to obtain the bank's risky equity value, based on the assumption that the equity is a call option to purchase the bank's assets (omitting $millions in some formulas for simplicity). We solve first for d_1, then using the z-Table (or spreadsheet), solve for N(d_1). We follow by solving for d_2, N(d_2) and finally, equity value:

$$d_1 = \frac{ln\left(\frac{150}{140}\right) + \left(0.02 + \frac{1}{2}.4^2\right) \times 3}{0.4 \times \sqrt{3}} = 0.5326;\ N(d_1) = 0.7028$$

$$d_2 = 0.5326 - 0.4 \times \sqrt{3} = \ -.16;\ N(d_2) = 0.4364$$

$$Equity\ Value = c_0 = 150{,}000{,}000 \times 0.7028 - \frac{140{,}000{,}000}{e^{.02\times 3}} \times 0.4366 = 47{,}894{,}763$$

Note: see the description for using the z-table following all parts of the problem solution.)

b. Next, we begin the process of valuing the firm's debt, which can be regarded to be risky due to the bank's option to fail. We first use put-call parity to value the risk premium associated with bank deposit default, assuming that the firm's shareholders might put the firm's assets to creditors by refusing to pay debt obligations:

$$p_0 = 47{,}894{,}763 + 140{,}000{,}000 \times .94176 - 150{,}000{,}000 = 29{,}741{,}798$$

The value 0.94176 is the 3-year continuous time discount function ($e^{-rT} = e^{-.02\times 3}$) at a discount rate equal to 2%.

c. Independently of the risk component of debt, the firm has an outstanding obligation of \$140,000,000, payable in three years in an environment with a riskless discount rate equal to 2%:

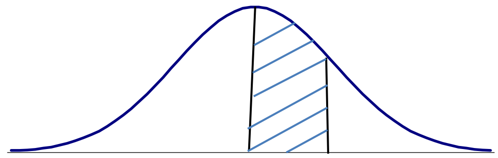
$$Xe^{-r_f T} = 140{,}000{,}000 \times e^{-.02\times 3} = 140{,}000{,}000 \times 0.94176 = 131{,}847{,}035$$

Thus, if the bank's deposits had been riskless, or if they were insured by a fully reliable insurer, they would be worth \$131,847,035 at the present time.

d. The bank succeeds and depositors receive face value X = \$140,000,000. Shareholders receive the residual value c_3 = \$165,000,000-\$140,000,000 = \$25,000,000. The bank insurer stake in this scenario is p_3 = 0.

e. Assets are not sufficient for depositors to receive face value X = \$140,000,000, though, the insurer will add \$35,000,000 to the \$105,000,000 asset value to ensure that depositors are fully paid. Thus, depositors receive all the assets of the bank plus an additional \$35,000,000 from the insurer. Depositors receive the \$140,000,000 that they were promised. Shareholders, as usual, receive the larger of zero or residual value c_3 = MAX[\$105,000,000-\$140,000,000, 0] = 0. Thus, shareholders receive nothing; the bank has failed. The insurer stake in the bank, which is equivalent to a short position on a put on assets, is p_3 = MIN[0, \$105,000,000-\$140,000,000] = -\$35,000,000. Hence, the insurer is obliged to fulfill shareholder obligations to depositors by paying \$35,000,000 to depositors.

Using the z-table



The Normal Density Function
The z-Table

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0159	.0199	.0239	.0279	.0319	.0358
0.1	.0398	.0438	.0478	.0517	.0557	.0596	**.0636**	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0909	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	**.2019**	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2356	.2389	.2421	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2703	.2734	.2764	.2793	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3437	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3906	.3925	.3943	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4986	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Since d_2 = -0.16 is negative, subtract .5 from 0.0636 to obtain 0.436 (ignore the minor rounding differences). Thus, N(d_2) = 0.436.

The areas given here are from the mean (zero) to *z* standard deviations to the right of the mean. To get the area to the left of *z*, simply add .5 to the value given on the table.

Since d_1 = 0.53 is positive, add .5 to 0.2019 to obtain 0.7 (ignore the minor rounding differences). Thus, N(d_1) = 0.7.