Chapter 5: Pricing and Hedging with Forward and Futures Contracts

A. Pricing Forward Contracts

As we discussed earlier, a forward contract represents an agreement that specifies the delivery of given quantity of an asset at a specific future date for a given price. Forward and futures prices specifying delayed delivery fluctuate over time. In this section, we will discuss four theories describing the relationship between futures and cash (or spot) prices over time. These four theories are all consistent with the empirical observation that forward prices and cash prices converge as settlement (delivery) dates approach.

The Expectations Hypothesis

In Chapter 2, we discussed the Pure Expectations Theory describing the relationship between long and short term interest rates. An analog to this very simple theory can be applied to explaining the relationship between cash (spot) and forward prices:

(1) $$F_T = E_0[S_T]$$

where F_T is the forward price of the asset to be delivered at time T, the contract settlement date, and $E_0\,[S_T]$ is expected value at time 0 (now) of the spot or cash price for the underlying asset at time T. One problem with theory is that we can never know with certainty what spot prices investors are expecting as expected prices are a more theoretical construct.

Forward Contracts and Basis

How might one obtain an expected spot price for some future date T? In the absence of asset storage costs, asset consumption needs, dividend, and market inefficiencies, pricing the forward contract relative to the asset spot price might be inferred as follows in a riskless or risk-neutral environment:

(2) $$F_T = S_0 e^{rT}$$

S_0 is the spot price (time 0 price) of the asset, and r is the relevant discount or interest rate. In effect, the price process of the forward contract to purchase an asset at time T at price F_T is similar to that of borrowing S_0 dollars at time zero and repaying S_0e^{rT} at time T. The time 0 and time T cash flows are known or locked in once both spot and forward contracts are locked in.

Suppose, for example, that an investor buys an asset in the spot market for S_0. She could simultaneously sell it forward at a known forward price. What should be this forward price in order to avoid arbitrage opportunities? Her hedged position, long in the commodity at a known price and short in the forward contract on the same commodity at a known price serves to lock in a return on her initial investment S_0. One might expect that her perfectly hedged position in spot and forward markets for the commodity should earn her the riskless rate of return of r over period T.

Note that Equation (2) implies that futures prices will converge to spot prices as settlement dates draw nearer. We will discuss the implications of this shortly. Also note that the difference between the local spot price of the underlying asset S_0 and its forward price F_T, known as the contract *basis* is calculated as $b_0 = S_0 - F_T$. Similarly, at any time t where $0 \leq t \leq T$, the contract *basis* is calculated as $b_t = S_t - F_T$.

Simple Illustration: Commodity Forward Contract

Suppose, for example, that the spot price for one ounce of gold is S_0 = $1,152.95. If the 2-year interest rate is 2%, we calculate the 2-year forward price for gold as follows:

$$F_T = S_0 e^{rT}$$

$$\$1200 = \$1{,}152.95e^{.02\times 2}.$$

In effect, the counterparty taking the long position in gold agrees to purchase one ounce for $1,200 at time T from the counterparty taking the short position in gold. If the counterparty with the short position in gold currently owns one ounce of gold, the forward contract provides her the riskless opportunity (and obligation) to sell her gold for $1,200 in one year, consistent with the absence of arbitrage opportunities given the spot price of $1,152.95 and the riskless rate equal to .02. We might say that the gold market is in *contango* now since the forward price exceeds the spot price. We will discuss contango in greater detail shortly.

Thus far, we have calculated the forward price from the spot price. We could just as easily have calculated the spot price from the forward price. That is, if the forward price were $1,200, then the spot price would be $1,152.95 given the riskless rate of .02 and the absence of arbitrage opportunities. Both scenarios are consistent with the statement that we made above:

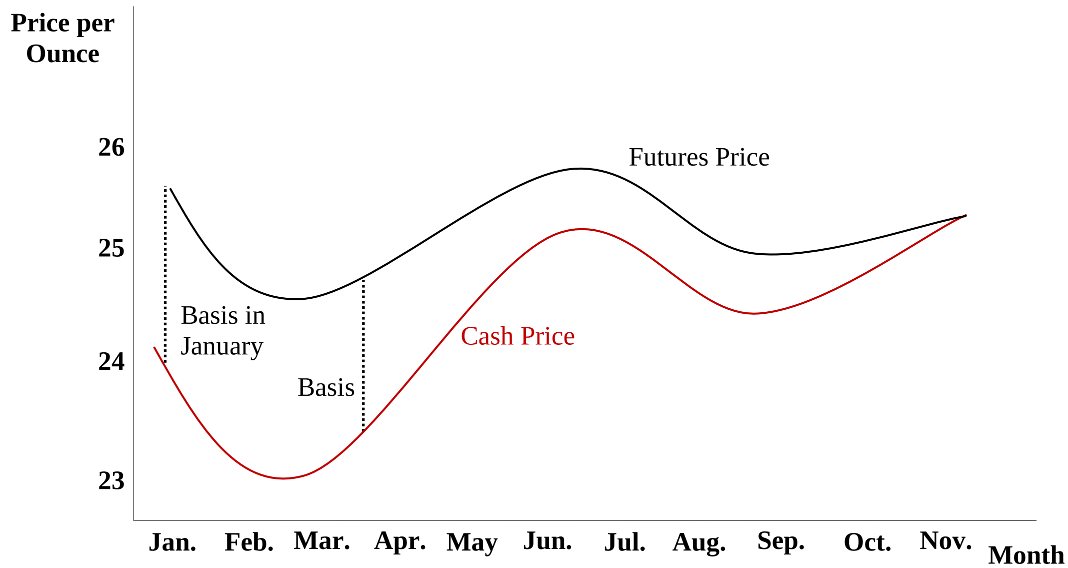
$$0 = \$1{,}200 - \$1{,}152.95e^{.02\times 2}$$

Again, no money changes hands at time zero when the counterparties enter into the forward contract. At time T = 2, the long counterparty receives one ounce of gold and the short counterparty receives $1,200. These figures are consistent with the expectations hypothesis, a cash price equal to $1,152.95 and a 2-year interest rate of .02. These contract terms hold in the absence of credit (default) risk, though other risks and complications might affect the relationship between the spot and forward prices and the veracity of the expectations hypothesis.

Contango

The expectations theory describing the relationship between cash and forward prices is not inconsistent with contango or backwardation (when the cash price exceeds the forward price). Contango and backwardation provide more detail on the nature of the expected relationship.

Positive rates in Equation (2) above is a form of *contango* (sometimes called *forwardation*), a situation in which the futures price of a commodity or financial instrument is higher than the spot price. Contango is common in forward and futures markets in which the underlying commodities are not consumed and when current demand is not particularly relatively high relative to anticipated demand. Contango enables traders who are net long in spot markets and short in forward or futures contracts to earn profits based on the riskless return and spot prices. The contango hypothesis might be based on the observation that commodity or instrument buyers are willing to pay a premium to lock in prices, and might pay a higher price than the expected spot price to achieve that result. There are a variety of forces that might drive contango, including high carry costs (e.g., refrigeration for frozen orange juice concentrate) for the underlying asset and the time value of money.



Calculating the Basis Example:
Cash (Spot) Price in January: 24
November Futures Price in January: 25.5
Basis in January: -1.5 ("1.5 under November" in trading nomenclature)

Figure 1: Cash Price vs. Futures Price: November Silver Contract in Contango Market

Illustration: Contango and Silver Markets

Now, let us consider a different example concerning silver markets and contango. In Figure 1, a snapshot diagram, the basis for the November silver contract is followed over time from January to November. With a cash price of silver at $24 per ounce in January and the November futures price for silver at 25.5 in January, we calculate that the basis for the November contract in January is $S_0 - F_T$ = $24 - 25.5$ = -1.5, or "1.5 under November" in trading lingo. Moving forward in time, the cash price for silver evolved between January and November, as did the futures price. As the settlement date for the November contract drew nearer, the basis became less negative, rising to zero by the settlement date in November. We can see that the market for November contracts on silver was in contango for the entire period from January to November since the futures price remained above the spot price for the entire year.

Now, we consider a third example from a different perspective depicted by the time series of prices in Table 1. Paying particular attention to the Prior Settle column (provides closing trading prices from the prior day), notice that gold prices are generally in contango, with a December 2020 spot price of 1861.7 (not on the table) and with futures contract prices rising as settlement dates fall further into the future. [1] Markets might remain in contango if spot prices and near-term futures prices rise towards convergence with longer-term forward prices.

[1] A second definition for *settle* or *settlement price* used in derivatives markets is the price used for determining profit or loss for the day, and for calculating margin requirements. This price would be the price on the final forward or futures trade for that day.

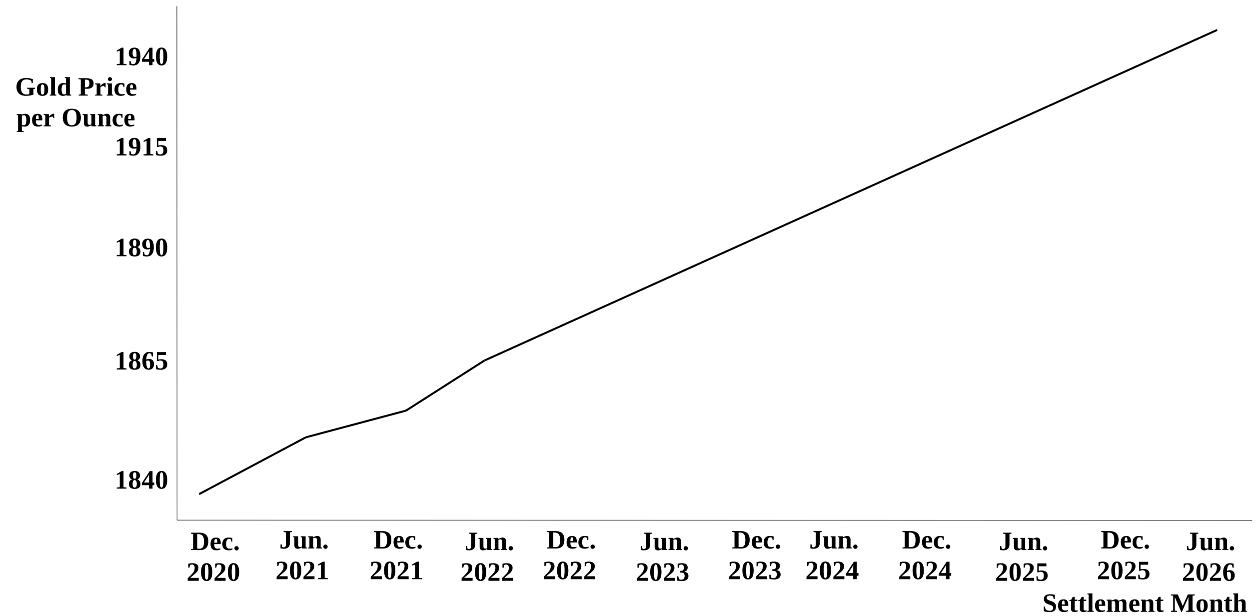
MONTH	LAST	CHANGE	PRIOR SETTLE	OPEN	HIGH	LOW	VOLUME
Dec-20	1861.8	25.9	1835.9	1836.5	1861.8	1820.8	1,049
Jan-21	1862.8	25.3	1837.5	1838.4	1865.2	1823	526
Feb-21	1866.2	26.2	1840	1841.1	1868	1824.8	148,612
Mar-21	-	-	-	-	-	-	0
Apr-21	1870.5	26.5	1844	1846.7	1871.7	1829.2	4,049
Jun-21	1872	25.3	1846.7	1848.5	1874.3	1831.8	1,438
Aug-21	1874.1	25.1	1849	1847.9	1876.5	1834.2	580
Oct-21	1845.5	-5.6	1851.1	1845	1845.5	1845	60
Dec-21	1878.4	24.9	1853.5	1853.2	1880.8	1841.4	731
Feb-22	1873.2	17.6	1855.6	1856.9	1873.2	1850	12
Apr-22	-	-	1858.5	-	-	-	0
Jun-22	-	-	1864.1	-	-	-	0
Aug-22	-	-	1865.6	-	-	-	0
Oct-22	-	-	1869.3	-	-	-	0
Dec-22	-	-	1873	-	-	-	0
Jun-23	-	-	1882.4	-	-	-	0
Dec-23	-	-	1891.7	-	-	-	0
Jun-24	-	-	1901.2	-	-	-	0
Dec-24	-	-	1910.3	-	-	-	0
Jun-25	-	-	1919.1	-	-	-	0
Dec-25	-	-	1928.2	-	-	-	0
Jun-26	-	-	1937.2	-	-	-	0
Dec-26	-	-	-	-	-	-	0

December 7, 2020; Source: CME Group,
https://www.cmegroup.com/trading/metals/precious/gold.html

Table 1: Gold Futures Quotes, per Ounce

Illustration: The Futures Curve and Contango in Gold Markets

The *futures curve* (a graphical representation at a given date of a time series various futures prices against settlement dates) is said to be in contango when the futures price of the nearest contract settlement in time is lower than the futures prices of the next following, with forward or futures contract prices below the underlying instrument's spot price, and with the two converging on contract settlement date. When futures markets are in contango when longer-term futures prices are higher than shorter-term futures prices. Figure 2 depicts a futures curve based on the gold price data from Table 1. Because forward prices in December 2020 rise with more distant settlement dates through June 2026, this futures curve is said to be in contango. Thus, contango exists in a futures market when futures prices increase progressively with later settlement dates.



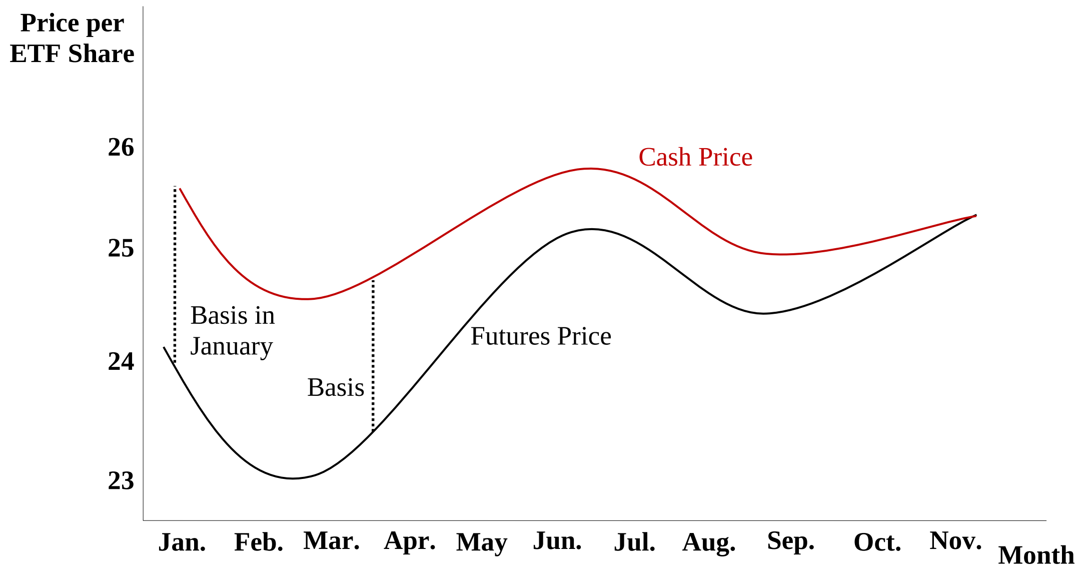
See Table 1 for numerical values
Gold futures settlement dates range from December 2020 to December 2026
All prices are from December 8, 2020, when the spot price is 1835.8
Figure 2: The Gold Futures Curve: December 2020 to June 2026 in Contago

Backwardation

A market is said to be in *backwardation* when the futures price is less than the spot price. Backwardation might suggest that investors expect the underlying asset price to fall over time. Suppose, for example, that bad or dry weather during the growing season might cause spot prices and short term to rise well above long term futures prices due to short term wheat shortages.

The process of convergence of higher forward prices to lower spot prices is known as *normal backwardation* or simply backwardation. Again, one might expect that forward prices will converge to spot prices as contract settlement dates draw nearer. Backwardation implies that an investor who owns and stores the commodity is effectively paying for the privilege to do so, which might make sense if the commodity is to be consumed, is in high current demand, shortage, or if the commodity or financial instrument makes a payment (such as a dividend for a stock or ETF). Another investor with a long position in a forward or futures contract does not have to pay the storage cost, and can hold money instead, but would not receive any ownership benefits.

Consider Figure 3 below, in which the basis for a given ETF on the November contract is followed over time from January to November. Looking back in time, with a cash price of the ETF at $25.5 in January and the November futures price for the ETF at 24, we calculate that the basis for the November contract in January is $S_0 - F_T = 25.5 - 24 = 1.5$, or "1.5 over November" in trading lingo. As the cash price for the ETF evolved between January and November, so did the futures price. As the settlement date for the November contract drew nearer, the basis became smaller, falling to zero by the settlement date in November. We can see that the market for November contracts on ETF shares was in backwardation for the entire period from January to November since the futures price remained below the spot price for the entire year.



<u>Calculating the Basis Example</u>:
Cash (Spot) Price in January: 25.5
November Futures Price in January: 24
Basis in January: 1.5 ("1.5 over November" in trading nomenclature)

<u>**Figure 3:** Cash Price vs. Futures Price: November ETF Contract in Backwardation Market</u>

A common cause of backwardation in commodities' futures markets are shortages of the underlying commodities in spot market. As implied by Figure 5.3, futures investors long in the underlying commodity or instrument will tend to profit from increases in futures prices over time as they converge with spot prices. While not depicted here, the futures curve for a futures market in backwardation would be downward sloping (see Figure 5.2 for a futures curve in contango). Thus, it is likely that the futures curve for different contracts on an ETF maintaining a high dividend yield would be downward sloping, sometimes referred to as an inverted futures curve. Economists, particularly such as John Maynard Keynes from the first half of the 20th century often argued that backwardation occurred because commodity producers were willing to lock in selling prices of those commodities assets at discounts on settlement dates in order to reduce their cash flow risks.

West Texas Crude and Backwardation

Suppose that a weather crisis were to hamper the production of West Texas Intermediate crude oil, causing the current supply of this oil to significantly and rapidly drop. The price of this commodity would skyrocket as traders, distributors and retailers quickly buy oil, driving the from, say, $45 per barrel to $60 per barrel. But, if the market believes that the weather crisis is temporary, futures prices of contracts settling years later might not be affected. Thus, is the spot price increases, and forward prices increase, but by less as settlement dates are later, oil markets would be in backwardation.

<u>The Net Hedging Hypothesis</u>

The Net Hedging Hypothesis essentially argues that if orders for long positions in forward or futures contracts dominate in the market, the result will be contagion, as forward or futures contract prices increase relative to spot contracts. Conversely, as orders to short forward or futures contracts increase relative to long orders, the result will be normal backwardation, as forward or futures contract prices decline relative to spot contracts.

B. Forward and Futures Market Complications

We discussed above the expectations hypothesis concerning the relationship between cash and futures prices above along with scenarios of contango and backwardation. Factors that cause these market conditions complicate our pricing models beyond what we presented in Equations 1 and 2 above but may well intensify conditions of contango and backwardation.

Dividends

In our models above, we generally avoided discussions of complications such as asset storage costs, dividends and other cash flows. For example, consider an equity index forward contract in which the underlying stocks pay dividends. These dividends, when paid (technically, on ex-dividend dates), reduce stock prices by amounts roughly comparable to dividend amounts. The holder of the long position in the forward contract will purchase a stock that is worth less on the settlement date because it went ex-dividend before. Here, we propose simple adjustments for such complications. First, consider the effect of income spun off by the asset prior to time T, where δ is the periodic income or dividend divided by the asset value, normally referred to as dividend yield:[2]

(3) $$F_T = S_0 e^{(r-\delta)T}$$

This type of dividend yield adjustment tends to be most appropriate when the index comprises a larger number of stocks whose ex-dividend dates are spread out over the course of time. A portfolio with a large number of stocks underlying an ETF might be a good candidate for this type of model. The equity dividend payout ratio δ reflects the proportion of equity value that the long position in the forward contract will not accrue from the equity waiting delivery at time T; instead, the holder of the ETF will retain this value. The dividend yield δ reflects asset value that will not be captured by the counterparty with the long position in the forward contract.

Carry Costs

Many assets require cash layouts for storage, transportation, insurance, financing, depreciation and other carry costs. For example, a commodity such as natural gas, incurs a negative cash flow because it requires storage in pressurized containers and may necessitate significant security expenditures. Similarly, many agricultural commodities will incur carry or storage costs and loss of value due to spoilage and rotting, or, in the case of cattle, feeding. The forward price F_T of an asset with carry cost κ (expressed as a proportion κ of asset value) is calculated as follows:

(4) $$F_T = S_0 e^{(r+\kappa)T}$$

Note that this "negative income" involves a simple sign change from our dividend illustration.

[2] See Problem 8 from Chapter 3 for a derivation of Equation (3), though this equation might already seem intuitively obvious to you. Equations (4) and (5) can be derived similarly.

The counterparty with the long position in the forward contract in gas does not incur the cost of storing the gas, making the right to receive gas at a later date more valuable than maintaining possession of the commodity over the relevant period. Not incurring storage costs until time *T* increases the forward price of the gas relative to the spot price to its prospective buyer.

<u>FX and Interest Rates: Interest Rate Parity</u>

In a sense, an FX (foreign exchange) forward contract has a similar complication. An FX contract provides for one counterparty to purchase one currency from a second counterparty in exchange for a second currency. An FX contract can provide for one counterparty to be long in one currency (e.g., buy Chinese yuan) and short in the other (e.g., sell dollars), with the second counterparty taking opposite positions in the two currencies. Each counterparty can earn interest on the currency that he shorts (perhaps the domestic currency from his perspective) until delivering it at time *T*. Suppose that *r*(*d*) is the riskless rate in the country issuing the currency associated with the short position in the contract and that *r*(*f*) is the interest rate associated with the long position in the contract (the foreign currency):

(5) $$F_T = S_0 e^{(r(d)-r(f))T}$$

In this scenario, the long position in a currency forward precludes the counterparty from investing the long currency at the rate *r*(*f*) until time *T*, but provides the opportunity him to invest in the shorted currency at rate *r*(*d*) until time *T*. This relationship between spot and forward rates is known as *interest rate parity*. In a single discrete time period, this relation, which describes an international currency market equilibrium, might be rearranged and portrayed as follows:

(6) $$\frac{(1+r_d)}{(1+r_f)} = \frac{F_T}{S_0}$$

A violation of Equation (6) in actual capital markets should be expected to result in a shift in one or both of the affected interest rates, forward or spot prices of affected currencies or some combination thereof. Such a scenario is illustrated in Figure 4 below.

Illustration 1: USD and CNY

Suppose, for example, one counterparty to a two-year FX forward contract longs *CNY*1 (1 Chinese yuan) in terms of dollars at the rate *USD*0.16 (0.16 U.S. dollars per yuan). This counterparty agrees to purchase 1 yuan for USD0.16 and can invest the *USD*0.16 at rate *r*(*d*) until time *T* = 2. However, this counterparty loses the opportunity to invest *CNY*1 at rate *r*(*f*) until time *T* = 2. Based on interest rates and the forward rate, what should be the spot rate be in dollars per yuan? If the U.S. interest rate is *r*(*d*) = .02 and the Chinese rate is *r*(*f*) = .04, the spot rate S_0 = *USD*0.1665 is consistent with our pricing relation above:

$$0.16 = 0.1665e^{(.02-.04)\times 2}$$

Illustration 2: USD and GBP

Here, assume that exchange rates of dollars for pounds are 1.6 and 1.629629 in spot and one-year forward markets, respectively.[3] Assume that nominal interest rates are 12.5% in the U.S. and 12% in the U.K. We should be able to demonstrate an arbitrage opportunity because Equation (5) and (6) are violated:

$$F_T = S_0 e^{(r(d)-r(f))T}$$
$$1.629629 \neq 1.6e^{(.125-.12)\times 1} = 1.60802$$

$$\frac{(1+.125)}{(1+.12)} \neq \frac{1.629629}{1.60802}$$

Figure 4 depicts transactions and cash flows for an appropriate arbitrage portfolio in the single discrete period case. Figure 4 demonstrates that we can lock in a profit of $15.74 by engaging the above transactions. This is because the change in exchange rates did not coincide appropriately with the countries' relative interest rates. Equations (5) and (6) do not hold for this example and creates this arbitrage opportunity for FX investors.

Transaction Number

1. Borrow $1000 now in the U.S. at 12.5%; repay at Time One
2. Buy £625 now for $1000
3. Loan £625 at 12%; Collect £700 in proceeds at Time One
4. Sell £700 at Time One at F_1=1.629629 for $1140.74; Repay $1125 in U.S. debt

TIME ZERO POSITIONS

Transaction Number	Time Zero Pound Position	Time Zero Dollar Position
1.		+$1000
2.	+£625	-$1000
3.	-£625	
4.		
Totals	0	0

TIME ONE POSITIONS

Transaction Number	Time One Pound Position	Time One Dollar Position
1.		-$1125.00
2.		
3.	+£700	
4.	-£700	+$1140.74
Totals	0	+$15.74

Figure 4: Interest Rate Parity Violation

[3] This illustration and a few others in this chapter were taken from Teall [2018].

References

Teall, John L. (2018): *Financial Trading and Investing*, 2nd ed., Waltham, Massachusetts: Elsevier, Inc.

Exercises

1. Suppose that the spot price of gold is $1,100 per ounce. The current one-year riskless rate of interest is 3%. Assume that gold has no carry costs, is not consumed and no inefficiencies exist of gold forward or futures markets.
   a. Provide a forecast for the price of gold in one year based on a relevant expectations hypothesis. Assume that the demand for gold is not expected to experience any shocks.
   b. Provide an estimate for the one-year forward price for one ounce of gold. Assume that the demand for gold is not expected to experience any shocks.
   c. Justify your use of the expectations hypothesis that you used in your answer to parts a and b.
   d. What might cause the forward price of gold to deviate from your forecast?
   e. How might a higher inflation rate affect the forward or futures price of gold?

2. Why must a forward price equal the spot price on the settlement date of the contract? That is, why much forward prices converge to spot prices at settlement?

3. Suppose that an automobile manufacturer needs to regularly purchase palladium for use in exhaust systems. To protect itself from price uncertainty associated with this metal, the manufacturer regularly takes long positions in palladium futures contracts. How might the regular presence of contango cause the manufacturer to regularly lose money from its futures market transactions?

4. Is a contango scenario more likely with high interest rates or low interest rates?

5. Why do investors who maintain long positions in contracts in futures markets that are in contango incur losses when those contracts settle?

6. Suppose that a stock's price grows over time at rate μ (or r) as the firm produces profit, but the firm also pays a continuous dividend at a rate δ. Further suppose that $S_0 = 50$, $\mu = .05$ and $\delta = .02$. Write an equation that would give the forward price F_T for a contract that settles at any time T.

7. Some observers of derivatives markets agree that publicly traded corporations can use derivatives to mitigate operating risks, but that shareholders would be better off if managers simply allowed shareholders to use their own portfolio management and hedging techniques to do so. In fact, some critics will argue that corporate managers are too concerned about mitigating risks, that they do so in order to protect their own jobs and preserve their own income and stock value.
   a. Why might shareholders be in a better position to mitigate and hedge operating risks of corporations whose shares they own?
   b. Why might shareholders be better off if corporate managers hedge operating risks?

8. Assume that exchange rates of Swiss francs per dollar are CHF1.6/USD in spot markets. Assume that nominal interest rates are 10% in the U.S. and 12% in Switzerland.

a. What would be the forward rate of CHF per USD consistent with the spot rate and two-year interest rates in Switzerland and the U.S.?
b. Demonstrate an arbitrage opportunity if the two-year forward rate in Swiss francs were instead CHF1.8. That is, demonstrate how an investor can obtain an arbitrage profit by simultaneously using spot and forward markets to transact for securities and by borrowing and/or lending money. Further assume no transactions costs or barriers on transactions and that interest rates apply to both borrowing (short) and lending (long).

Solutions

1. a. Like currency, gold is easy to store and transport, divisible, and it is non-perishable. If a one-year selling price for gold can be locked in today with a forward contract, then the relationship between its forward and spot prices should be $F_0 = E_0[S_t] = S_0 e^{rt}$ with continuous compounding. Thus, we use the expectations hypothesis for the relationship between spot and forward rates for gold: $F_0 = E_0[S_t] = \$1{,}100 e^{.03\times 1}$ = \$1,133.
b. Again, use the expectations hypothesis as above: $F_0 = E_0[S_t] = \$1{,}100 e^{.03\times 1}$ = \$1,133.
c. Assuming that the market prices given (\$1,100 spot and 3% riskless rate) are determined by supply and demand, and that the assumptions that we noted in part a are true, this expectations hypothesis might provide for a reasonable relationship between the spot and forward prices for gold.
d. It is important to remember that the forward rate is based on actual market prices of forward contracts whereas an expected spot rate is merely a prediction by an investor, using prevailing information about prices and other factors such as risk preferences and uncertainty to establish. Hence, there are good reasons for the two values to diverge, including inefficiencies in markets for gold or violations in the assumptions that we noted in part a.
e. Higher inflation rates might be expected to increase forward and futures prices of gold.

2. Ignoring market frictions, forward contract prices must equal spot prices for the same underlying asset on the forward contract settlement date in order to eliminate arbitrage opportunities, which would be a violation of the Law of One Price. If the two prices were to be unequal, an investor could take a long position in the lower-priced contract and an offsetting short position in the higher-priced contract and pocket the price differences.

3. The most significant problem arising from contango comes from automatically rolling forward contracts, which means to hedge long term uncertainties with series of short-term contracts that are rolled over (replaced) on settlement dates. Investors who long commodity contracts when markets are in contango tend to lose some money when the futures contracts expire with prices that exceed prevailing spot prices. Even though the futures prices reflect trading losses for the manufacturer, their significance is offset by being able to pay lower prices for the underlying commodity (palladium). The real benefits of the hedge occurs when the market is not in contango; that is, to be able to use profits from the futures market transaction to offset having to pay more for the commodity in spot markets.

4. High interest rates: Higher interest rates lead to higher forward and futures rates relative to spot rates.

5. Investors who take long positions in futures contracts when markets are in contango tend to lose money because their futures contracts settle with prices that are higher than spot prices. If their futures positions are not covered by the underlying instruments or commodities, they need to purchase the underlying commodity at a higher price to deliver and sell at the lower price, or, more common, make an equivalent cash disbursement.

6. $F_T = 50e^{(.05-.02)T}$.

7.a. Shareholders have at their disposal a variety of risk and portfolio management techniques to manage risk, and don't need corporate managers to do so for them. For example, shareholders can hold large, well-diversified portfolios, take positions in a wide variety of highly liquid derivative securities to manage a variety of specific types of risk. Furthermore, when shareholders are left to themselves to manage the risks of their own portfolios, they can more easily fit their portfolios to their personal risk preferences than managers can.

b. When corporations take risks, they subject themselves to bankruptcy risks, and the wide variety of associated direct bankruptcy costs (e.g., legal, accounting and court fees, reorganization and liquidation costs) and indirect costs of bankruptcy (e.g., inability to transact with and loss of goodwill from clients and suppliers). Shareholders are normally not in a position to evaluated pertinent information as are managers, or to be able to easily and cost-effectively access financial institutions and contracts needed to mitigate risks.

8.a. Work with the following: $F_T = S_0 e^{(r(d)-r(f))T}$ = $F_T = 1.6e^{(.12-.10))\times 2}$ = 1.6×1.040811 = USD 1.665297

b. The following set of transactions, with the first three in any order, will produce an arbitrage profit:

Borrow CHF1000 now in Switzerland at 12%; repay at Time One
Buy USD625 at S_0=1.6 now for CHF1000
Loan USD625 at 10%; Collect proceeds at Time One
Sell USD622.22 at Time One at F_1 =1.8 for CHF1120

The time zero CHF amount can be any amount, though either the time zero USD amount must be 0.625 times the CHF amount or the time 1 USD amount must be 0.5556 times the CHF amount. Regardless, the investor borrows CHF, converts to USD, and lends USD. The time one amounts are determined by the forward and interest rates. Or, ,ore generally, borrow any amount of CHF in Switzerland at 12% say, CHF1), convert into dollars at the 1.6 spot rate, lend this sum in the U.S. at 10% for two years, and convert the U.S. loan proceeds in two years into CHF at 1.8, then pay back the original 2-year loan of CHF1 at 12%. This results in a profit of .102829 for each CHF franc borrowed at time zero, calculated as follows:

$$(1/1.6\times e^{0.1\times 2})\times \text{CHF}1.8-(1\times e^{(0.12\times 2)}) = \text{CHF}0.102829$$

This arbitrage portfolio requires zero net investment at time zero and produces a profit equal to CHF0.10289 for every franc borrowed. This arbitrage portfolio is based on *Interest Rate Parity*, which characterizes the arbitrage-free relationship among spot rates, forward rates, and interest rates. An alternative and slightly different solution can be obtained for the one-period discrete time scenario.