How Important is Endogenous Mobility for Measuring Employer and Employee Heterogeneity?\(^1\)

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Abstract

We study the effects of endogenous mobility on the linear decomposition of log wage rates into observable, personal, and employer heterogeneity. The Abowd, Kramarz and Margolis (1999) method for estimating such models under the assumption of exogenous mobility produces residuals that can be used to produce tests for the validity of the assumption. We propose two such tests. The first is based on estimated match effects that are computed from the average residual within person and employer. These are shown to have a relation to the wage rate received from subsequent employers through the firm effect even when controlling for the estimated individual effect and the firm effect on the old job. The second test is based on employer changes to the distribution of person effects in the firm’s workforce. The AKM residuals are used to compute the average residual within a firm during a particular year. This effect, which we label a productive workforce effect, is shown to have a relation with the firm’s adjustments to its workforce person-effect distribution after controlling for the firm effect. The model is applied to U.S. longitudinally linked employer-employee data from the Census Bureau’s LEHD Program.
1 Introduction

It is now well documented that both employer and employee heterogeneity contribute substantially to the statistical decomposition of job-level outcomes such as earnings at a particular employer and inter-employer mobility (see Mortensen (2003)). A common empirical model used to quantify this heterogeneity is the Abowd et al. (1999) decomposition into factors associated with time-varying observable characteristics, individual heterogeneity (person effects), employer heterogeneity (firm effects), and the statistical residual, AKM below. These models have been criticized for failing to account for endogenous mobility—specifically, the possibility that the error in the AKM decomposition might be structurally related to the assignment of workers to employers either through search dynamics, for example in Mortensen (2003), Postel-Vinay and Robin (2002), and Lentz (2010), coordination frictions (Shimer (2005)), or learning (Gibbons et al. (2005)).

One purpose of the research using longitudinally integrated employer-employee data is to study the nature of job assignment in the economy. The specific question to be answered is whether job assignment depends on unobserved productive characteristics of the workers and firms involved. Is the labor market characterized by assortative matching? Are search frictions without assortative matching sufficient explain the wage and employment outcomes? What kind of productive characteristics do firms and workers use to match? These question are interesting for several reasons. First, the manner in which firms and workers form an employment relation matters for our understanding of the efficiency of job search, matching, and the value of job flows and churning in the labor market. Second, if labor is not really allocated at random within relatively homogeneous groups, then we must further challenge the idea that markets provide a reasonable analogy for the exchange of labor in the economy. Finally, the identification of structural effects in the econometric analysis of labor market outcomes like wages or separation hazards depends on assumptions about this matching
process. Better information about this process should inform these empirical exercises.

In this paper we study the effects of endogenous mobility on the linear decomposition of log wage rates into observable, personal, and employer heterogeneity. The AKM method for estimating such models under the assumption of exogenous mobility produces residuals that can be used to produce tests for the validity of the assumption. We propose two such tests. The first is based on estimated match effects that are computed from the average residual within person and employer. These are shown to have a relation to the wage rate received from subsequent employers through the firm effect even when controlling for the estimated individual effect and the firm effect on the old job. The second test is based on employer changes to the distribution of person effects in the firm’s workforce. The AKM residuals are used to compute the average residual within a firm during a particular year. This effect, which we label a productive workforce effect, is shown to have a relation with the firm’s adjustments to its workforce person-effect distribution after controlling for the firm effect. The model is applied to U.S. longitudinally linked employer-employee data from the Census Bureau’s LEHD Program.

Section 2 lays out the statistical framework for the AKM decomposition and shows where the information used in the tests originates. Section 3 briefly describes the data, which have been documented elsewhere, and the assumptions used in our implementation. Section 4 derives the formal tests and establishes their properties. Section 5 provides the results and discussion. Finally, section 6 concludes.

2 Statistical Model

We begin by summarizing the network structure of the labor market embodied in the AKM decomposition as analyzed by Abowd and Schmutte (2010). The estimated linear decompo-
sition of log wage rates is the least squares fit of the equation

\[ y = X\beta + D\theta + F\psi + \varepsilon \]  \hspace{1cm} (1)

where \( y \) is the \([N \times 1]\) stacked vector of log wage outcomes \( y_{it} \) sorted by \( t \) then \( i \); \( X \) is the \([N \times k]\) design matrix of observable individual and employer time-varying characteristics (the intercept is normally suppressed, with \( y \) and \( X \) measured as deviations from overall means); \( D \) is the \([N \times I]\) design matrix for the individual effects; \( F \) is the \([N \times J - 1]\) design matrix for the employer effects (non-employment is suppressed). \( \varepsilon \) is the \([N \times 1]\) vector of statistical errors whose properties will be elaborated below; \( \begin{bmatrix} \beta^T & \theta^T & \psi^T \end{bmatrix}^T \) are the unknown effects \([k \times 1], [I \times 1] \) and \([J - 1 \times 1] \), resp., associated with each of the design matrices. Identification of \( \begin{bmatrix} \beta^T & \theta^T & \psi^T \end{bmatrix}^T \) is ensured through imposition of the connectedness conditions derived in Abowd et al. (2002). The basic properties of the least squares solution for the parameters in equation (1) are orthogonality of each design matrix with respect to the estimated residual:

\[ \hat{\beta}^T X^T \hat{\varepsilon} = 0, \quad \hat{\theta}^T D^T \hat{\varepsilon} = 0 \quad \text{and} \quad \hat{\psi}^T F^T \hat{\varepsilon} = 0, \]  \hspace{1cm} (2)

implying that the estimated individual and employer effects are also orthogonal to the residual. The computational details are also discussed in Abowd et al. (2002).

One property of the conditions given in 2 is that there are many other functions of the design matrices \( D \) and \( F \) that should also be orthogonal to the true residuals \( \varepsilon \) under exogenous mobility but which are not used in solving for \( \hat{\theta} \) and \( \hat{\psi} \) by imposing 2. Since these functions of the design matrices are not orthogonal to the estimated residuals \( \hat{\varepsilon} \) when those residuals are produced from the AKM decomposition, they can be used as the basis for meaningful diagnostic tests. We derive two such tests below, implement them, and interpret their results in the context of endogenous mobility biases.
3 Data

We use matched employer-employee data from the Longitudinal Employer-Household Dynamics (LEHD) Program at the U.S. Census Bureau. The LEHD Infrastructure File system data are built from state Unemployment Insurance (UI) wage records collected through a voluntary federal/state cooperative program managed by the U.S. Census Bureau. Details of the LEHD integrated employer-employee data can be found in Abowd, Stephens, Vilhuber, Andersson, McKinney, Roemer and Woodcock (2009). These data have been used extensively in applications of the AKM decomposition. See Abowd et al. (2006; 2002; 2003); Abowd, Kramarz, Pérez-Duarte and Schmutte (2009); Schmutte (2010); Woodcock (2008) for details.

The research file used in this study contains data for 28 states covering the period 1990–2004. From this research file, we use three sub-samples in different stages of estimation and testing. Our estimation sample for the AKM decomposition consists of all 982 million dominant job observations in this time period.\footnote{The dominant job is the one from which the individual earns the most labor market earnings in a particular year.} To implement the tests, we discretize estimated person effects, firm effects and residuals onto a fixed support. The quantiles that define the support points are calculated from a point-in-time snapshot of the distribution of dominant jobs in progress as of April 1, 2002. That distribution is restricted to full-time, full-year jobs held by individuals age 18-70. Finally, in testing, we use all 465 million dominant job observations for workers 18-70 that occur between 1999 and 2002. Test 1 uses data for about 104 million job changers during 1999-2004, inclusive. Test 2 uses data for about 4 million firms alive in 2001.
4 Tests

Although most researchers assume that endogenous mobility is an important feature of longitudinally integrated employer-employee data, there are currently no formal tests that can be computed from estimates made under the null hypothesis of exogenous mobility. In this section, we attempt to fill this void.

If endogenous mobility is present, then the residuals from the AKM model should vary systematically with the estimated components of the model. Some of this variability cannot be detected from the model estimates because of orthogonality conditions necessary to compute the least squares estimates and their associated residuals. When forming test statistics, we are constrained by the moment conditions embodied in these equations. For example, \[ \sum_{\{t:i=h\}} \hat{\varepsilon}_{it} = 0 \] for all \( i \), since \( \theta_i \) already captures the average residual for worker \( i \).

Our first proposed test estimates whether the residuals at a worker’s last job vary systematically across \( \theta_i \) decile, \( \psi_{J(i,t-1)} \) decile, and \( \psi_{J(i,t)} \) decile cells. Specifically, the test measures whether the individual’s average residual (within deciles of the residual distribution) from the most recently completed job predicts the transition from \( \psi_{J(i,t-1)} \) decile to \( \psi_{J(i,t)} \) decile, given \( \theta_i \) decile. Our second proposed test estimates whether changes in the firm’s skill (\( \theta_i \) for all \( i \) where \( J(i,t) = j \)) distribution from \( t \) to \( t+1 \) varies significantly by the employer-average residual decile, given the \( \psi_{J(i,t)} \) decile.

4.1 Basic Data Setup and Definitions

Given the fitted version of the AKM decomposition, which has been computed under the null hypothesis of exogenous mobility, we select the sample of individuals and employers active at the beginning of 2002, quarter 2 (April 1, 2002). For this sample, we compute

\footnote{Rothstein (2010) attacks the related problem of testing for endogenous mobility in the context of value-added models using longitudinally-linked education data. His method requires estimating a more general model, and so is different from what we propose here.}
deciles from the estimated \( \hat{\theta}_i, \hat{\psi}_{J(i,t)} \), and \( \hat{\varepsilon}_{it} \) as described in section 3. Using the estimated deciles, we discretize each component of the decomposition onto 10 fixed points of support.

We adopt the following notation:

\[
Q(z) = a \text{ denotes quantile } a \text{ for } z \in \{\theta, \psi, \varepsilon\}
\]

and

\[\#Q(z) \text{ denotes then number of quantiles for } z \in \{\theta, \psi, \varepsilon\}.\]

In the tests presented below, we use deciles, so \( \#Q(z) = 10. \)

### 4.2 Test Statistic 1: Match Effects Test

Under the hypothesis of exogenous mobility, the match effect for a given individual–employer pair can be estimated using the average residual for the most recent completed job at \( j \) by \( i \).

We denote these match effects as \( \varepsilon_{it-1} \) for those individuals who change employers between periods \( t-1 \) and \( t \). Formally, we have

\[
\varepsilon_{it-1} = \sum_{\{s|J(i,s)=j \land s<t \land J(i,s) \neq J(i,t)\}} \hat{\varepsilon}_{is}
\]

An individual for whom \( \varepsilon_{it-1} > 0 \) received wage payments while employed at \( J(i, t-1) = j \) that exceeded their expected value, again under the hypothesis of exogenous mobility. The opposite is true for individuals for whom \( \varepsilon_{it-1} < 0. \)

#### 4.2.1 Derivation of the Match Effects Test

To form a test statistic that captures the potential for \( \varepsilon_{it-1} \) to be predictive of the next employer type, we calculate the count of all \((i,t)\) pairs where \( J(i, t-1) \neq J(i, t) \) (job
changers) in quantiles of the components $\hat{\theta}_i$, $\hat{\psi}_{J(i,t-1)}$, $\hat{\psi}_{\psi}(i,t)$, and $\hat{\varepsilon}_{it-1}$:

$$n_{abcd} = \sum_{\{i,t\mid J(i,t-1) \neq J(i,t)\}} 1 \left\{ Q(\hat{\theta}_i) = a \land Q(\hat{\psi}_{\psi}(i,t-1)) = b \land Q(\hat{\psi}_{\psi}(i,t)) = c \land Q(\hat{\varepsilon}_{it-1}) = d \right\}.$$  

(3)

The joint probability of observing $n_{abcd}$ is

$$\pi_{abcd} = \Pr \left\{ Q(\theta_i) = a \land Q(\psi_{\psi}(i,t-1)) = b \land Q(\psi_{\psi}(i,t)) = c \land Q(\varepsilon_{it-1}) = d \right\}.$$  

Exogenous mobility implies that the match effect from period $t-1$ should not be predictive of the transition from $\psi_{\psi}(i,t-1)$ to $\psi_{\psi}(i,t)$ for an individual with $\theta_i$. This hypothesis can be formalized as conditional independence of the outcome $Q(\theta_i) = a \land Q(\psi_{\psi}(i,t-1)) = b \land Q(\psi_{\psi}(i,t)) = c$ from $Q(\varepsilon_{it-1}) = d$. In terms of the joint probabilities we compute

$$X^2_{\nu_1} = \text{Test} \left( \pi_{abcd} = \pi_{abc} + \pi_{++d} \right)$$  

(4)

where the subscript + denotes the marginal distribution with respect to the indicated dimension, and the degrees of freedom are given by $\nu_1 = \left( \#(Q(\theta_i)) \times \#Q(\psi_{\psi}(i,t-1)) \times \#Q(\psi_{\psi}(i,t)) - 1 \right) \times \left( \#Q(\varepsilon_{it-1}) - 1 \right)$.

4.2.2 Computation of the Match Effects Test

We implemented test (4) by direct calculation of the likelihood ratio. The population of job changers consists of individuals $i$ for whom $J(i,t-1) \neq J(i,t)$ for $t = 1999, ..., 2003$. The entire population of individuals and employers was used to compute the quantiles of the $\hat{\theta}_i$, $\hat{\psi}_{\psi}(i,t-1)$, $\hat{\psi}_{\psi}(i,t)$, and $\hat{\varepsilon}_{it-1}$ distributions. Then the counts (3) were tabulated using all observations in the job-changer population.
4.3 Test Statistic 2: Productive Workforce Test

Our second test considers the implications of exogenous mobility for the employer’s choice of workforce distributions over $\theta_i$ where $J (i, t) = j$. The average amount by which wages deviate from their expectations, under exogenous mobility, for a given workforce at a point in time can be computed as the average residual for all employees at $J (i, t) = j$ in year $t$

$$\tilde{\varepsilon}_{jt} = \frac{\sum_{\{i|J(i,t)=j\}} \tilde{\varepsilon}_{it}}{\sum 1 \{i|J (i, t) = j\} }.$$ 

An employer for whom $\tilde{\varepsilon}_{jt} > 0$ has paid higher than expected wages in period $t$; and the opposite is true for $\tilde{\varepsilon}_{jt} < 0$. Although there could be many reasons for this, we will refer to $\tilde{\varepsilon}_{jt}$ as a measure of workforce productivity, which will be appropriate if workforce productivity increases when $\tilde{\varepsilon}_{jt}$. However, the exogenous mobility hypothesis is silent about the meaning of $\tilde{\varepsilon}_{jt}$. What matters is its relationship to the within-employer distribution of $\theta_i$. If $\tilde{\varepsilon}_{js}$ is predictive of the within-employer distribution of $\theta_i$ for some period $t > s$, given $\psi_j$, then exogenous mobility fails because the distribution of future employment depends on residuals in the theoretical AKM decomposition.

To implement this test, consider two periods $s < t$ and all employers alive in period $s$. Compute the counts

$$n_{abc|s} = \sum_j \left\{ 1 \{Q (\psi_j) = a \land Q (\tilde{\varepsilon}_{js}) = c \} \times \sum_{\{i|J(i,s)=j\land Q(\psi_j)=a\}} Q (\theta_i) = b \right\},$$

and

$$n_{abc|t} = \sum_j \left\{ 1 \{Q (\psi_j) = a \land Q (\tilde{\varepsilon}_{js}) = c \} \times \sum_{\{i|J(i,t)=j\land Q(\psi_j)=a\}} Q (\theta_i) = b \right\}.$$ 

Note that the two counts are not independent because they condition on the same distribution
of employers alive in period $s$. Let

$$\pi_{abc|s} = \Pr \{ Q(\psi_j) = a \wedge (Q(\theta_i) = b|s) \wedge Q(\tilde{\epsilon}_{js}) = c \}$$

and

$$\pi_{abc|t} = \Pr \{ Q(\psi_j) = a \wedge (Q(\theta_i) = b|t) \wedge Q(\tilde{\epsilon}_{js}) = c \}.$$ 

Then, the appropriate test of the conditional independence of the within-employer workforce distribution over $\theta_i$ is

$$X^2_{\nu_2} = \text{Test} \left( \ln \left( \frac{\pi_{abc|s}}{\pi_{abc|t}} \right) = \ln \left( \frac{\pi_{ab+|s}}{\pi_{ab+|t}} \right) \right)$$

where $\nu_2 = (#Q(\theta_i) - 1) \times (#Q(\tilde{\epsilon}_{js}) - 1) + (#Q(\psi_j) - 1) \times (#Q(\theta_i) - 1) \times (#Q(\tilde{\epsilon}_{js}) - 1)$.

### 4.3.1 Derivation of the Productive Workforce Test Statistic

To see why the test in equation (5) is correct, consider

$$\ln \left( \frac{\pi_{abc|s}}{\pi_{abc|t}} \right) = (\mu_{a|s} - \mu_{a|t}) + (\mu_{b|s} - \mu_{b|t}) + (\mu_{c|s} - \mu_{c|t})$$

$$+ (\gamma_{ab|s} - \gamma_{ab|t}) + (\gamma_{ac|s} - \gamma_{ac|t}) + (\gamma_{bc|s} - \gamma_{bc|t})$$

$$+ (\rho_{abc|s} - \rho_{abc|t})$$

where the notation $\mu_{z|t}$ denotes main effects of $z \in \{ Q(\psi_j), Q(\theta_i), Q(\tilde{\epsilon}_{js}) \}$ given period $t$, $\gamma_{yz|t}$ denotes 2-way interactions of $y, z \in \{ Q(\psi_j), Q(\theta_i), Q(\tilde{\epsilon}_{js}) \}$ given period $t$, and $\rho_{xyz|t}$ denotes 3-way interactions of $x, y, z \in \{ Q(\psi_j), Q(\theta_i), Q(\tilde{\epsilon}_{js}) \}$ given period $t$. The change in main effects of $Q(\psi_j)$, $(\mu_{a|s} - \mu_{a|t})$, must be identically 0 since the population of employers is the same in both periods (only those alive in period $s$). Similarly, the change in main effects of $Q(\tilde{\epsilon}_{js})$, $(\mu_{c|s} - \mu_{c|t})$, must be identically 0 since the workforce
productivity distribution is only measured at period $s$. The change in interaction of $Q(\psi_j)$ and $Q(\bar{\epsilon}_{js})$, $Q(\theta_i)$, $(\gamma_{ab|s} - \gamma_{ab|t})$, must also be identically 0 for the same reason. This leaves two sets of parameters that are unconstrained by the null hypothesis—the change in main effects of $Q(\theta_i)$, $(\mu_{b|s} - \mu_{b|t})$, with $df = (#Q(\theta_i) - 1)$ and the change in interaction of $Q(\psi_j)$ and $Q(\theta_i)$, $(\gamma_{ab|s} - \gamma_{ab|t})$, with $df = (#Q(\psi_j) - 1) \times (#Q(\theta_i) - 1)$. The parameters affected by the null hypothesis are the change in interaction of $Q(\theta_i)$ and $Q(\bar{\epsilon}_{js})$, $(\gamma_{bcs} - \gamma_{bc|t})$, with $df = (#Q(\theta_i) - 1) \times (#Q(\bar{\epsilon}_{js}) - 1)$ and the change in interaction of $Q(\psi_j)$, $Q(\theta_i)$ and $Q(\bar{\epsilon}_{js})$, $(\rho_{abcs} - \rho_{abc|t})$, with $df = (#Q(\psi_j) - 1) \times (#Q(\theta_i) - 1) \times (#Q(\bar{\epsilon}_{js}) - 1)$. Under the null hypothesis $(\gamma_{bc|s} - \gamma_{bc|t}) = 0$ and $(\rho_{abc|s} - \rho_{abc|t}) = 0$ with $df = \nu_2 = (#Q(\theta_i) - 1) \times (#Q(\bar{\epsilon}_{js}) - 1) + (#Q(\psi_j) - 1) \times (#Q(\theta_i) - 1) \times (#Q(\bar{\epsilon}_{js}) - 1)$.

### 4.3.2 Computation of the Productive Workforce Test Statistics

We use the method of moments to conduct test (5). The observations are firms $j$ with positive employment in $s$. For each firm compute

$$x_j = \left[ \begin{array}{c} \frac{n_{j1t}}{n_{j+t}} - \frac{n_{j1s}}{n_{j+s}} \\ \frac{n_{j2t}}{n_{j+t}} - \frac{n_{j2s}}{n_{j+s}} \\ \vdots \\ \frac{n_{j(#Q(\theta_i)-1)t}}{n_{j+t}} - \frac{n_{j(#Q(\theta_i)-1)s}}{n_{j+s}} \end{array} \right]$$

where

$$n_{jqt} = \sum_{\{i|J(i,t)=j\}} 1(Q(\theta_i) = q).$$

and $x_j$ is $[(#Q(\theta_i) - 1) \times 1]$. For each value of $a$ and $c$ compute the vector of means and the covariance matrix

$$\bar{x}_{ac} = \frac{\sum_{\{j|Q(\psi_j)=a\wedge Q(\bar{\epsilon}_{js})=c\}} n_{j+s} x_j}{\sum_{\{j|Q(\psi_j)=a\wedge Q(\bar{\epsilon}_{js})=c\}} n_{j+s}}$$
\[ V_{ac} = \frac{\sum_{\{j|Q(\psi_j)=a \land Q(\bar{\epsilon}_js)=c\}} n_{j+s} (x_j - \bar{x}_{ac}) (x_j - \bar{x}_{ac})'}{\sum_{\{j|Q(\psi_j)=a \land Q(\bar{\epsilon}_js)=c\}} n_{j+s}}. \]

\[ N = \sum_j 1(j \exists i : J(i, s) = j) \]

For each value of \( a \) compute the expected mean under the null hypothesis

\[ \bar{x}_a = \frac{\sum_{\{j|Q(\psi_j)=a\}} n_{j+s} x_j}{\sum_{\{j|Q(\psi_j)=a\}} n_{j+s}}. \]

Then,

\[ X^2_x = N \sum_{a,c} (\bar{x}_{ac} - \bar{x}_a)' V_{ac}^{-1} (\bar{x}_{ac} - \bar{x}_a). \]

5 Results

Both tests strongly reject the null hypothesis of exogenous mobility. The match effects test (4) has \( X^2 = 7,438,692 \) with 8,991 degrees of freedom. Using conventional criteria, this test has a p-value less than \( 10^{-6} \). From a Bayesian viewpoint and assuming equal prior odds on the null and alternative, the change in the Bayes Information Criterion associated with going from the null hypothesis to the unconstrained alternative hypothesis is 166,049, which also indicates that the data strongly favor some model in which the AKM residuals are related to job mobility.

The productive workforce test (5) has \( X^2 = 172,295 \) with 900 degrees of freedom. Again, using conventional criteria, this test has a p-value less than \( 10^{-6} \). The change in the BIC is 13,714, which also strongly favors some model in which the AKM residuals are related to job mobility.
5.1 Discussion of Results for Match Effects Test

The individual components of our tests can be used to study the basic correlates of the departures from exogenous mobility. Figure 1 shows the conditional distribution of $Q\left(\hat{\psi}_{J(i,t-1)}\right)$ given $Q(\bar{\varepsilon}_{it-1})$ for job changers; that is, the joint distribution of the employer effect in the old job and the match effect in the same job. Under the null hypothesis, we expect this graph to show no variation along the $Q(\bar{\varepsilon}_{it-1})$ dimension. The distribution for $Q(\bar{\varepsilon}_{it-1}) = 1$ should be identical to the distribution for all other deciles. This is clearly not the case. For the extreme values of the match effect distribution, $Q(\bar{\varepsilon}_{it-1}) = 1$ and $Q(\bar{\varepsilon}_{it-1}) = 10$, the job changer is much more likely to have originated in a low $\hat{\psi}_{J(i,t-1)}$ decile, $Q\left(\hat{\psi}_{J(i,t-1)}\right) = 2$ or 3; whereas for the central deciles of the match effect, $Q(\bar{\varepsilon}_{it-1}) = 5$ or 6, the job changer is much more likely to have come from a high $\hat{\psi}_{J(i,t-1)}$ decile, $Q\left(\hat{\psi}_{J(i,t-1)}\right) = 7$ to 9. Figure 1 thus confirms that there is information in the correlation of the AKM residuals with the individual and employer effects even though the model was computed under the null hypothesis of exogenous mobility.

Figures 2, 3 and 4 shed additional light on the correlation between the match effect and job mobility. The figures plot the conditional distribution of $Q\left(\hat{\psi}_{J(i,t)}\right)$ given $Q\left(\hat{\psi}_{J(i,t-1)}\right)$ for different quantiles of the match effect, $Q(\bar{\varepsilon}_{it-1})$. Under exogenous mobility, we expect these figures to be identical. Figure 2 shows the transition probabilities for $Q(\bar{\varepsilon}_{it-1}) = 1$ while Figures 3 and 4 display the same graph for $Q(\bar{\varepsilon}_{it-1}) = 5$ and 10, respectively. All three graphs have a ridge running from $Q\left(\hat{\psi}_{J(i,t)}\right) = 1$ given $Q\left(\hat{\psi}_{J(i,t-1)}\right) = 1$ to $Q\left(\hat{\psi}_{J(i,t)}\right) = 10$ given $Q\left(\hat{\psi}_{J(i,t-1)}\right) = 10$, and most of the mass of the transition rate distribution is to the right of this ridge. In general, job changers improve their employer effect, a result first documented in Schmutte (2010). Our tests indicate that this tendency is strongest for individuals in the middle of the match effect distribution.
5.2 Discussion of Results for Productive Workforce Test

Figures 5, 6, and 7 illustrate why the productive workforce test of exogenous mobility rejects the null. Each of these figures show the change in the distribution of the workforce by individual effect, \( Q(\hat{\theta}_i) \) between 2001 and 2003, conditional on the quantile of the employer effect in 2001, \( Q(\hat{\psi}_{J(i,2001)}) \), and the quantile of the productive workforce (average firm residual) in 2001, \( Q(\tilde{\varepsilon}_{i2001}) \). In terms of the notation of the testing section, they plot

\[
\hat{\pi}_{abc|2003} - \hat{\pi}_{abc|2001} = \Pr \{ Q(\psi_j) = a \wedge (Q(\theta_i) = b)|2003 \} \wedge Q(\tilde{\varepsilon}_{js}) = c \} - \Pr \{ Q(\psi_j) = a \wedge (Q(\theta_i) = b)|2001 \} \wedge Q(\tilde{\varepsilon}_{js}) = c \}
\]

Figure 5 is the plot for \( Q(\tilde{\varepsilon}_{js}) = 1 \), the lowest decile of the average residual for employers in 2001. Figure 6 plots the fifth decile, and Figure 7 plots the tenth decile. In all three figures, the plots for employers of all deciles \( Q(\hat{\psi}_{J(i,2001)}) \) of the employer effect distribution are very similar. However, employers near the median of the productive workforce measure in 2001 (Figure 6) did very little adjustment of the proportion of their employees in deciles two through nine of the individual effect distribution. They did reduce employment in the lowest decile of \( \hat{\theta}_i \) and increased employment in the highest decile of \( \hat{\theta}_i \). Employers from the lowest decile of the firm average residual distribution, reduced employment in all deciles of the \( \hat{\theta}_i \) distribution, but most profoundly in the lowest deciles, especially decile 1. Employers in the highest decile of the firm average residual distribution look similar to those in the lowest decile, not those near the median. Notice that since the population for these tests excludes new firms (entrants between 2001 and 2003), it is possible (and indeed happens) that the firms responsible for the increases in employment in each of the deciles of \( \hat{\theta}_i \) are the entering firms. While this phenomenon is interesting, it does not influence the test for exogenous mobility.
6 Conclusion

We propose two new tests for the validity of the assumption of exogenous mobility in estimates of the AKM decomposition of wages into observable effects, individual heterogeneity, and employer heterogeneity. The basic theory underlying the tests is the fact that the first order conditions for estimating the AKM decomposition do not force the residuals to be orthogonal to every linear function of columns of the design matrix. Consequently, there is information in the residuals that can be used to uncover potential violations of exogenous mobility. Using LEHD data, both of our tests reject the null hypothesis of exogenous mobility using frequentist or Bayesian criteria. The magnitude of the bias that endogenous mobility might induce on the estimates produced by the AKM decomposition is unknown. Recent research indicates that endogenous mobility bias might be quite severe (Lopes de Melo (2008); Gruetter and Lalive (2009)). We are actively developing methods to estimate and correct for endogenous mobility bias.
Bibliography


A Figures

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