

ANSWER KEY PART I

1. Consider a firm operating as a monopoly and producing a new version of a computer memory chip. The firm has hired a consultant and is seeking advice about pricing and advertising strategy for the chip. With the data that the firm provided, the consultant was able to estimate the demand curve for the chips:

$$Q(P,A) = 1000 - 10P + 40A^{0.25}$$

Where:

$Q(P, A)$ = Quantity of chips demanded per month

P = Price per chip

A = Total dollar advertising expenditures, measured in dollars

A) Suppose the firm were to introduce the chip at the price of \$100. Fill in the E-K columns in the spreadsheet posted on the course website called "HW1Spreadsheet"

The table is constructed by entering \$100 increments in advertising expenditures down Column C, and holding price constant down Column D.

For each combination of price and advertising levels, calculate:

- Quantity demanded (Q) by substituting price and advertising expenditures into the demand formula (Column E)
- Total sales (total revenue) (Column F)
- Price Elasticity (Column G)
- Advertising Elasticity (Column H)
- Dorfman-Steiner Ratio (Column I)
- Advertising-To-Sales Ratio (Column J)
- Absolute Difference between Column I and Column J.

Find the optimal advertising expenditures. Highlight (or bold) the row with the optimal price and advertising levels. Briefly discuss your results and attach the printed table.

ANSWERS

- A) See the spreadsheet posted with the answers on the course website. There are essentially 2 ways to arrive at the same answer: 1) by using the arc rule of elasticity (by taking the discrete changes in strategic variables) or 2) by taking derivatives of demand function and using equilibrium values of Q, P and A from given or calculated entries in the spreadsheet. The spreadsheet that is posted on the course website followed the (2) option. If you click on each of the cells, you can back out the formulas used to calculate each entries.

To summarize:

Column E: Q was calculated by plugging given levels of A and P into demand function: $Q=1000-10P+40A^{0.25}$

Column F: $TR=P*Q$ = Column D * Column E

Column G: Price elasticity = [derivative of demand with respect to $P=10$]* $[P/Q]$. Take the be absolute value

Column H: Advertising elasticity = [derivative of demand with respect to $A=10*A^{-0.75}$]* $[A/q]=(10*A^{0.25})/Q$

Column I: D-S condition=Advertising elasticity/Price elasticity = column H / column G

Column J: Ad-to-sales ratio = A/TR = column C / column F

The optimal advertising-price level will be found when Dorfman-Steiner condition holds, i.e. when Column I = Column J, or Column I minus Column J is as close to zero as possible.

The optimal A=1600 for the given P=100.