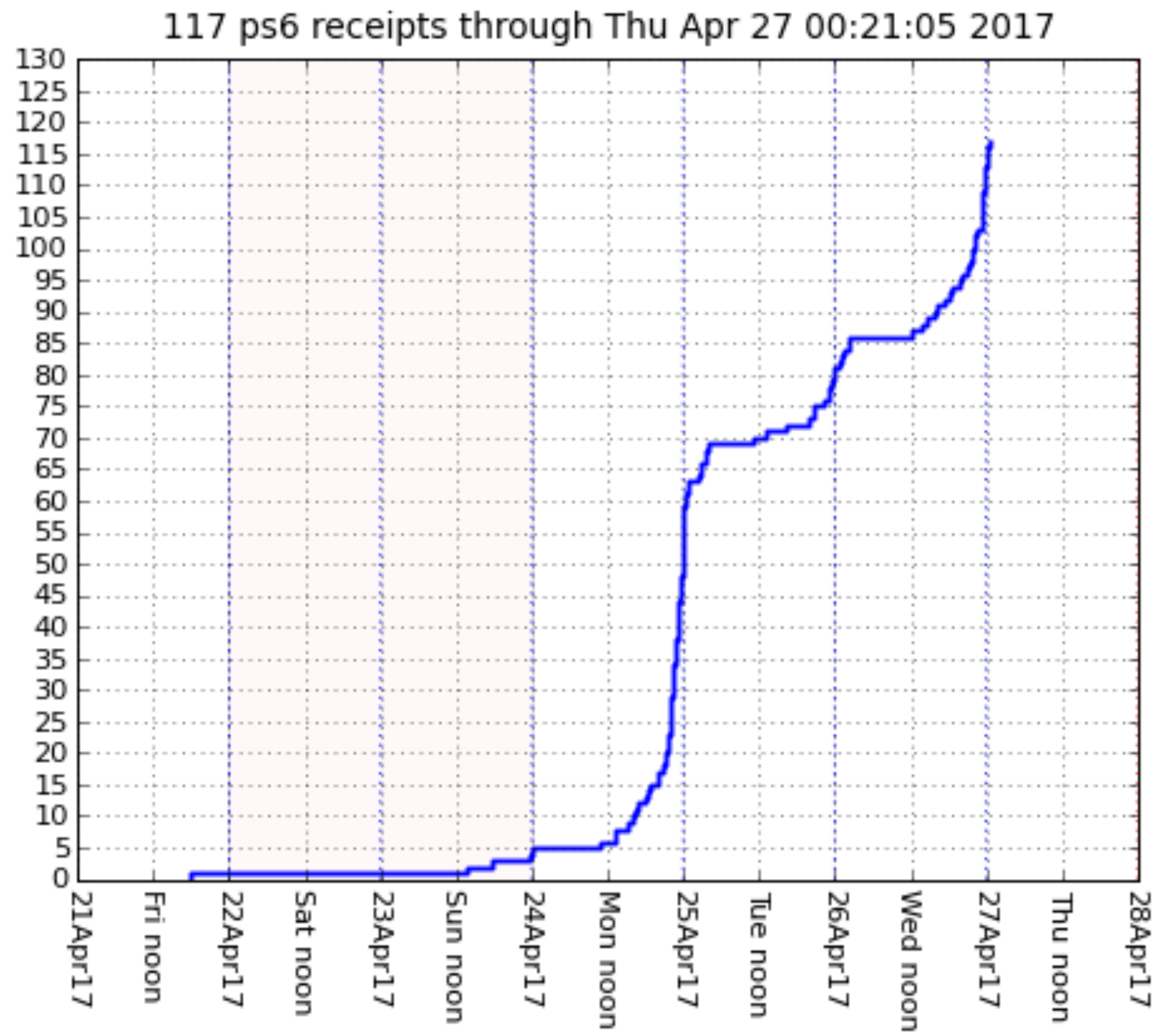
# Info 2950, Lecture 23

27 Apr 2017




Prob Set 7: due Wed 3 May

Prob Set 8: due 11 May (end of classes)

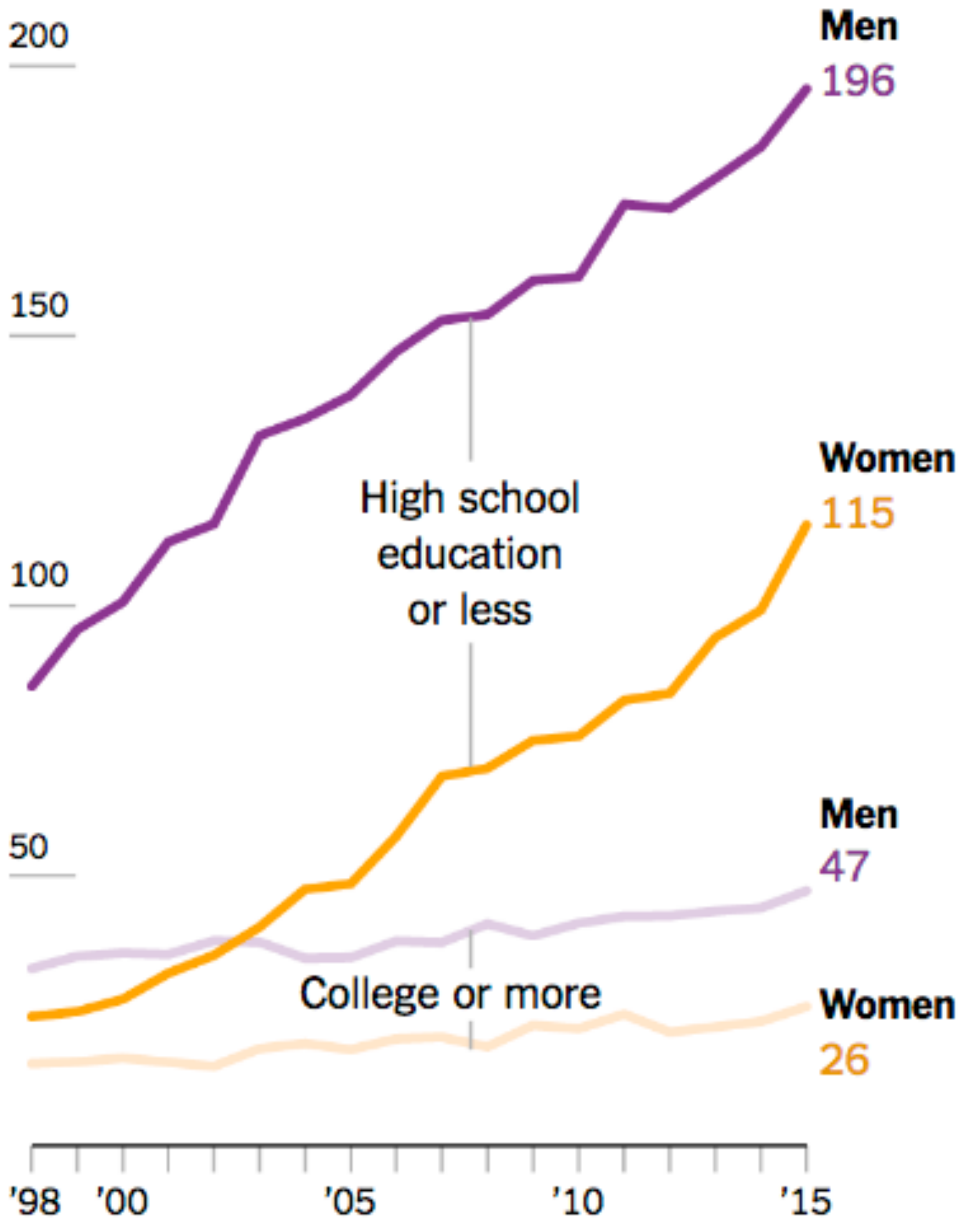
The Opinion Pages | CONTRIBUTING OP-ED WRITER

# Reaching Out to the Voters the Left Left Behind

Thomas B. Edsall APRIL 13, 2017

### White 'Deaths of Despair'

Deaths per 100,000 non-Hispanic whites aged 50 to 54, from suicide, alcohol or other drugs, by education level.




Source: Brookings Papers on Economic Activity

By The New York Times

mortality rates for whites were stable or declining in all counties with populations of one million or more — the counties, in other words, that voted decisively for Clinton. Conversely, white mortality rates rose by roughly one percent a year in all counties of less than a million — the counties that voted for Trump.

Bill Bishop, co-author of the book "The Big Sort" and a founder of The Daily Yonder, makes the case that the political split in America is not an urban-rural divide. Instead, he argues, it is between the largest cities and the rest of America.

ps7#4:

https://wonder.cdc.gov/ucd-icd10.html

CDC Centers for Disease Control and Prevention

CDC 24/7: Saving Lives, Protecting People™

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About Underlying Cause of Death, 1999-2015

https://www.nytimes.com/2017/04/13/opinion/reaching-out-to-the-voters-the-left-left-behind.html

Defined as the Pearson correlation for the ranks, the Spearman correlation is written

$$\rho = \frac{\mathrm{Cov}[r,s]}{\sigma[r]\sigma[s]} \ , \tag{1}$$

where $\mathrm{Cov}[r,s] = E[(r - E[r])(s - E[s])]$

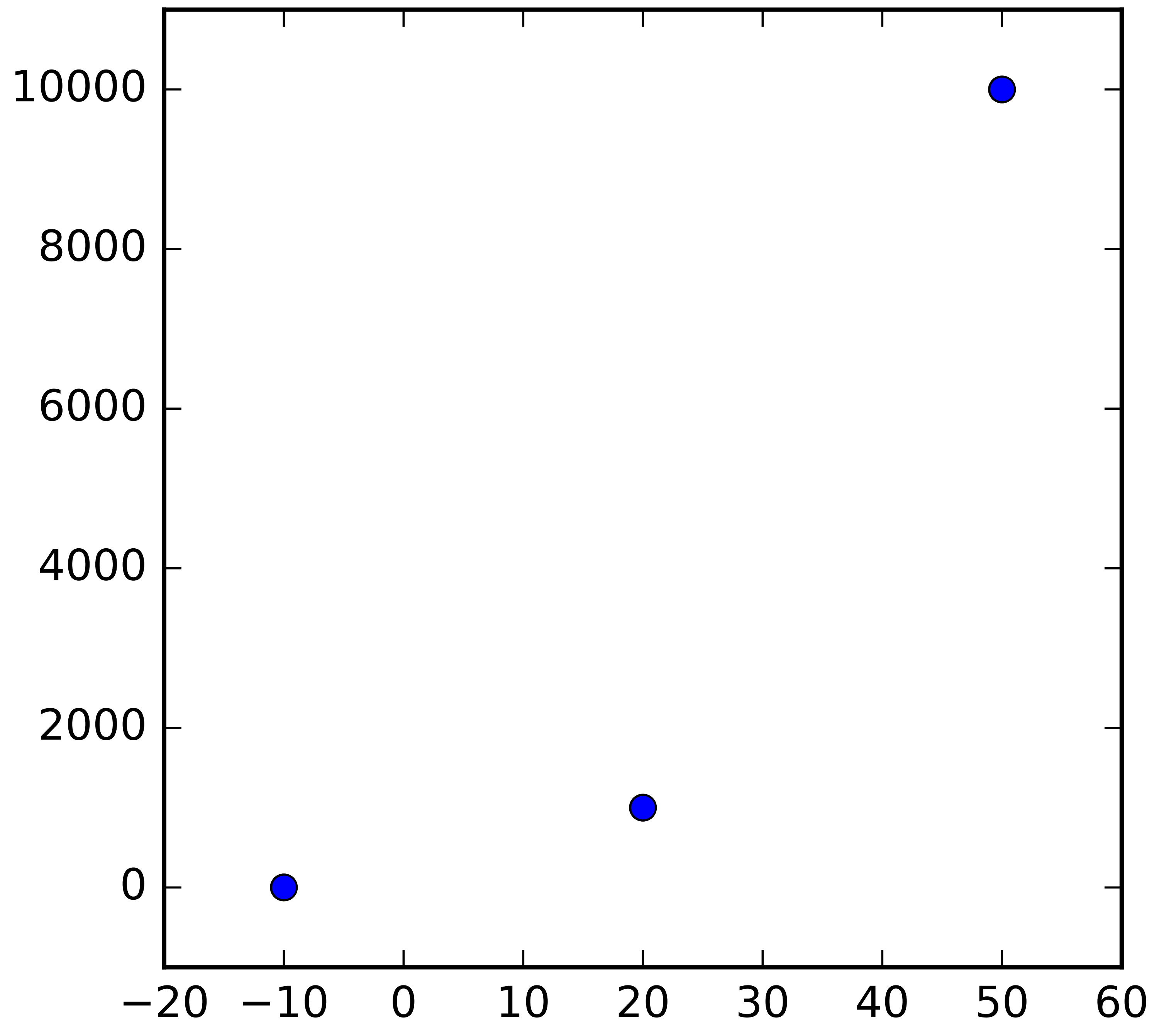
(generalizing the $\mathrm{Var}[x] = E[(x - E[x])^2]$, with $\mathrm{Cov}[x,x] = \mathrm{Var}[x]$).

The formula for the Spearman correlation coefficient is given at
http://en.wikipedia.org/wiki/Spearman's_rank_correlation_coefficient
in terms of the difference $d_i = r_i - s_i$ between ranks, in this easily calculable form:

$$\rho = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)} \ . \tag{2}$$

It is straightforward to verify that (1) reduces to (2) (see linked notes)

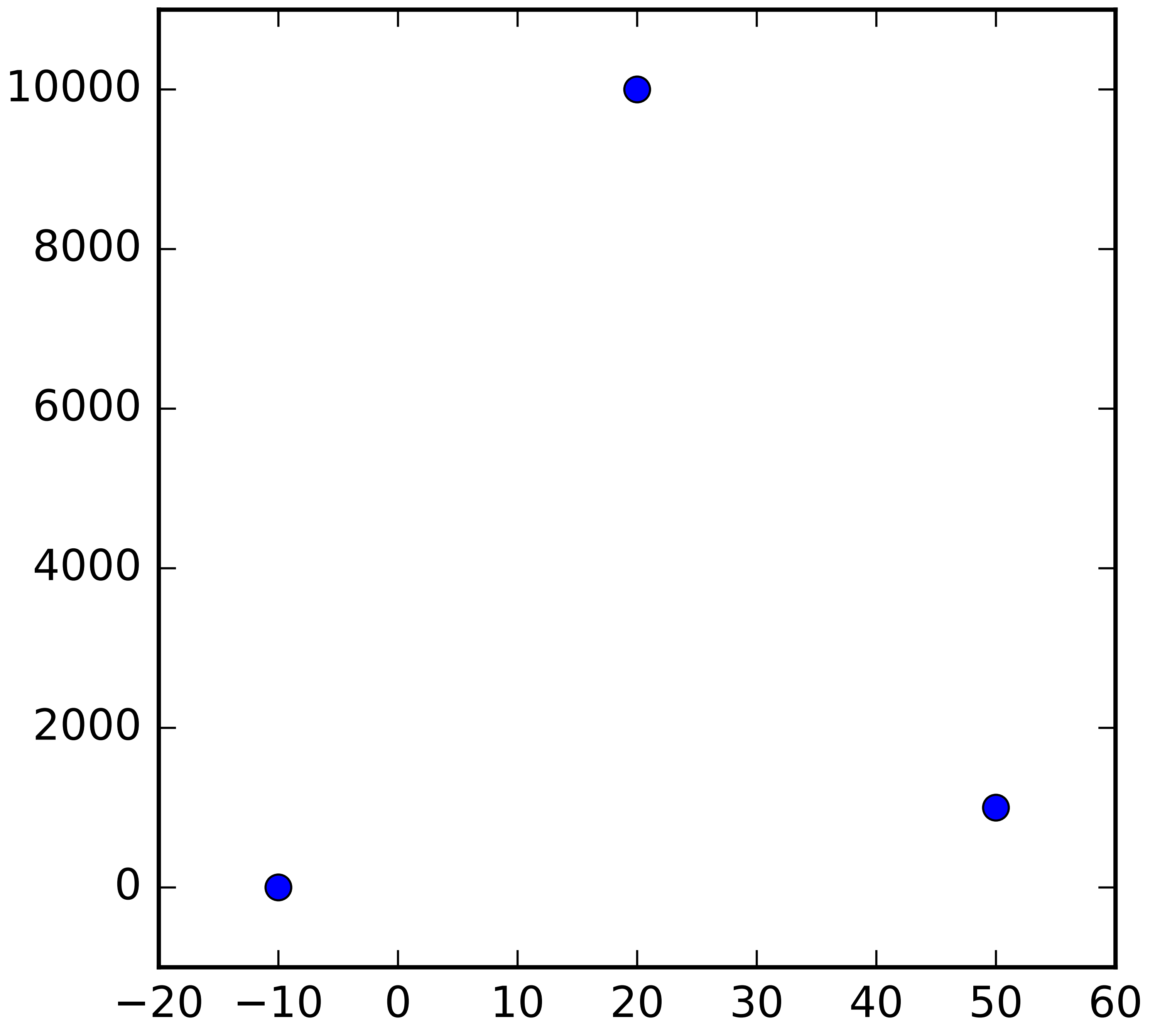
xdata=[-10, 20, 50]
ydata=[.5, 1000, 10000]




pearsonr(xdata,ydata) = .91

spearmanr(xdata,ydata) = 1
pearsonr([1,2,3],[1,2,3]) =1

xdata=[-10, 50, 20]
ydata=[.5, 1000, 10000]

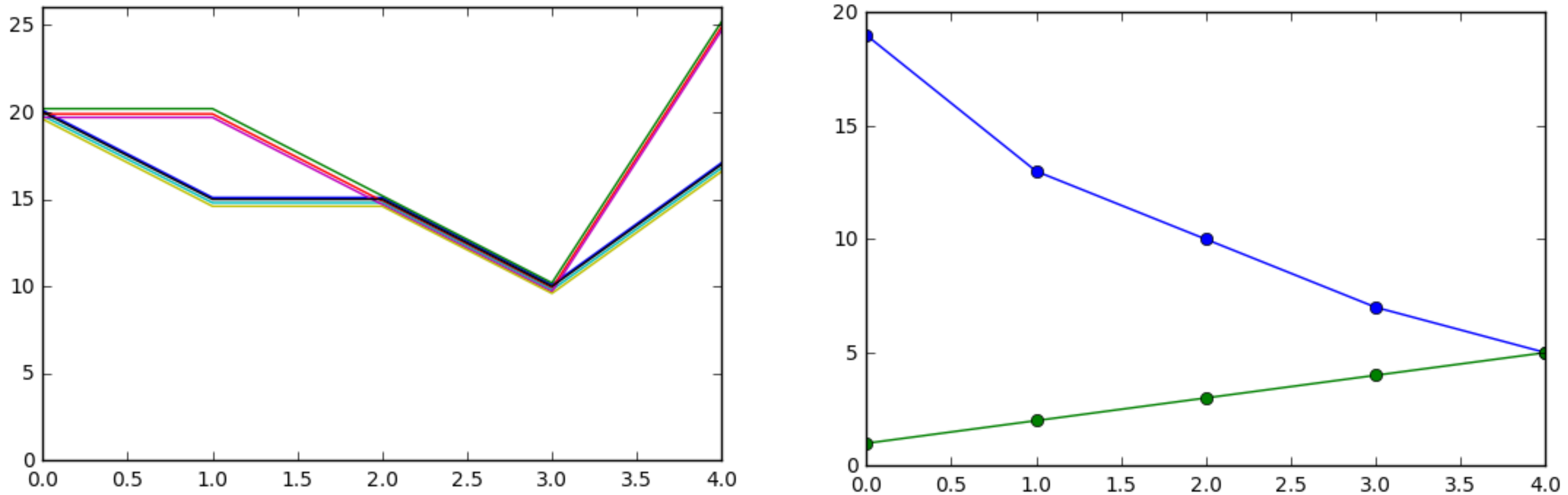



pearsonr(xdata,ydata) = .091

spearmanr(xdata,ydata) = .5
pearsonr([1,3,2],[1,2,3]) = .5

var([1,2,3])=2/3

cov([1,2,3],[1,3,2]) = ((1-2)(1-2) + (2-2)(3-2) + (3-2)(2-2)) / 3
= 1/3

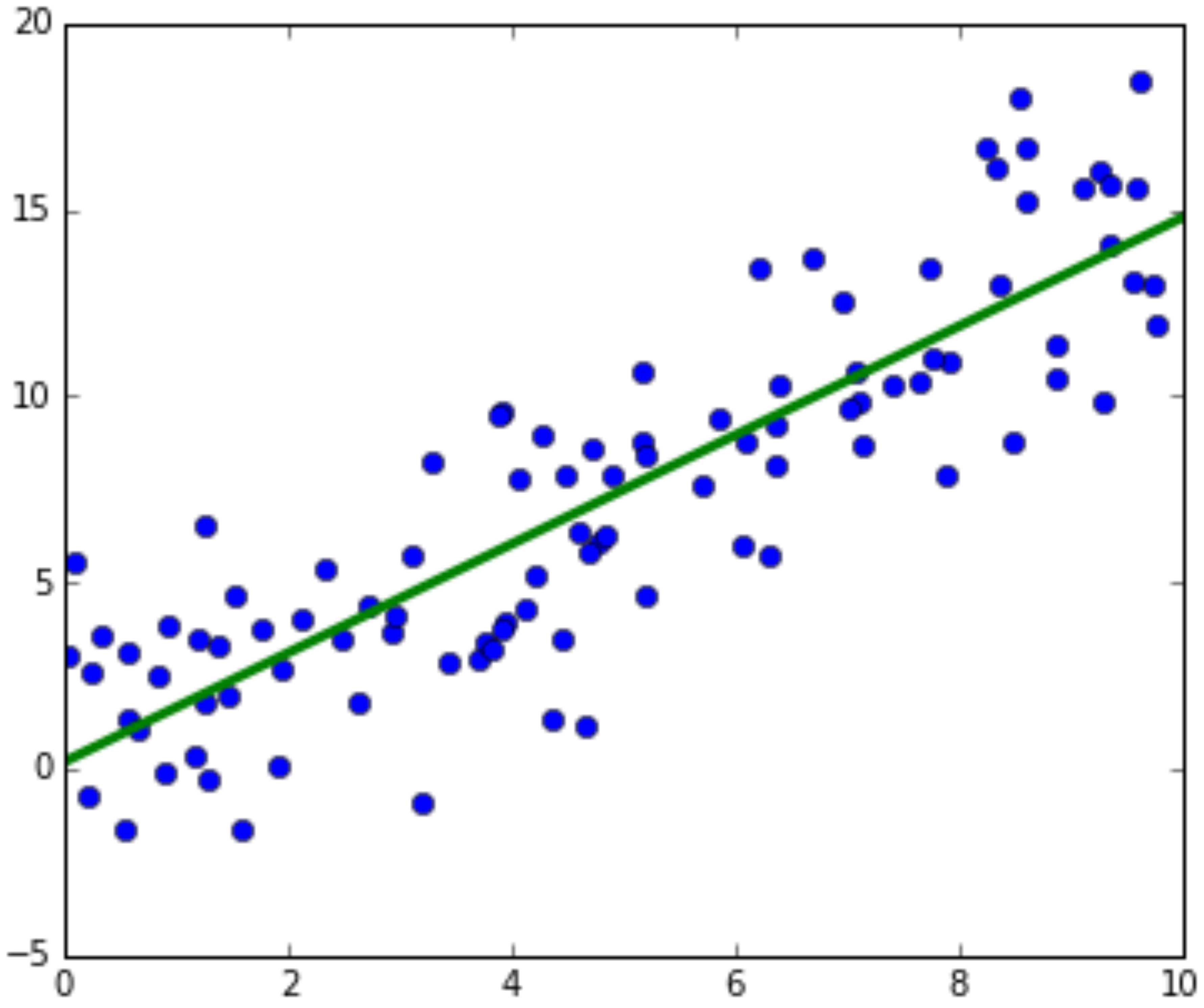

25
20
15
10
5
0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
20
15
10
5
0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0

Linear Regression (Least Square Fit)

The best line $y=ax+b$ is determined by parameters a,b that minimize the sum

$$\sum_{i=1}^{n} ((ax_i + b) - y_i)^2$$

over the n data points $(x_i, y_i)$

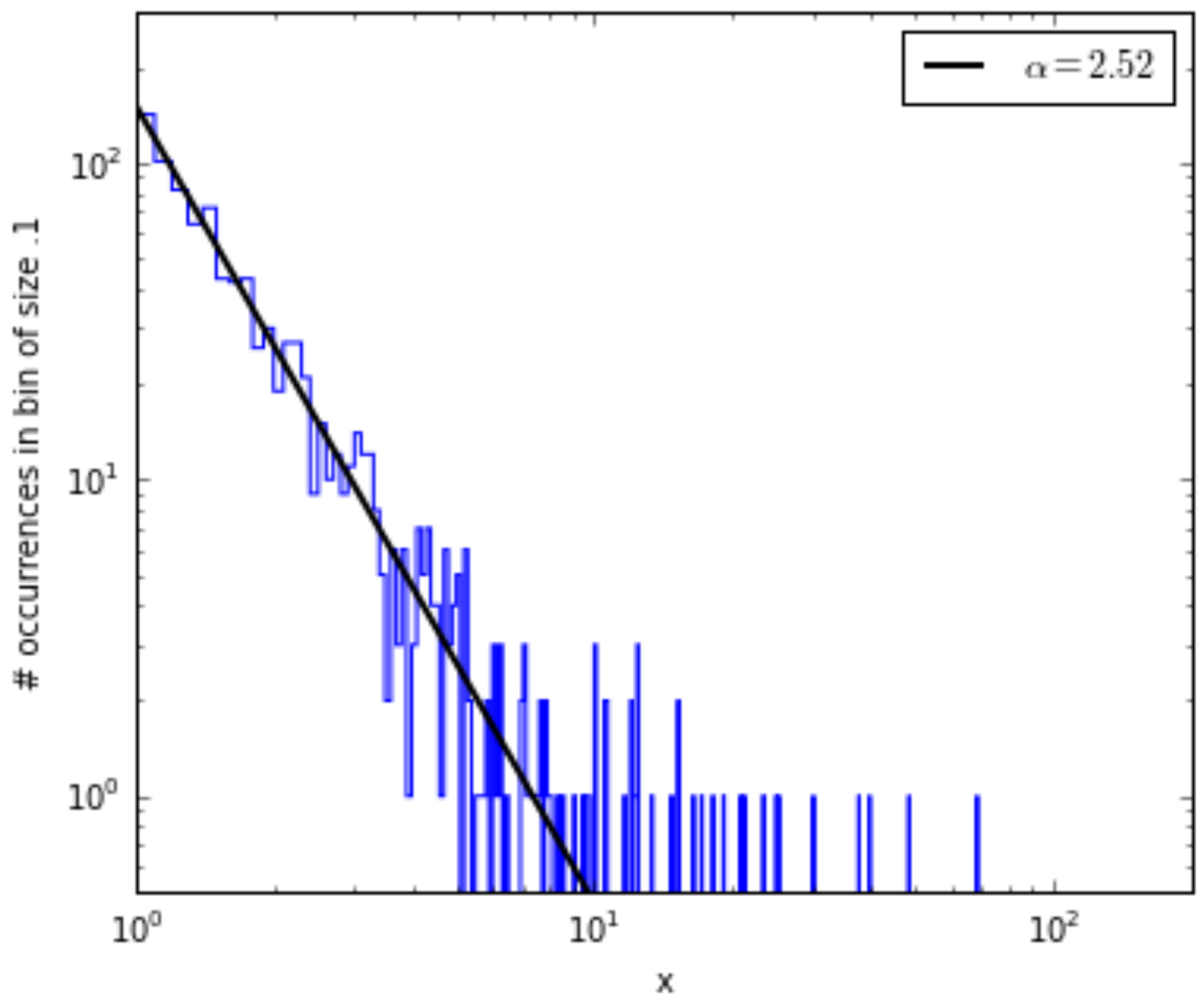



The solution was

$$b = E[y] - aE[x]$$

$$a = \frac{Cov(x,y)}{Var(x)}$$

Fit power law data




$p(x) = Cx^{-k}$

C is determined by the normalization that the sum over the probabilities (in this case the integral from some $x_{min}$ to ∞) is equal to 1.

The overall probability of the data for some given k is

$$p(x \mid k) = p(x_1 \mid k)\, p(x_2 \mid k)\, p(x_3 \mid k) \ldots p(x_n \mid k)$$

so the best fit is given by the value of k that maximizes the probability of the data. Setting to zero the derivative of the above with respect to k gave

$$k = 1 + n \Big/ \sum_{i=1}^{n} \ln(x_i / x_{min})$$

In each case, there's a model M, with some parameters, intended to describe some data x.

How to determine the model parameters?

A. Maximum Likelihood Estimate (MLE)

find the parameters that maximize p(x l M)

B. Maximum a Posteriori Estimate (MAP)

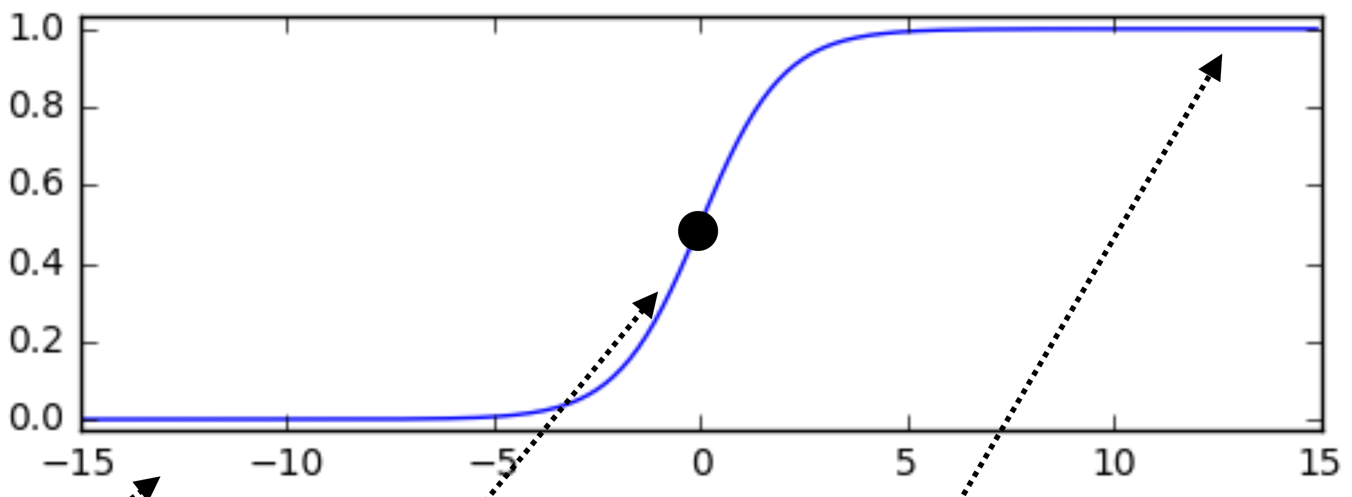
find the parameters that maximize p(M l x)

# Logistic Regression

modification of regression scheme for predicting
a binary outcome rather than a linear relation

```
def logit(x): return 1/(1+exp(-x))
```

logistic function: $$\mathrm{logit}(x) = \frac{1}{1+e^{-x}}$$




```
x=np.arange(-15,15,.1)
ylim(-.03,1.03)
plot(x,logit(xd))
```

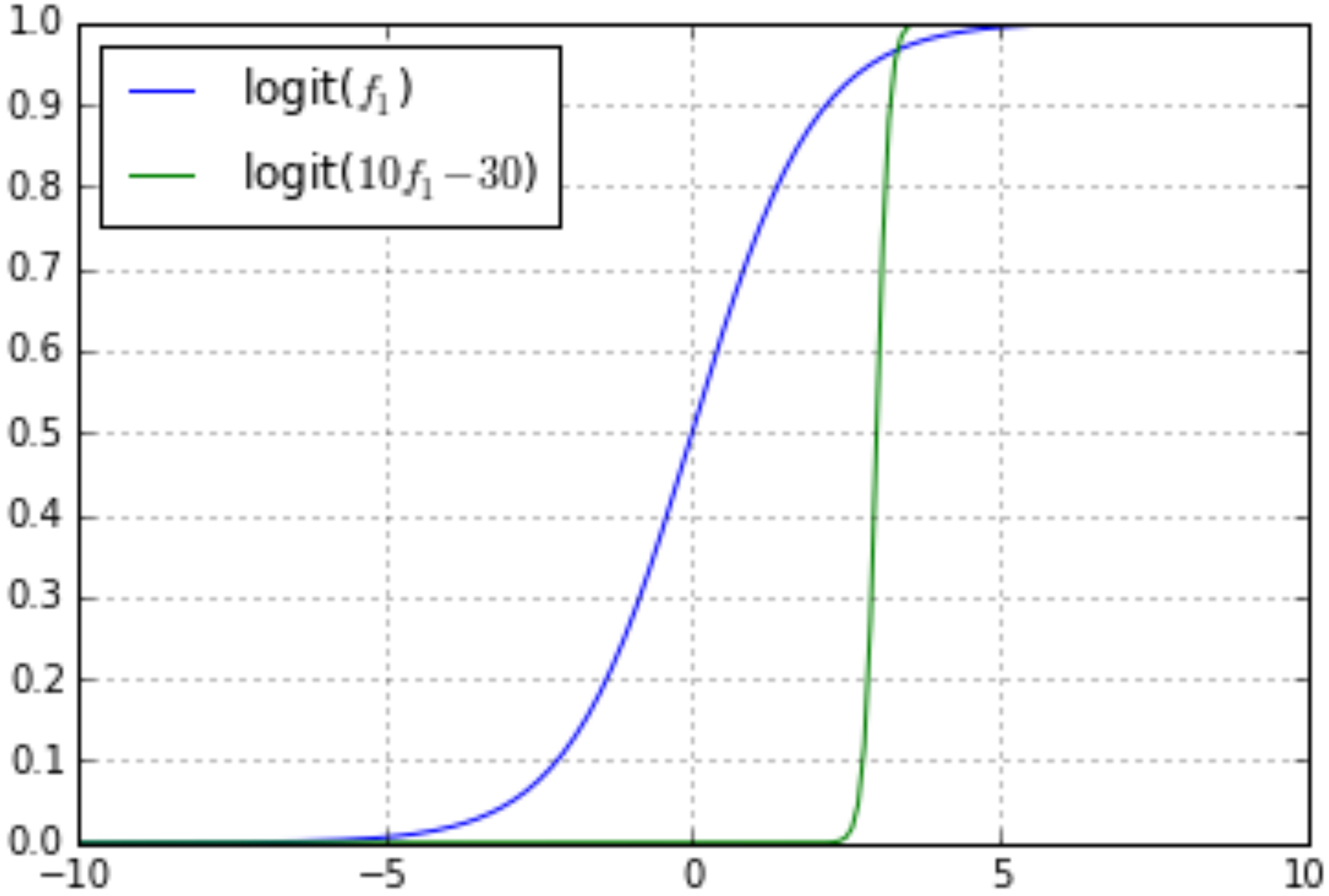
For large positive values of the argument x, $e^{-x}$ becomes very small,
so the function evaluates to 1,

whereas for large negative values of x, $e^{-x}$ becomes very large
and the function evaluates to zero.

The value for x=0 is logit(0)=.5,
and that marks the middle of the transition region between 0 and 1.

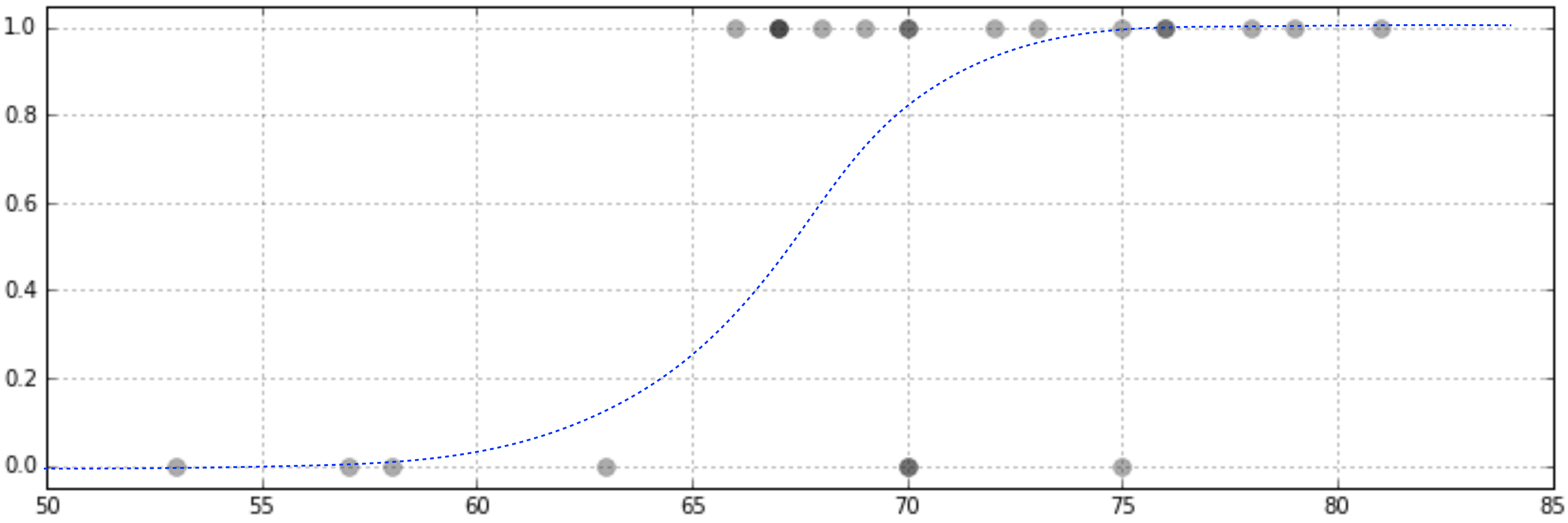
$x$ can be considered to be any linear combination of feature values $f_i$ being used to make the binary prediction, $x = \alpha_1 f_1 + \ldots + \alpha_n f_n + \beta$, where the weights $\alpha_i$ and $\beta$ are determined by some training procedure (to provide the best fit to labelled training data). Intermediate values of the logit function can be interpreted as the probability of the label being 1 for those values of the features.

For a single feature $x = \alpha f_1 + \beta$, the logit is equal to .5 for $f_1 = -\beta/\alpha$, and the width of the transition region is proportional to $1/\alpha$:




$$\text{logit}(x) = \frac{1}{1 + e^{-x}}$$

# Logistic Regression




again determine parameters by MLE

# Markov chains

Consider a system with M states, labelled 1,2,3,…M
which undergoes a series of transitions
2,1,8,5,6, …

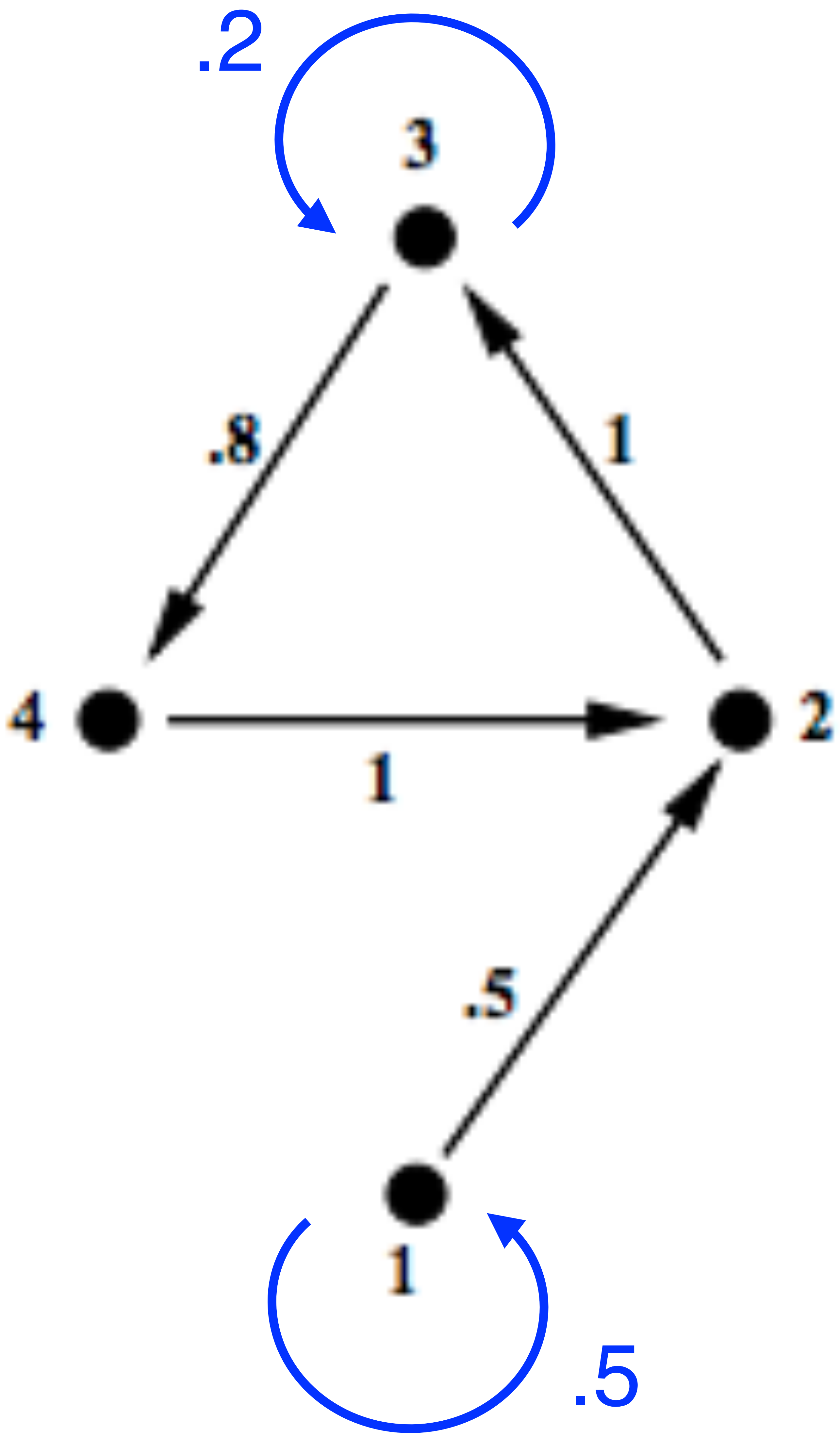
$X_i$ = state after i transitions

$X_0$ $X_1$ $X_2$ $X_3$ $X_4$ $X_5$ $X_6$ …

Markov chain property:

$$T_{ij} = p(X_{n+1} = j | X_n = i) = p(X_{n+1} = j | X_n = i, X_{n-1} = i-1, \ldots, X_0)$$

# States with probabilistic transitions




$$\mathbf{T} = \begin{pmatrix} .5 & .5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & .2 & .8 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

A state is recurrent if wherever you go from it, there's always a way back

If a state is not recurrent, then it's called transient.

A state is periodic if recurrent and there exists a number d such that probability of returning is zero except in steps of multiples of d

A recurrent class is a set of recurrent states which can all reach one another: they are said to communicate.

A Markov chain is ergodic if it has only one recurrent class and no periodic states. (These are the "good" Markov chains, where the states all communicate, and are inter reachable via any number of steps.)

# “Mark V Shaney"

```
def make_trigrams(filename):

   with open(filename) as f: words = f.read().split()

    trigrams = defaultdict(list)

    bigram=tuple(words[:2])

    for w in words[2:] + words[:2]:
    #keys of trigram dict are tuples, values are lists
        trigrams[bigram].append(w)
        bigram=(bigram[1],w)

    return trigrams
```

```
def random_text(trigrams, startwords, num_words=100):

    current_pair = random.choice(startwords)
    random_text = list(current_pair)

    # continue past num_words until ends in .
    while len(random_text)< num_words:
        next = random.choice(trigrams[current_pair])
        random_text.append(next)
        current_pair = (current_pair[1], next)
        # avoid long loops if too few periods in training text

    return ' '.join(random_text)
```

(We, both) sprang

(both, sprang) in

(sprang, in) and

in and away

and away we

away we went

we went to

went to the

to the ventilator

the ventilator and

ventilator and to

and to carry

to cary out

carry out my