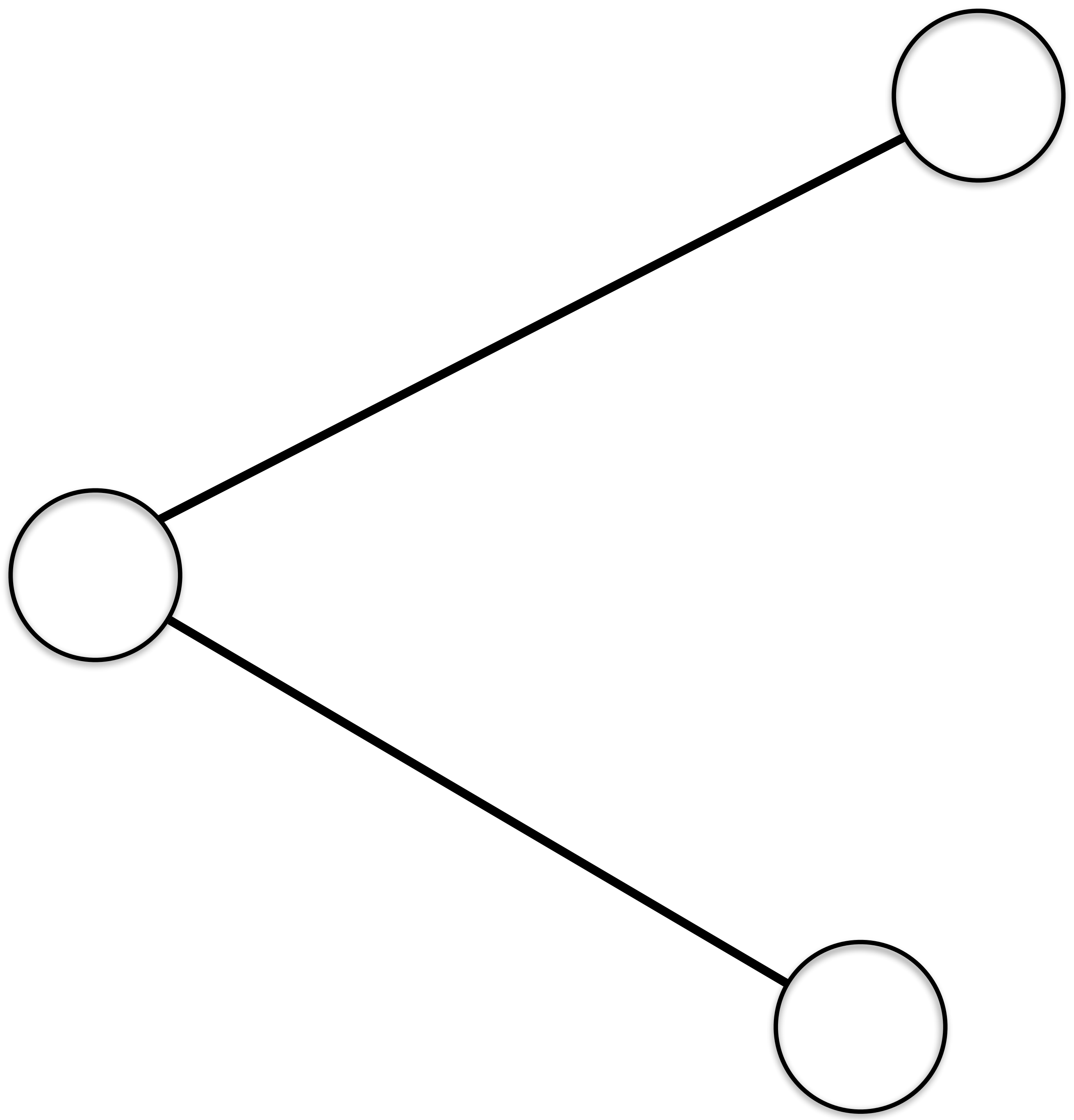
Info 2950, Lecture 18

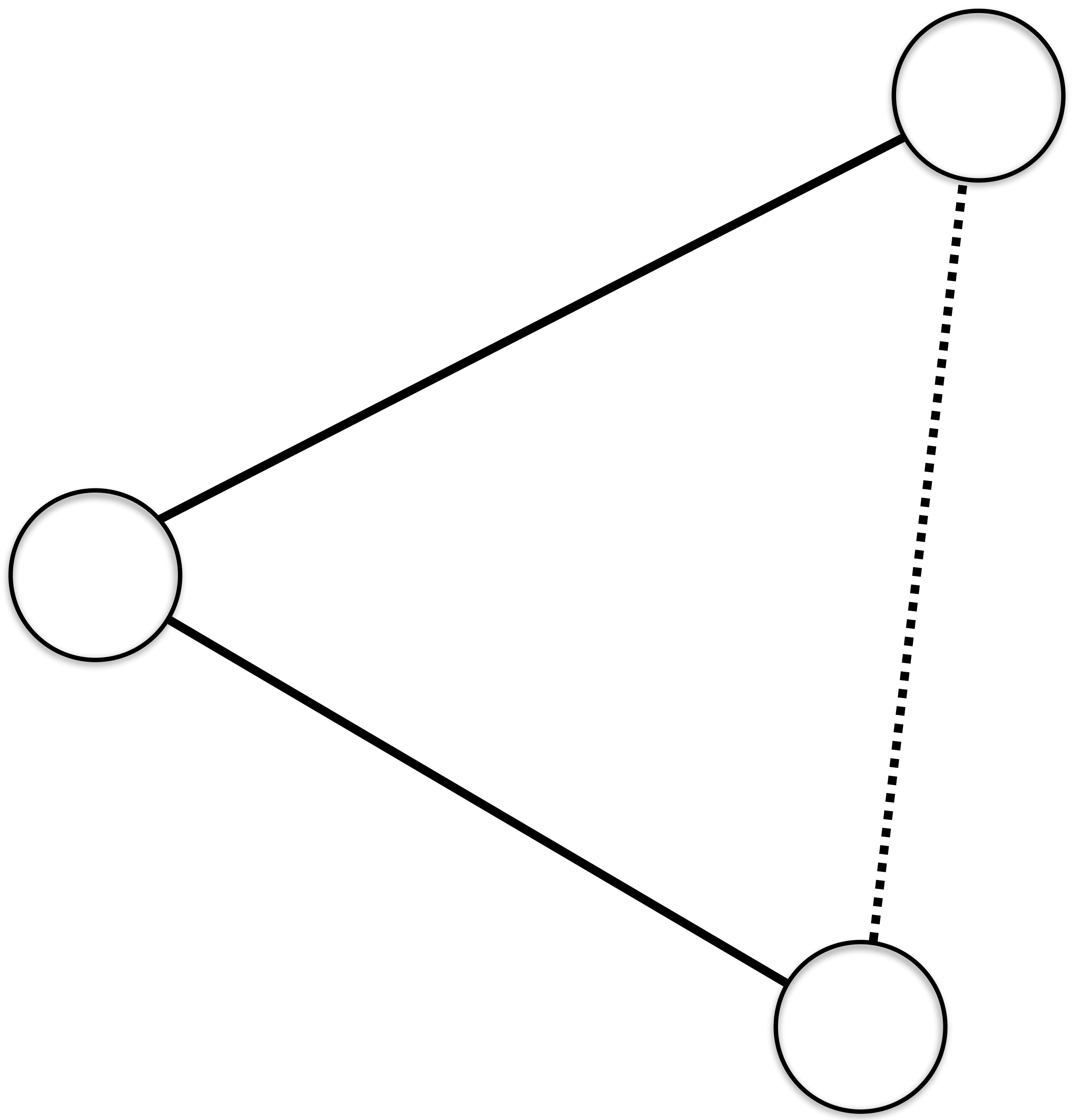
11 Apr 2017

Prob Set 6: out tonight (?)

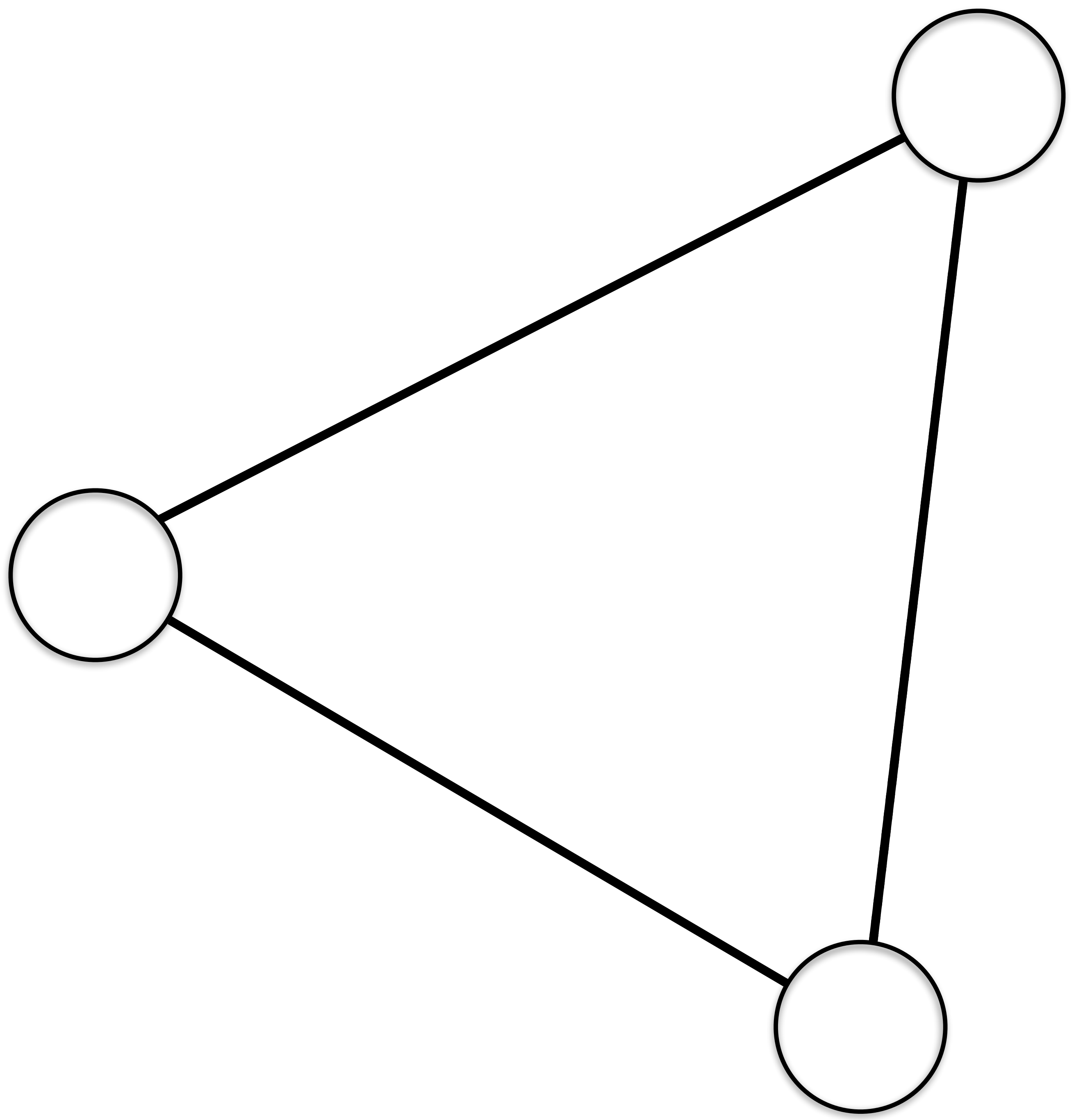
Triadic Closure



Triadic Closure



Triadic Closure



The clustering coefficient of a node A is defined as the probability that two randomly selected friends of A are friends with each other.

In other words, it is the fraction of pairs of A’s friends that are connected to each other by edge.

(ranges from 0 to 1)

For a node i of degree n, there are

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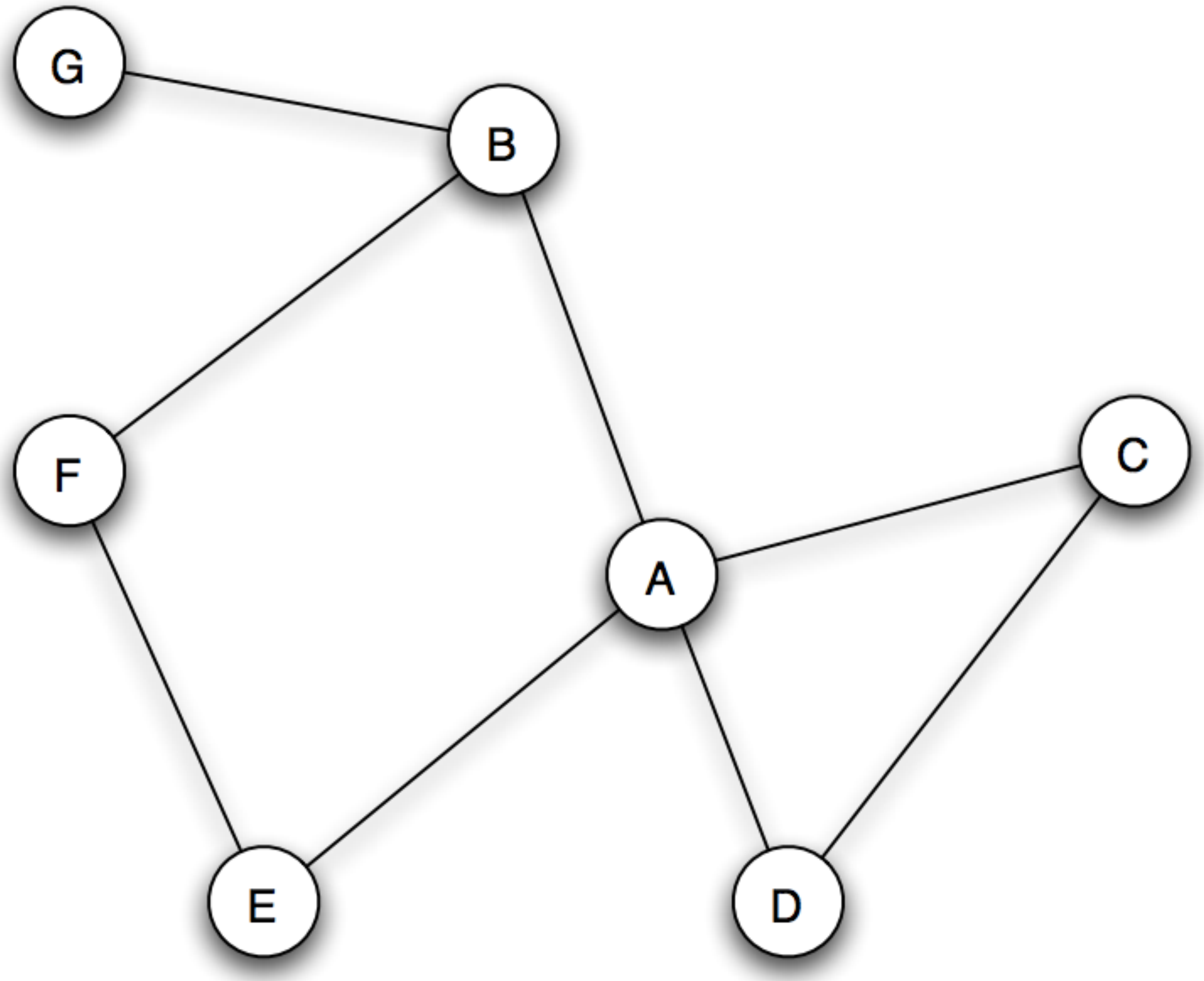
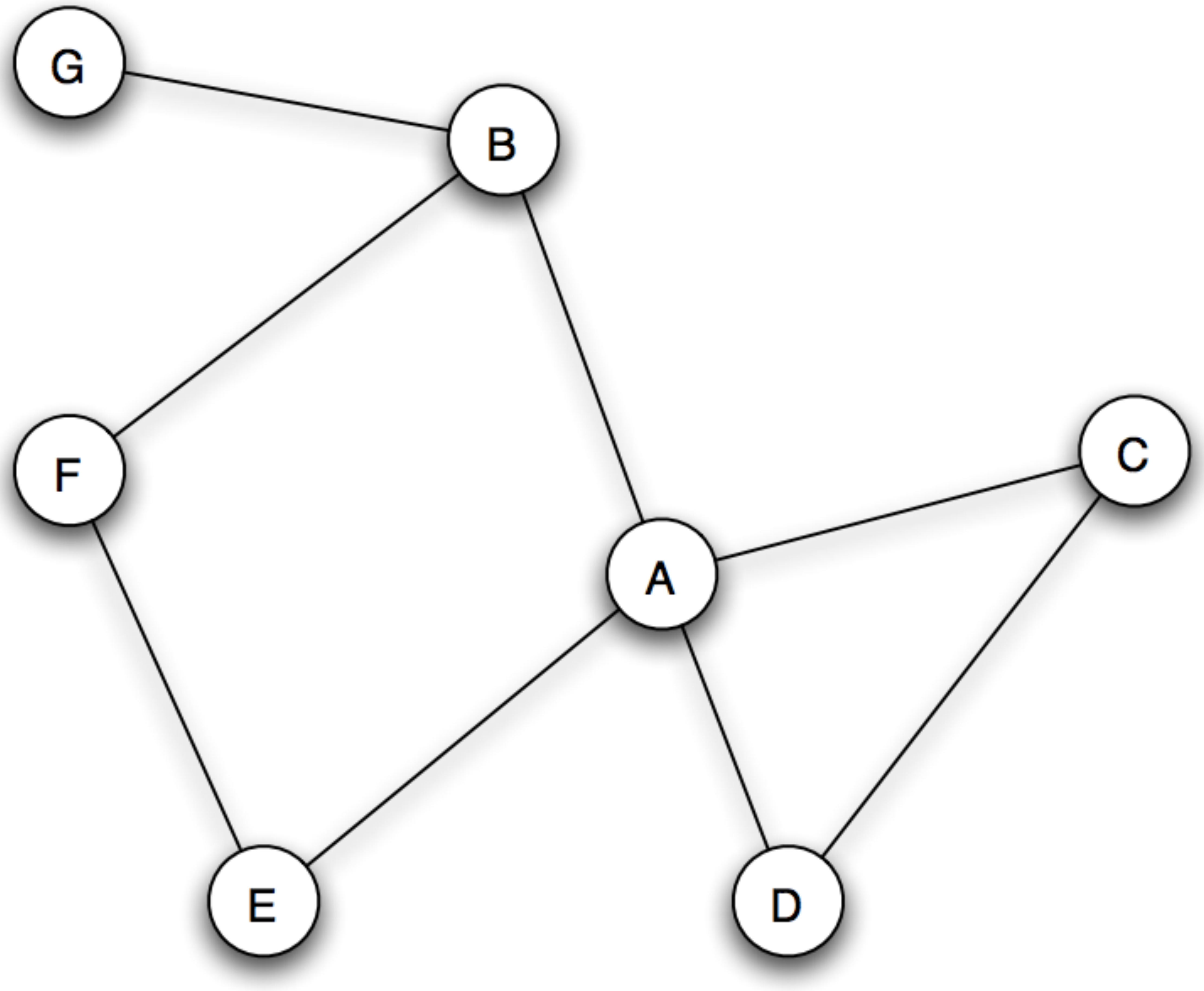
(ranges from 0 to 1)

For a node i of degree n, there are $\binom{n}{2} = n(n-1)/2$

possible friend pairs, and hence $C_i = m / (n(n-1)/2)$,
where m is the number of those pairs themselves friends

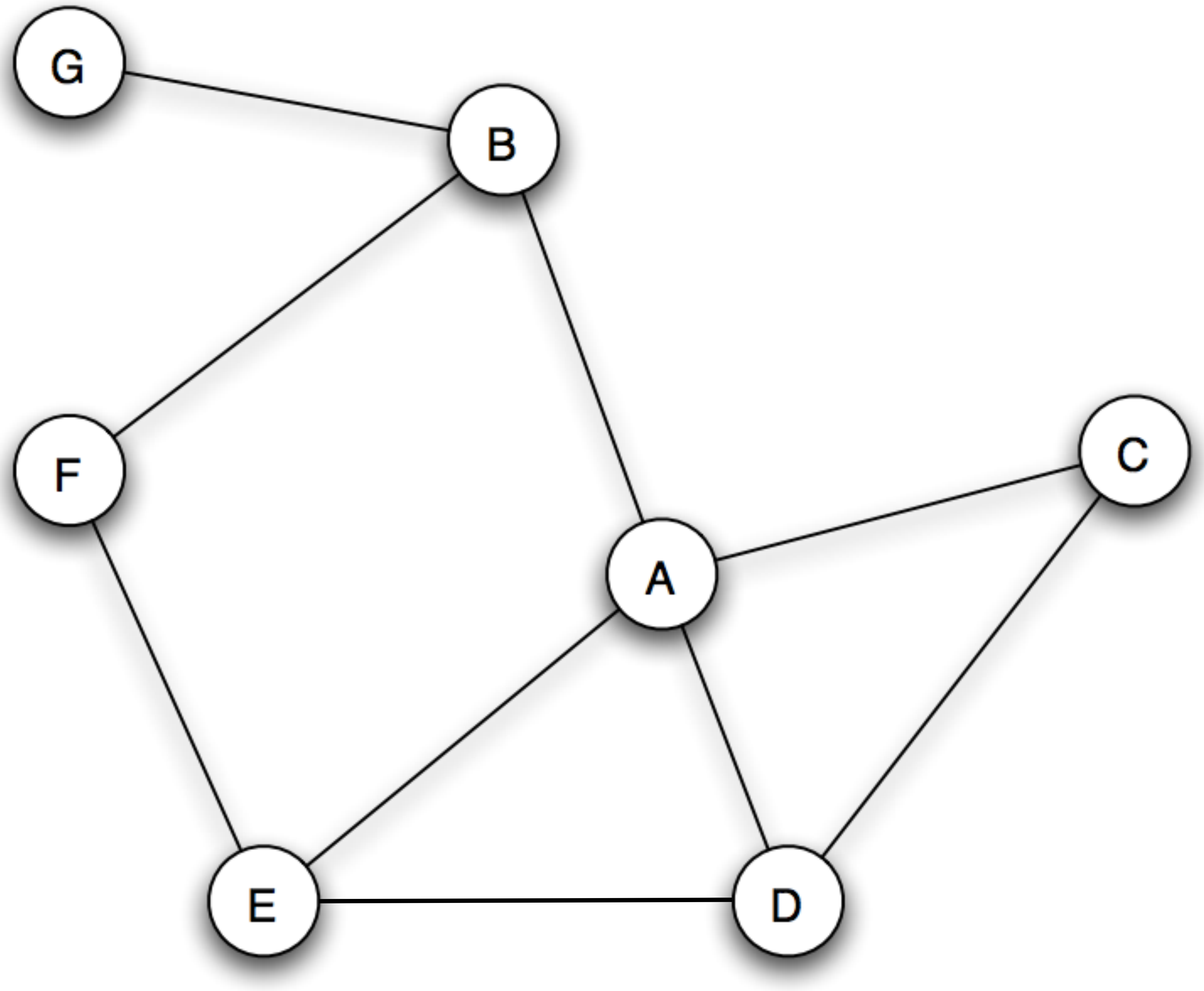


Easley/Kleinberg Figure 3.2



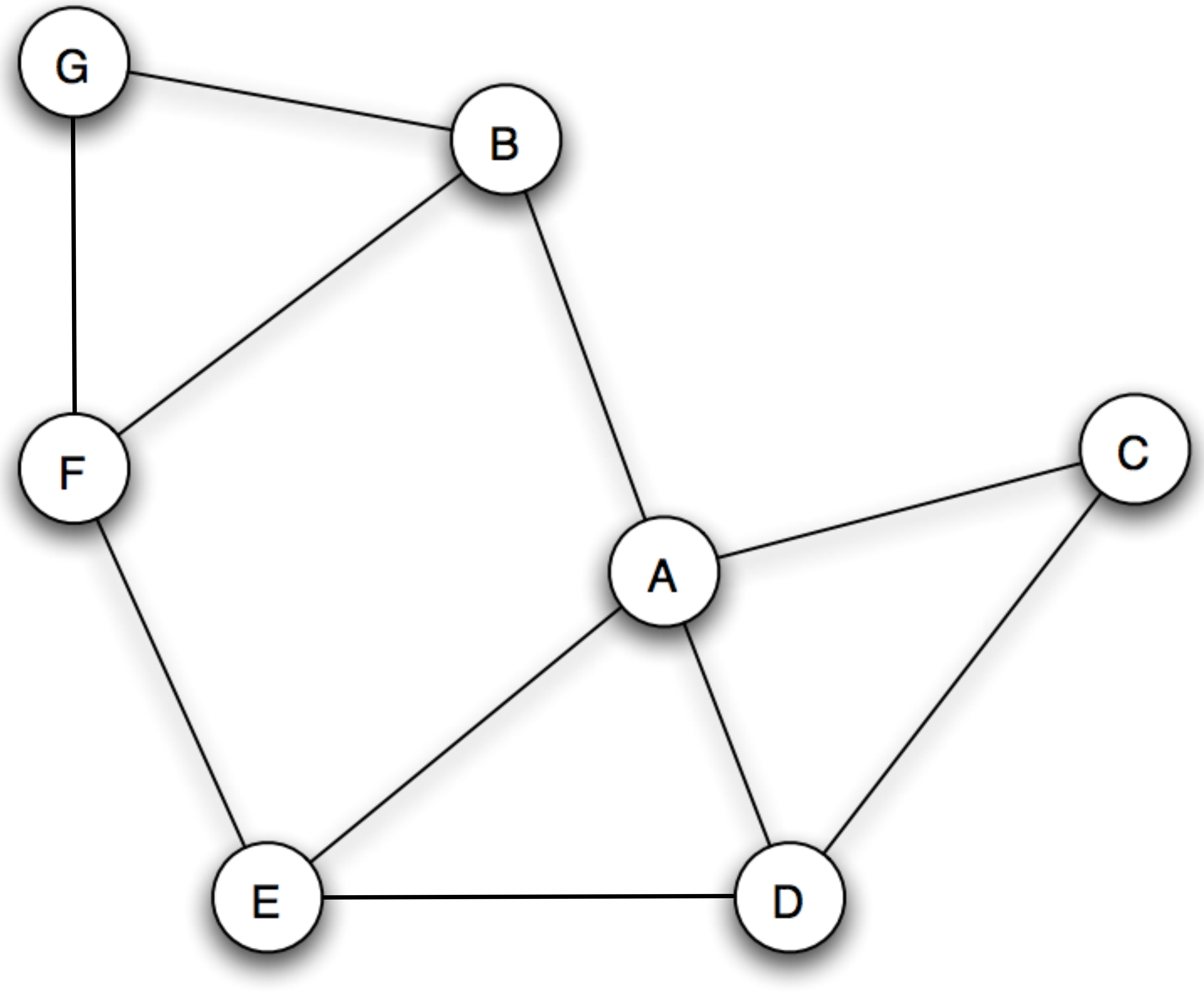
C(A) = 1/6

Easley/Kleinberg Figure 3.2



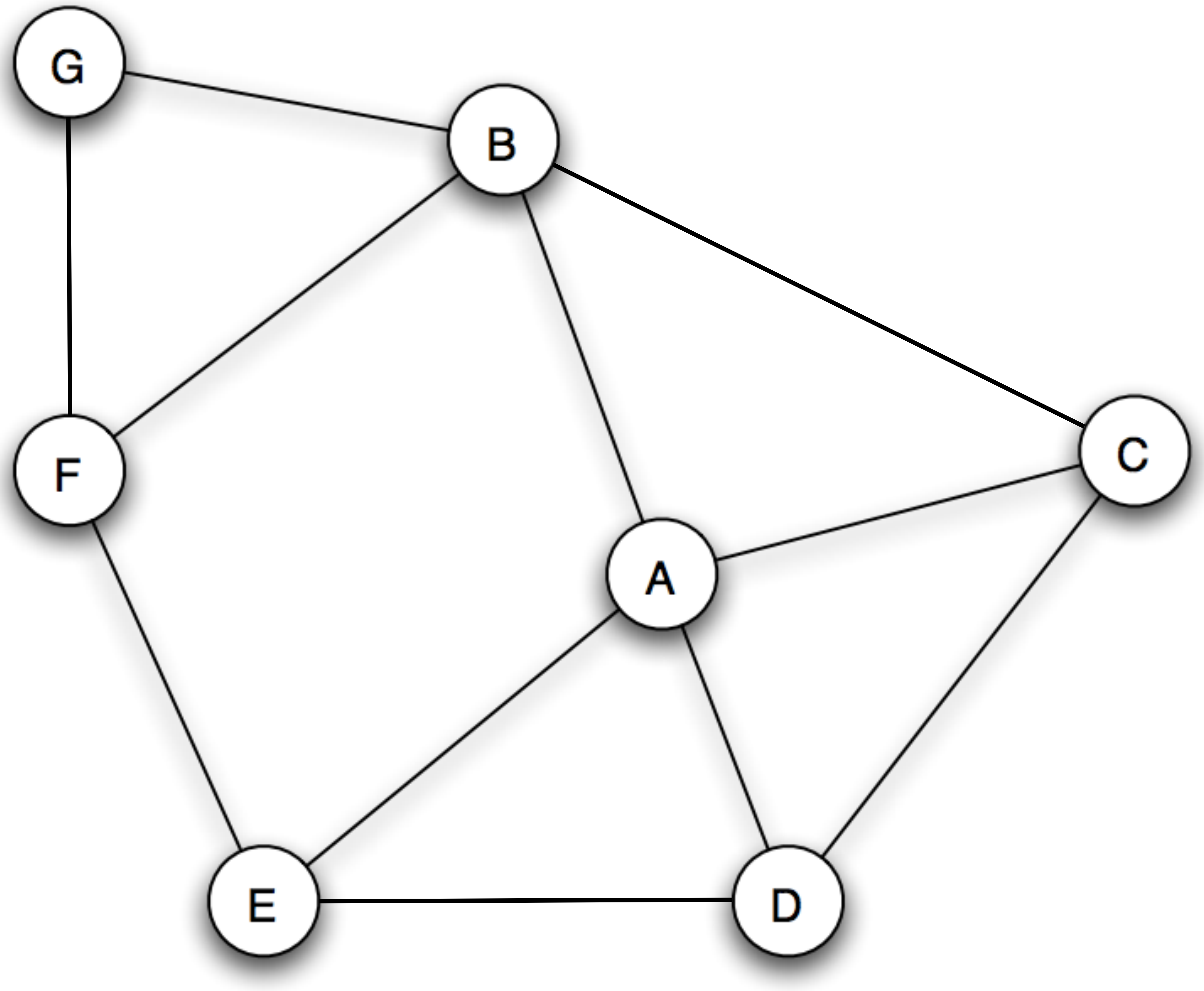
C(A) = 1/3

Easley/Kleinberg Figure 3.2



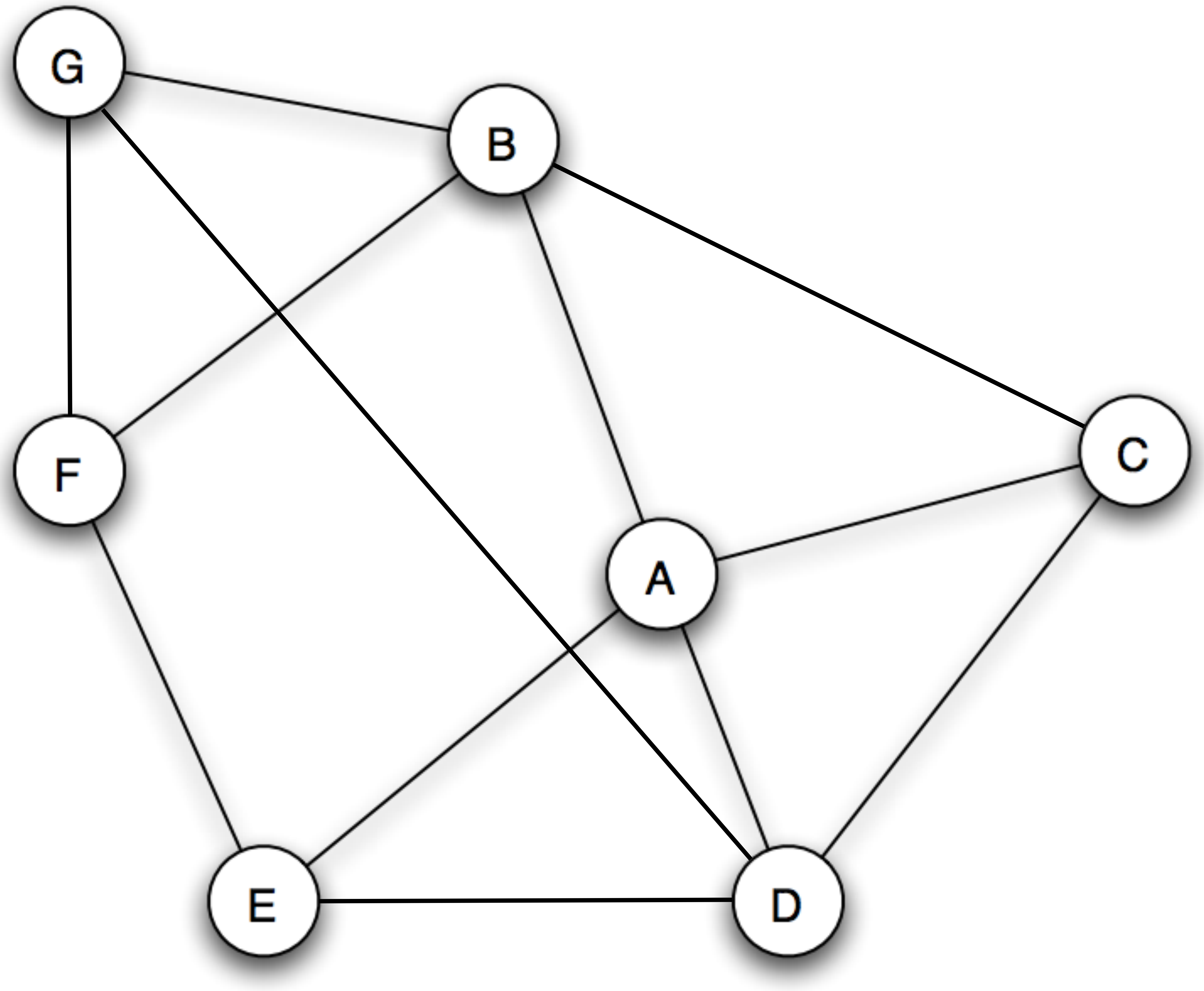
C(A) = 1/3

Easley/Kleinberg Figure 3.2



$C(A) = 1/2$

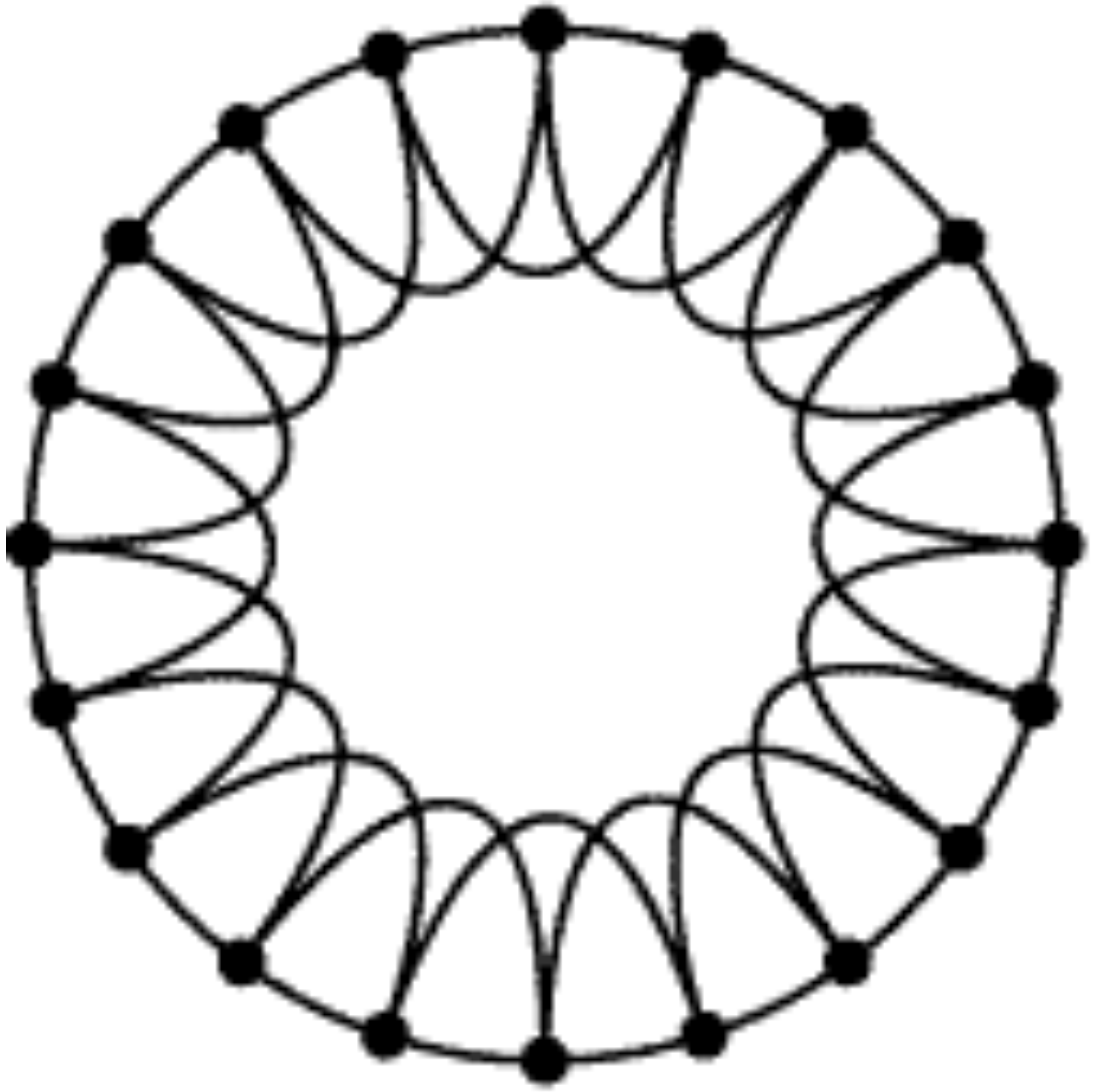
Easley/Kleinberg Figure 3.2



C(A) = 1/2

Easley/Kleinberg Figure 3.2

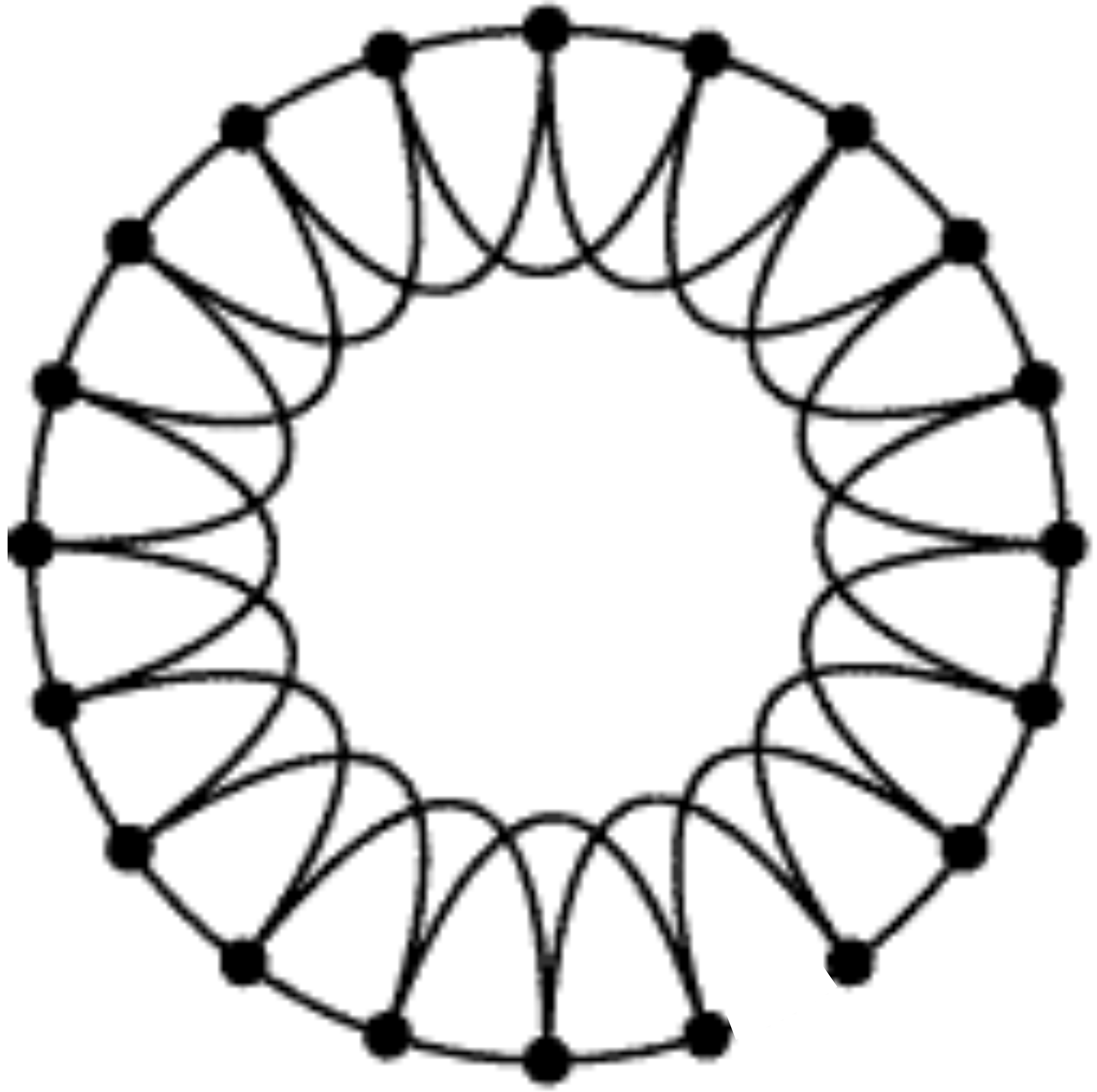
Watts Strogatz (1998)



D. Watts and S. Strogatz
Collective dynamics of
'small-world' networks
Nature 393, 440-442 (1998)

"rewire" with probability p

Watts Strogatz (1998)



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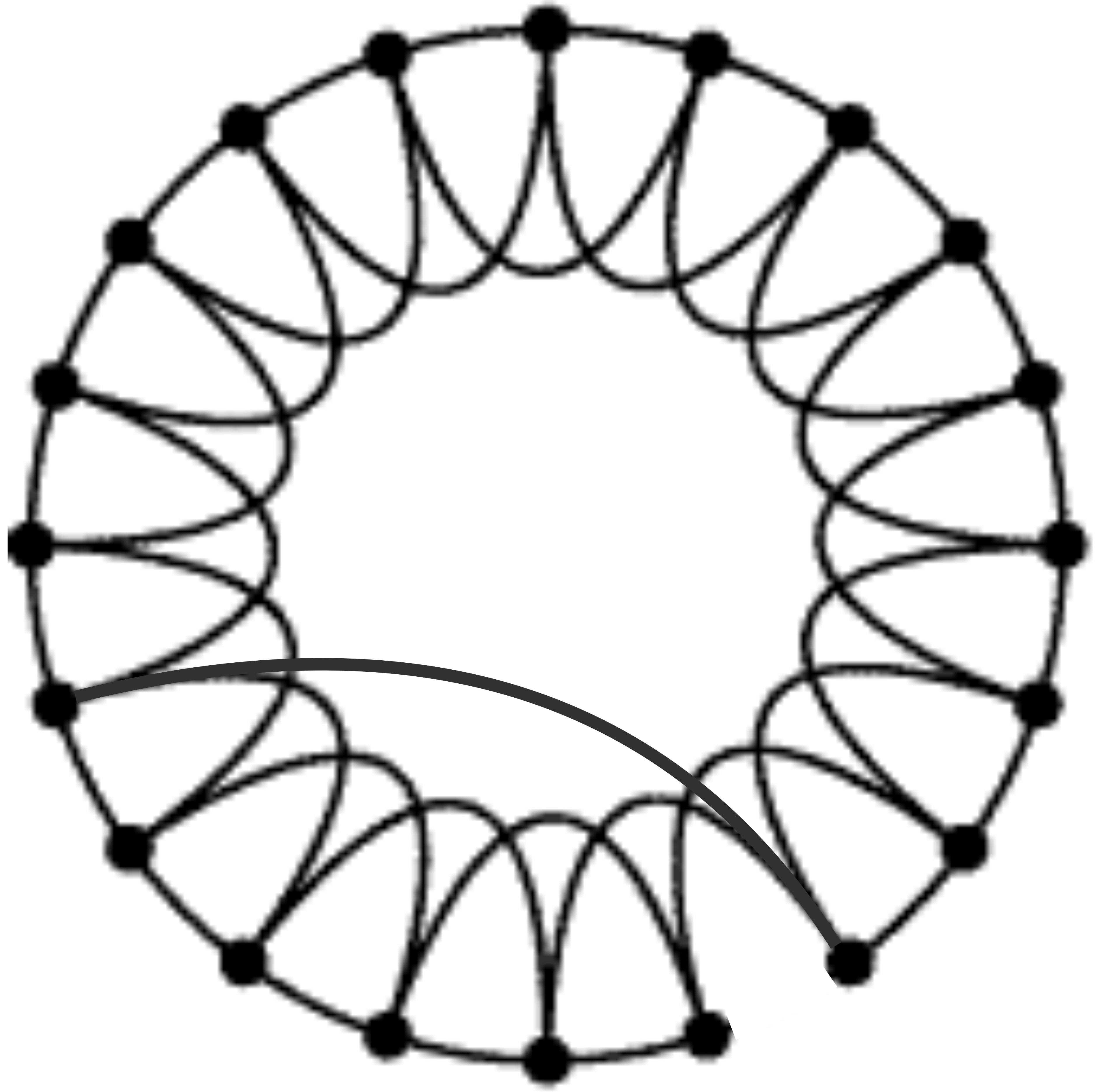

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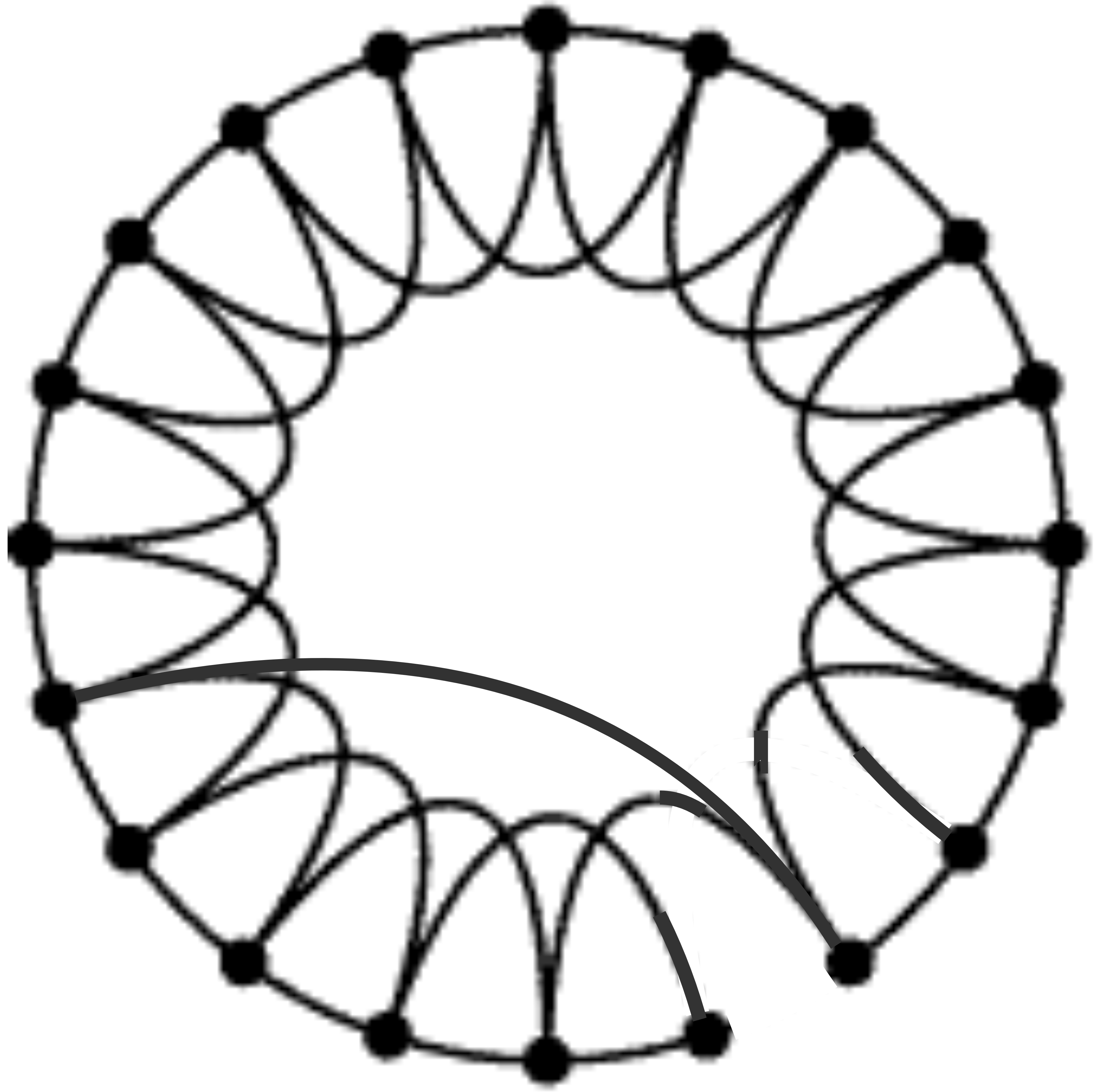

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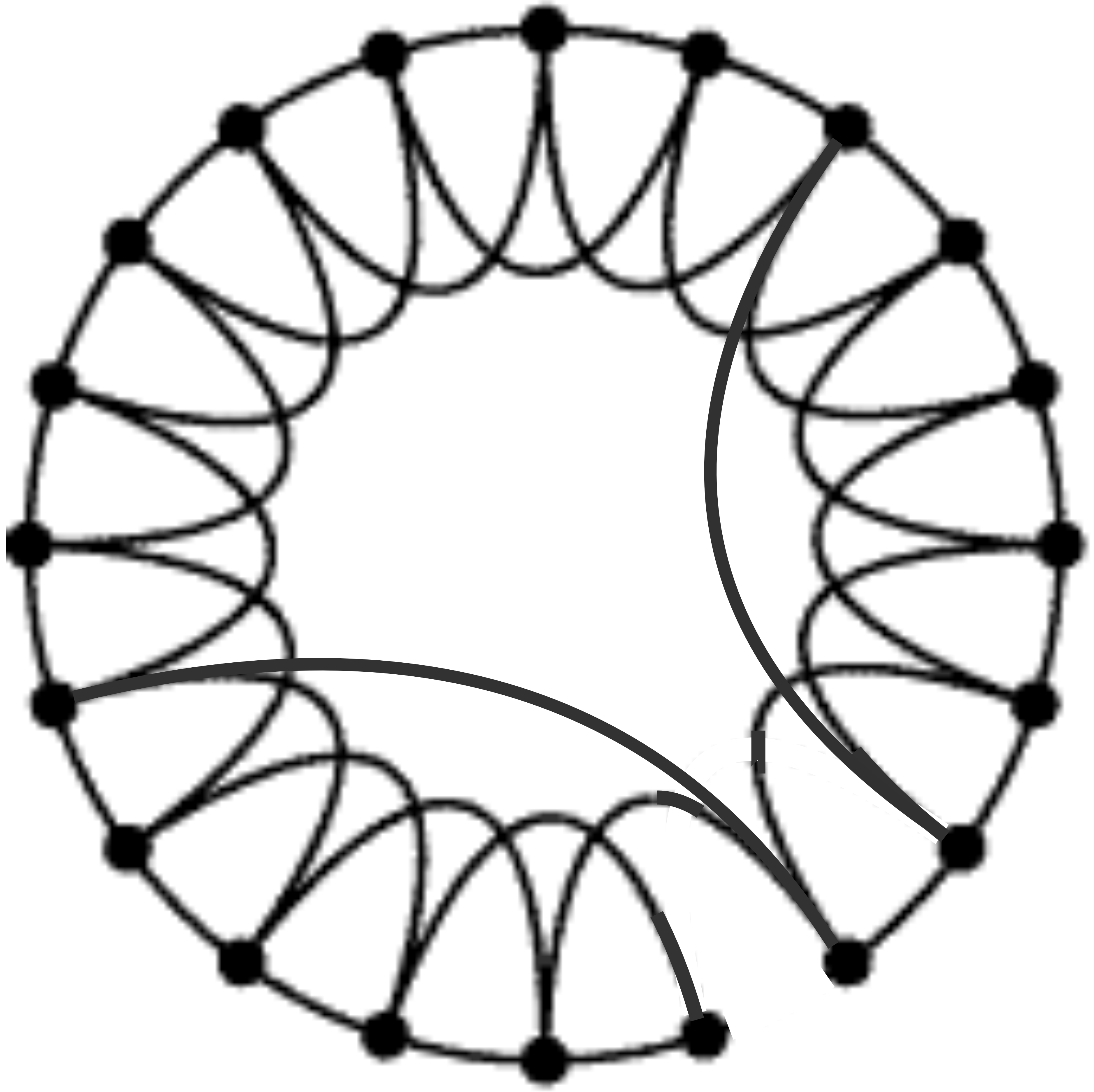

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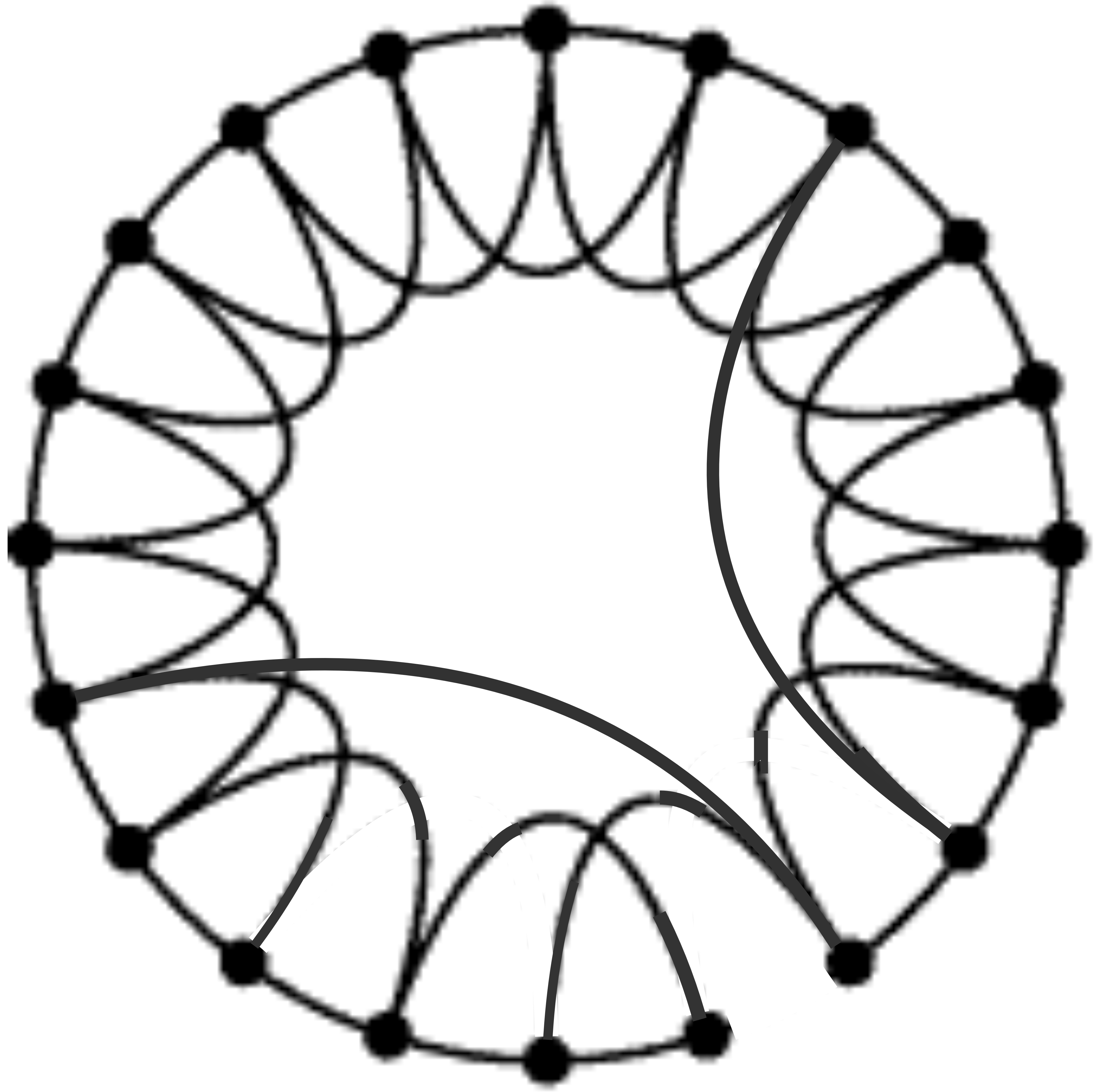

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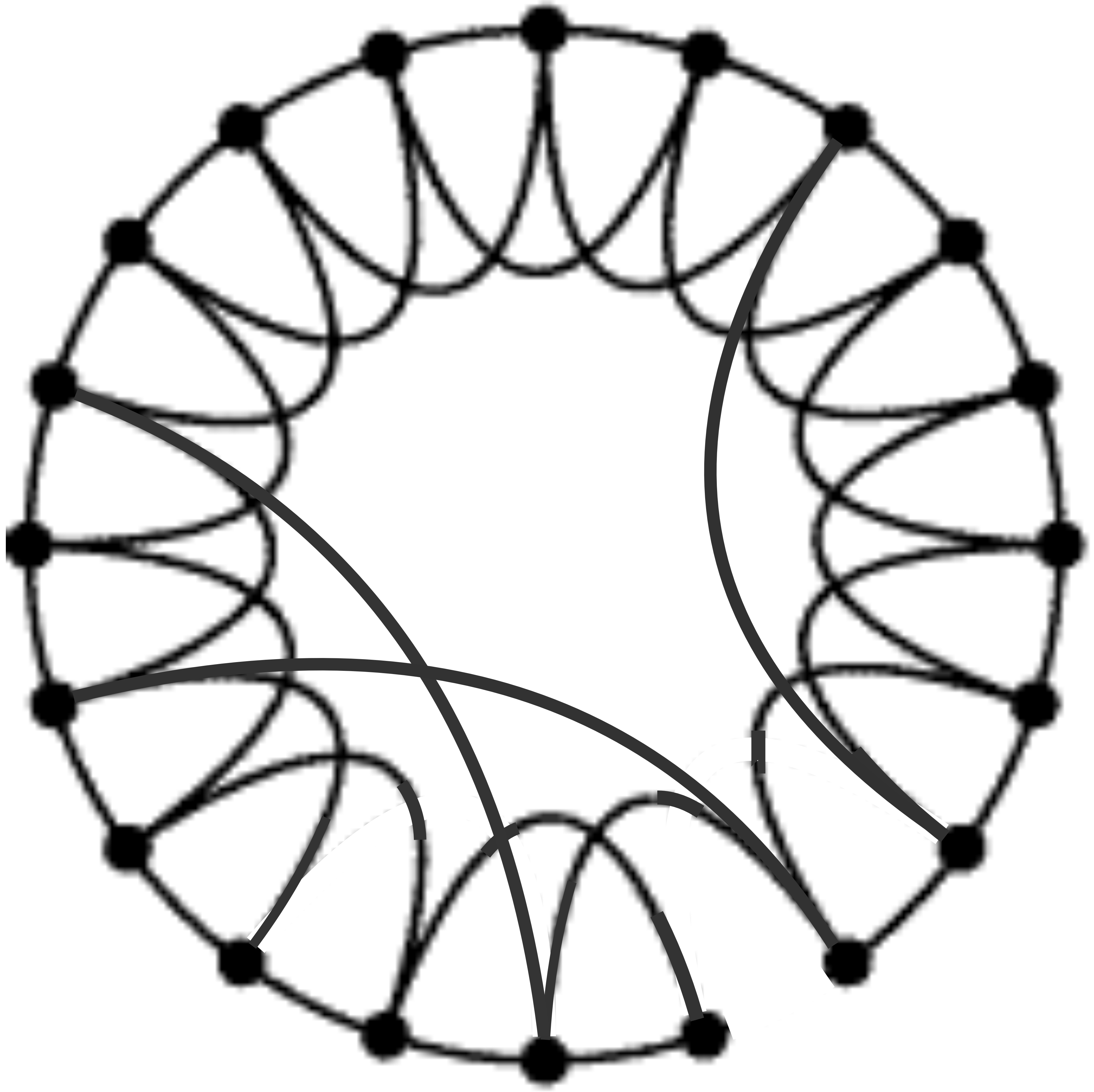

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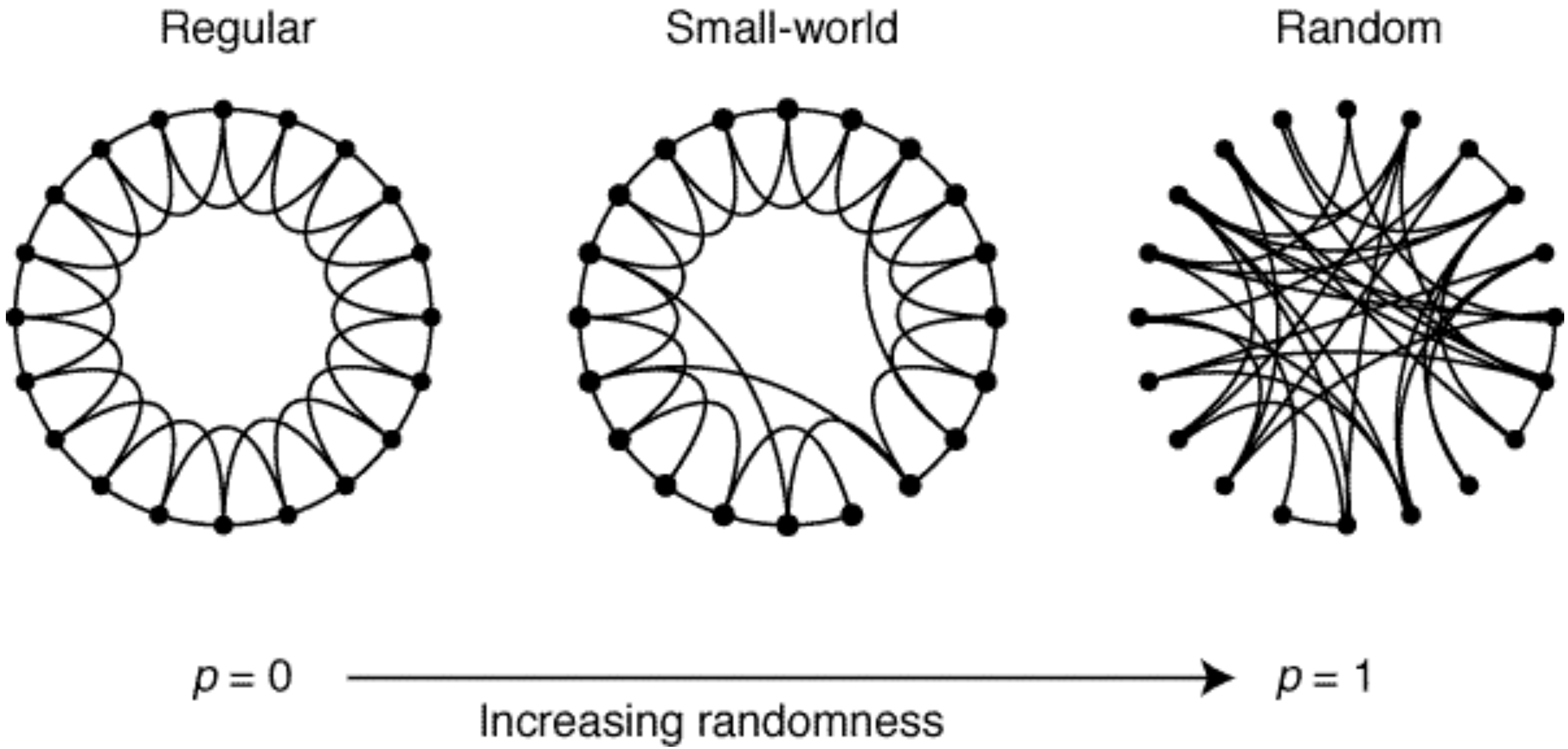
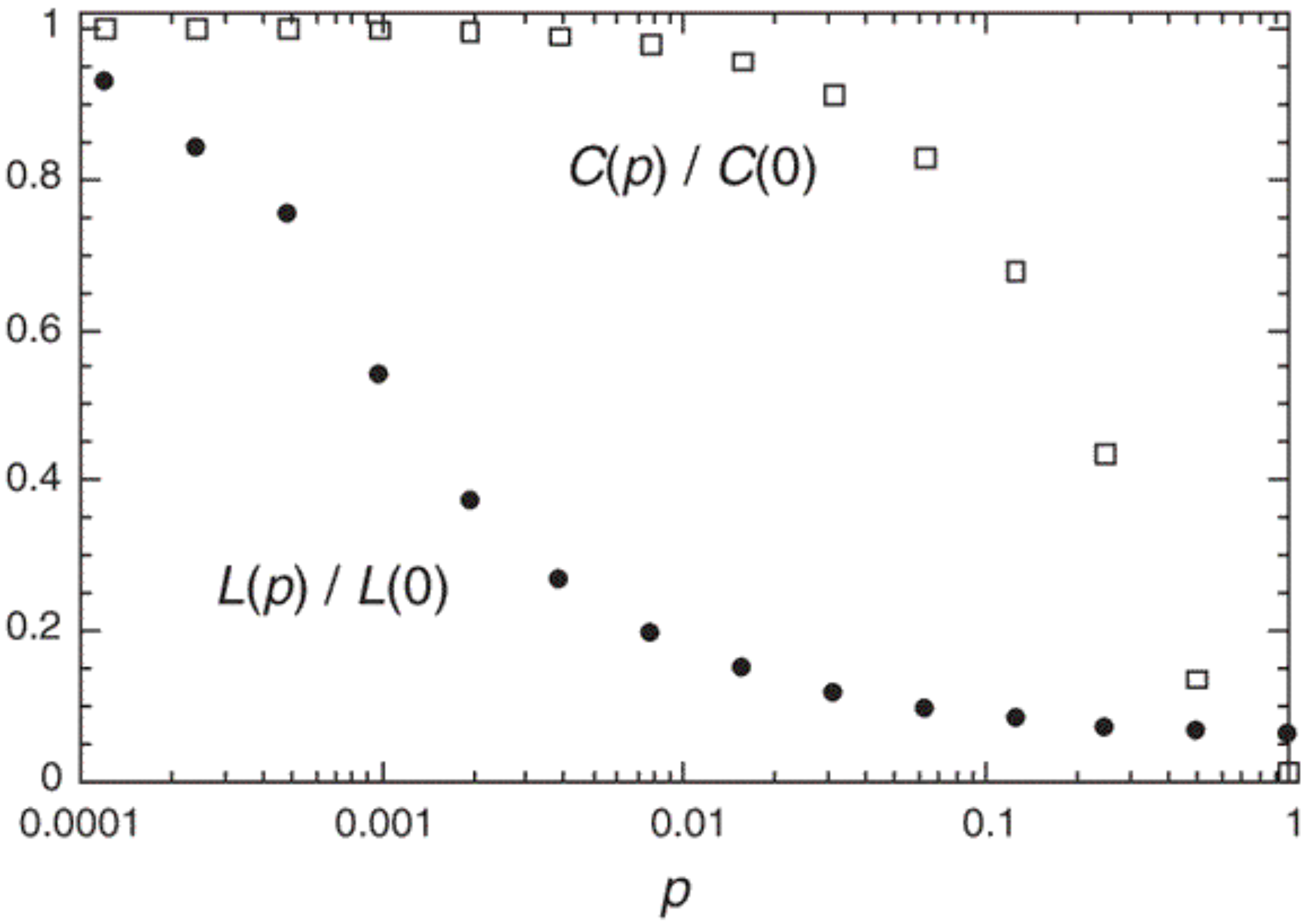
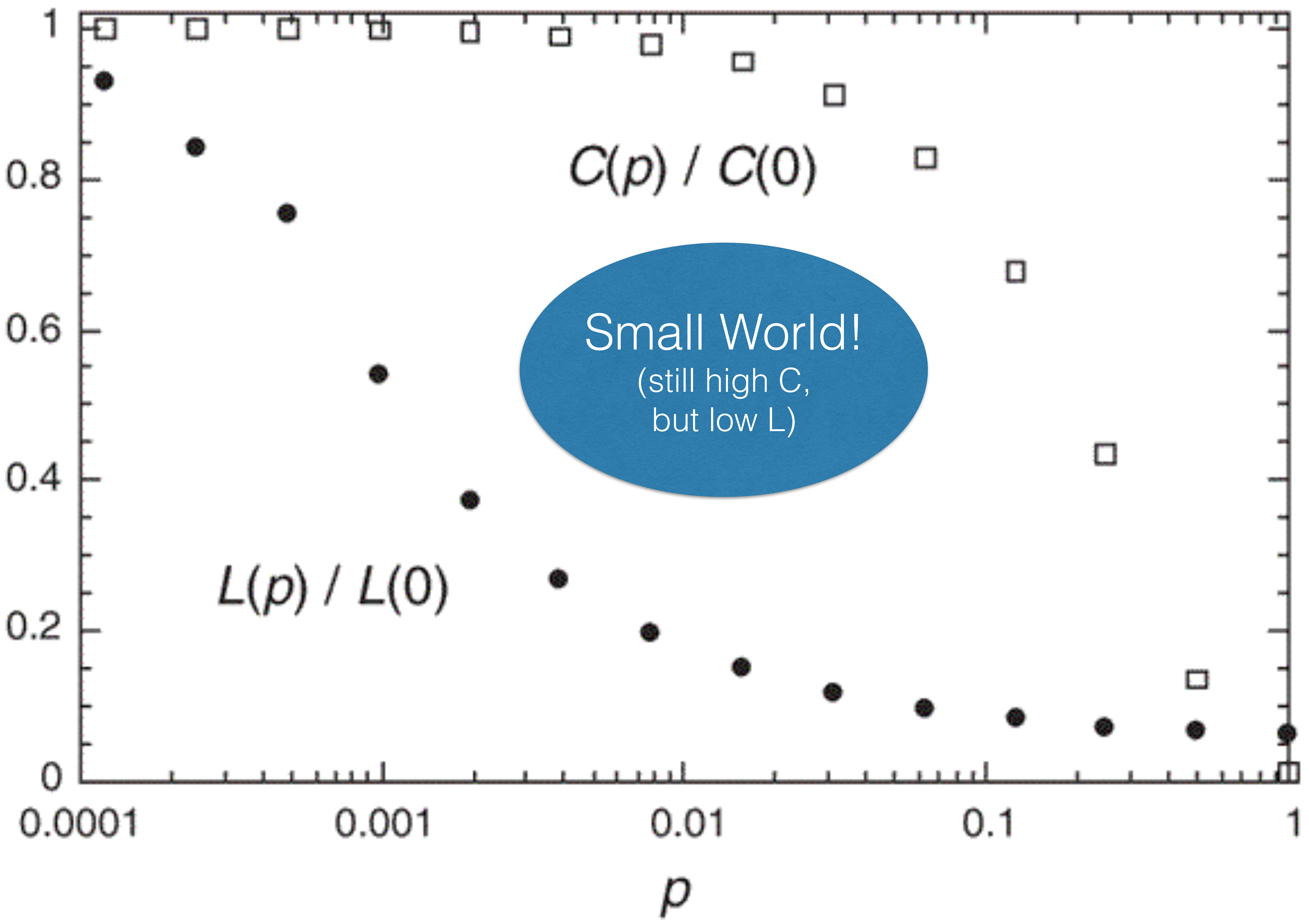

Regular
Small-world
Random
p = 0
Increasing randomness
p = 1



C(p) = average clustering coefficient,   L(p) = average path length



C(p) = average clustering coefficient, L(p) = average path length

Poisson Distribution

Bernoulli process with N trials, each probability p of success:

$$p(m) = \binom{N}{m} p^m (1-p)^{N-m} \ .$$

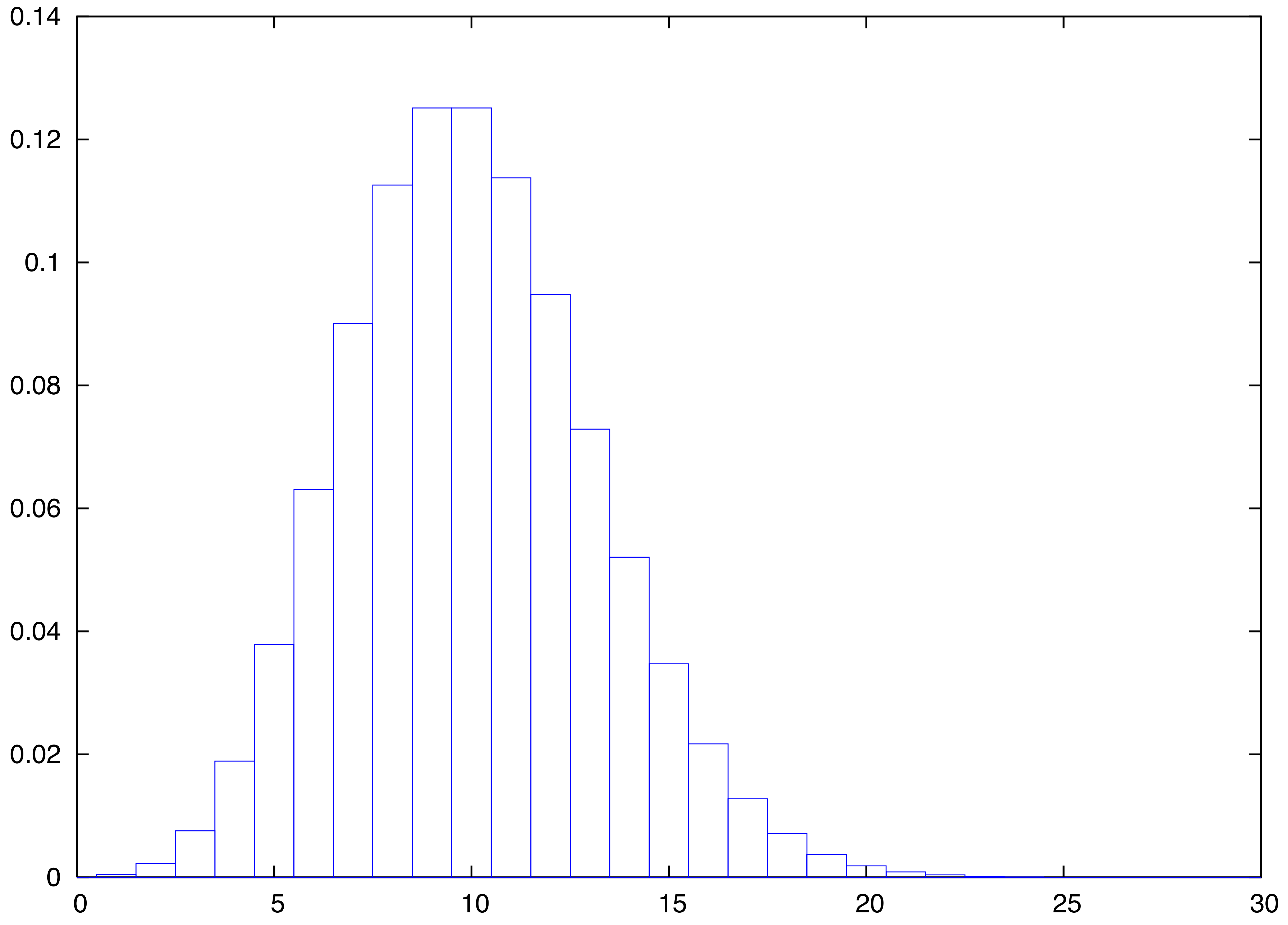
Probability $p(m)$ of m successes, in limit N very large and p small, parametrized by just $\mu = Np$ (μ = mean number of successes).
For $N \gg m$, we have $\frac{N!}{(N-m)!} = N(N-1)\cdots(N-m+1) \approx N^m$,
so $\binom{N}{m} \equiv \frac{N!}{m!(N-m)!} \approx \frac{N^m}{m!}$, and

$$p(m) \approx \frac{1}{m!} N^m \left(\frac{\mu}{N}\right)^m \left(1 - \frac{\mu}{N}\right)^{N-m} \approx \frac{\mu^m}{m!} \lim_{N\to\infty} \left(1 - \frac{\mu}{N}\right)^N = \mathrm{e}^{-\mu} \frac{\mu^m}{m!}$$

(ignore $(1-\mu/N)^{-m}$ since by assumption $N \gg \mu m$).
N dependence drops out for $N \to \infty$, with average μ fixed ($p \to 0$).
The form $p(m) = \mathrm{e}^{-\mu} \frac{\mu^m}{m!}$ is known as a Poisson distribution
(properly normalized: $\sum_{m=0}^{\infty} p(m) = \mathrm{e}^{-\mu} \sum_{m=0}^{\infty} \frac{\mu^m}{m!} = \mathrm{e}^{-\mu} \cdot \mathrm{e}^{\mu} = 1$).

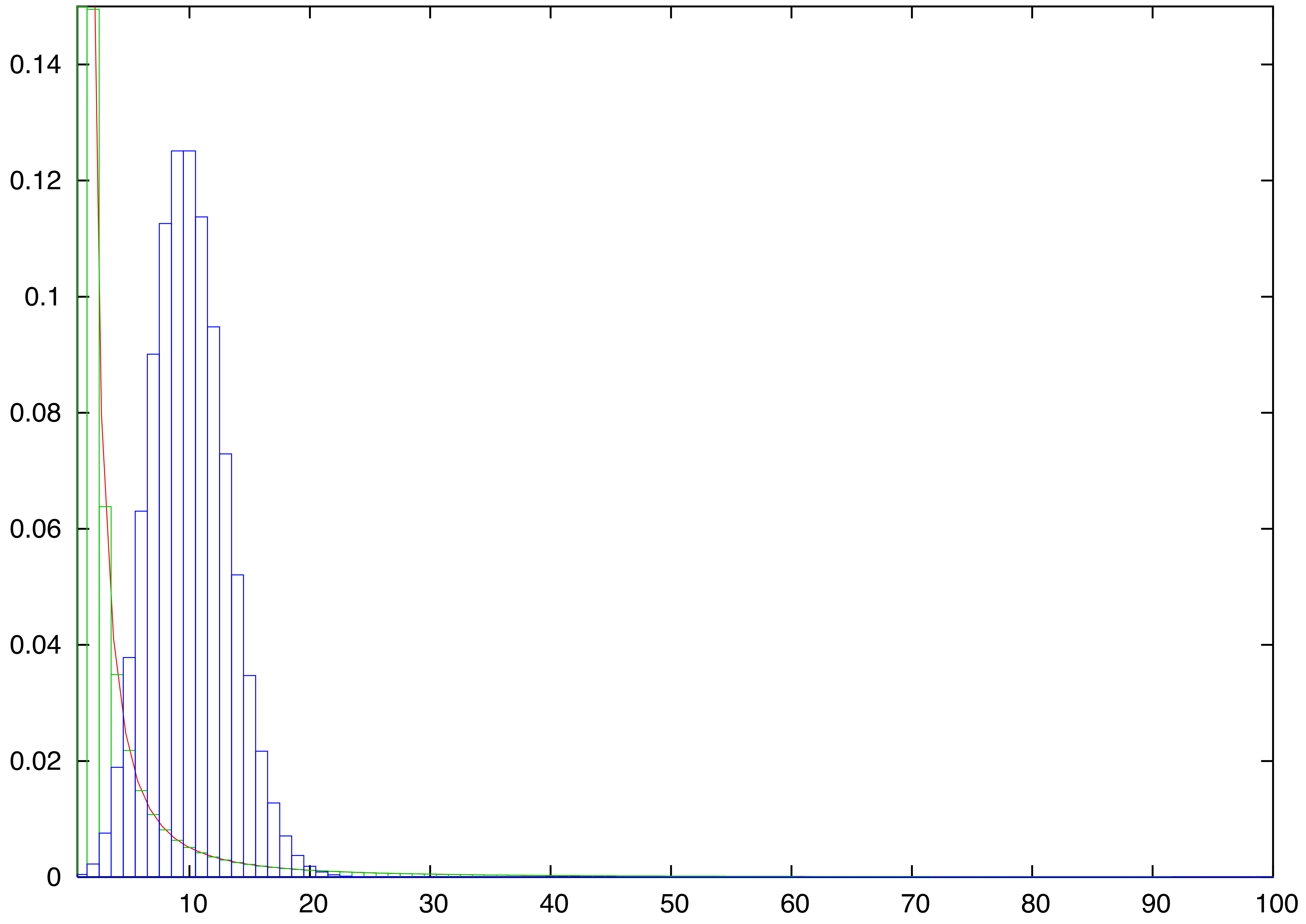
Poisson Distribution for $\mu = 10$

$$p(m) = \mathrm{e}^{-10}\frac{10^m}{m!}$$

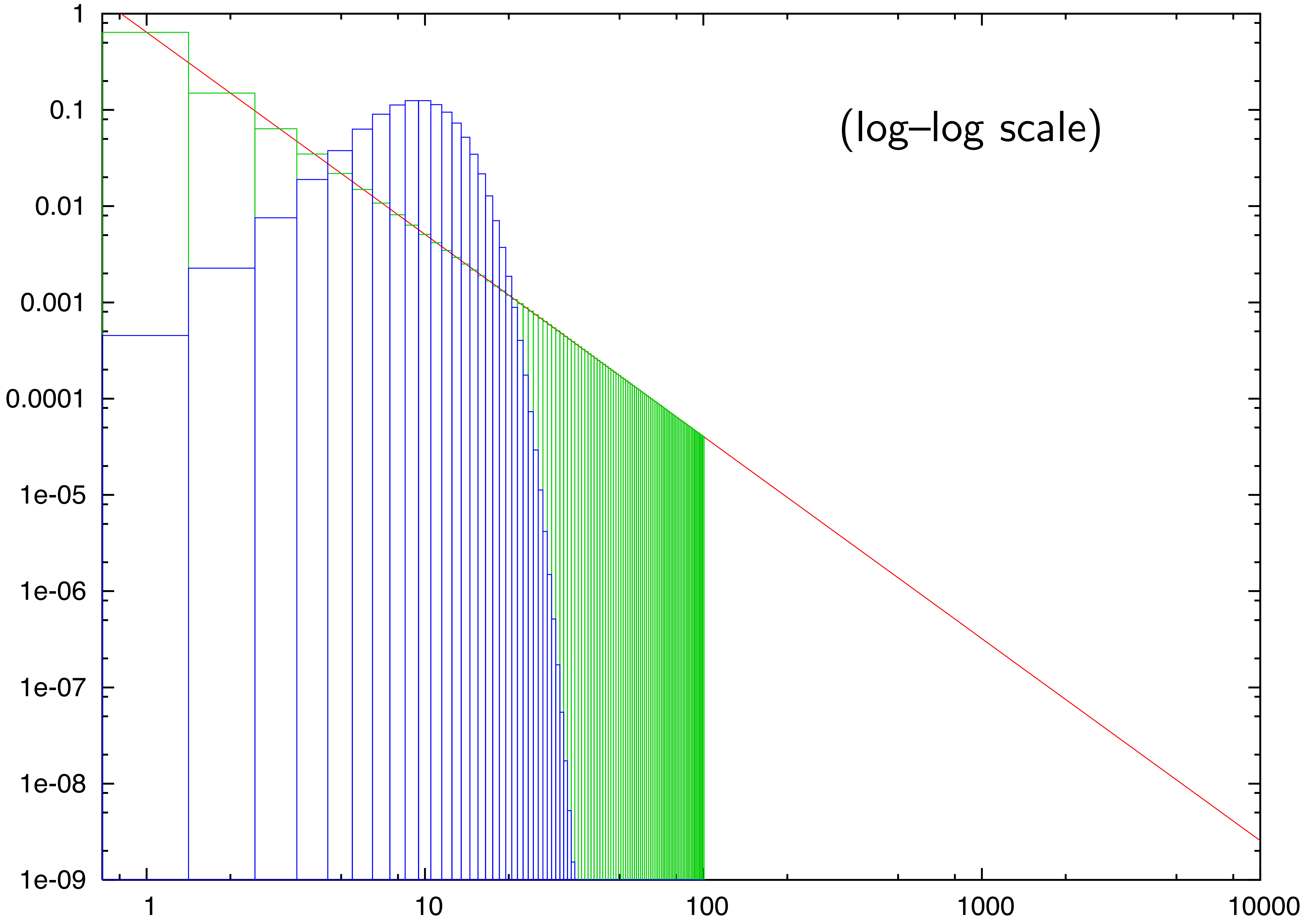


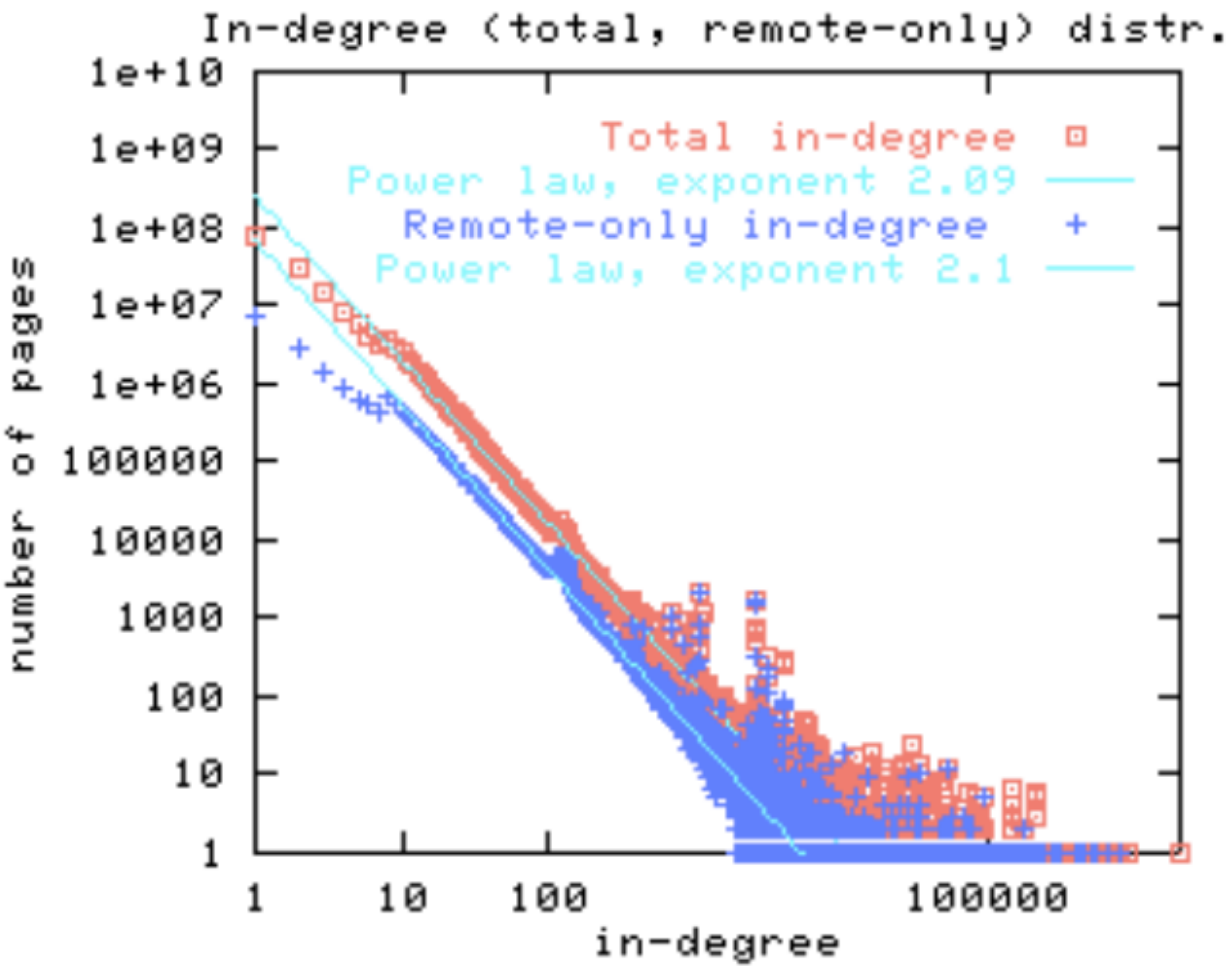
Compare to power law $p(m) \propto 1/m^{2.1}$

Power Law $p(m) \propto 1/m^{2.1}$ and Poisson $p(m) = e^{-10}\frac{10^m}{m!}$



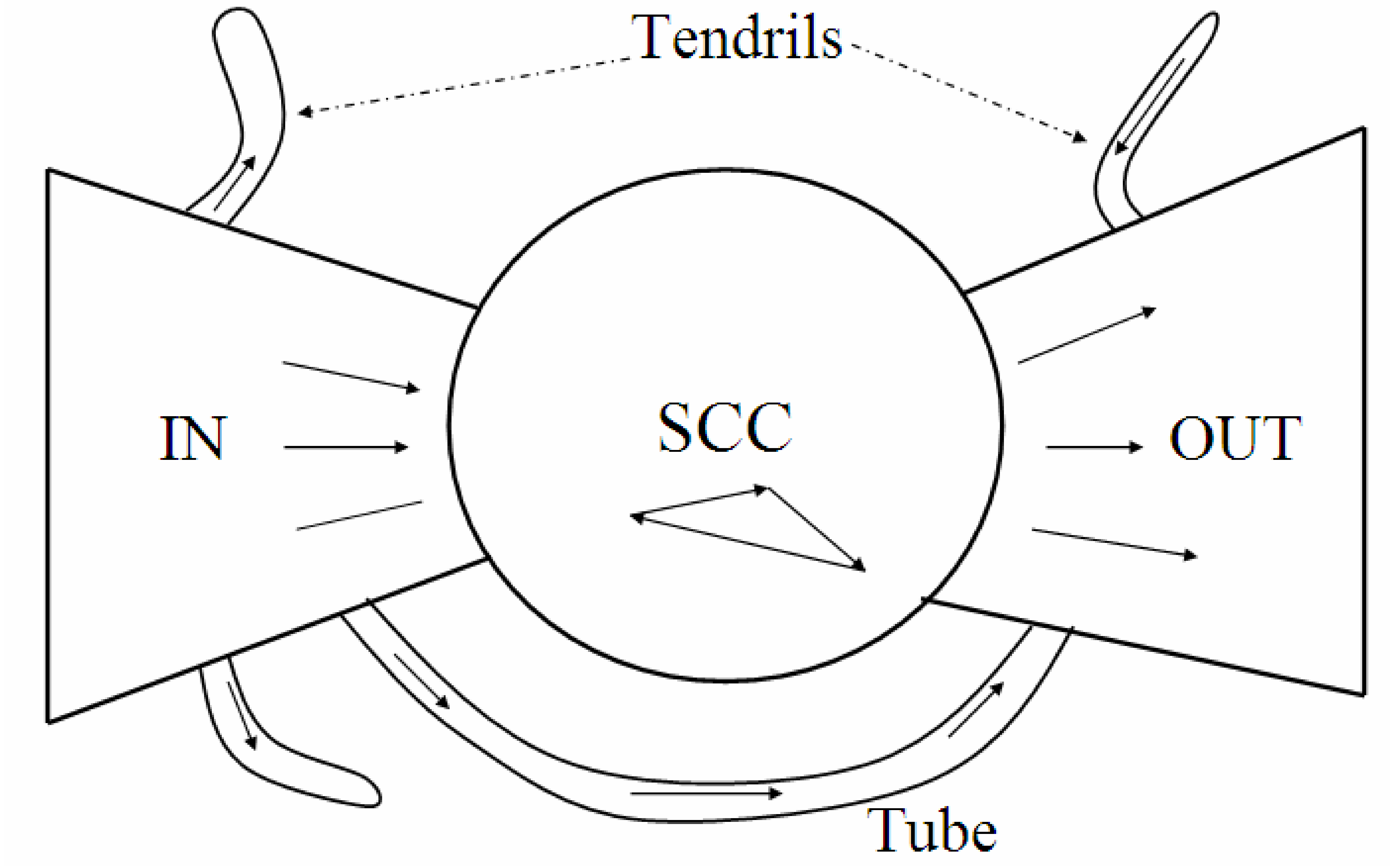
Power Law $p(m) \propto 1/m^{2.1}$ and Poisson $p(m) = e^{-10} \frac{10^m}{m!}$




In-degree (total, remote-only) distr.
Total in-degree
Power law, exponent 2.09
Remote-only in-degree
Power law, exponent 2.1
number of pages
in-degree
1e+10
1e+09
1e+08
1e+07
1e+06
100000
10000
1000
100
10
1
1
10
100
100000

“Bowtie” structure of the web

A.Broder,R.Kumar,F.Maghoul,P.Raghavan,S.Rajagopalan,S. Stata, A. Tomkins, and J. Wiener. Graph structure in the web. Computer Networks, 33:309–320, 2000.



- Strongly connected component (SCC) in the center
- Lots of pages that get linked to, but don't link (OUT)
- Lots of pages that link to other pages, but don't get linked to (IN)
- Tendrils, tubes, islands

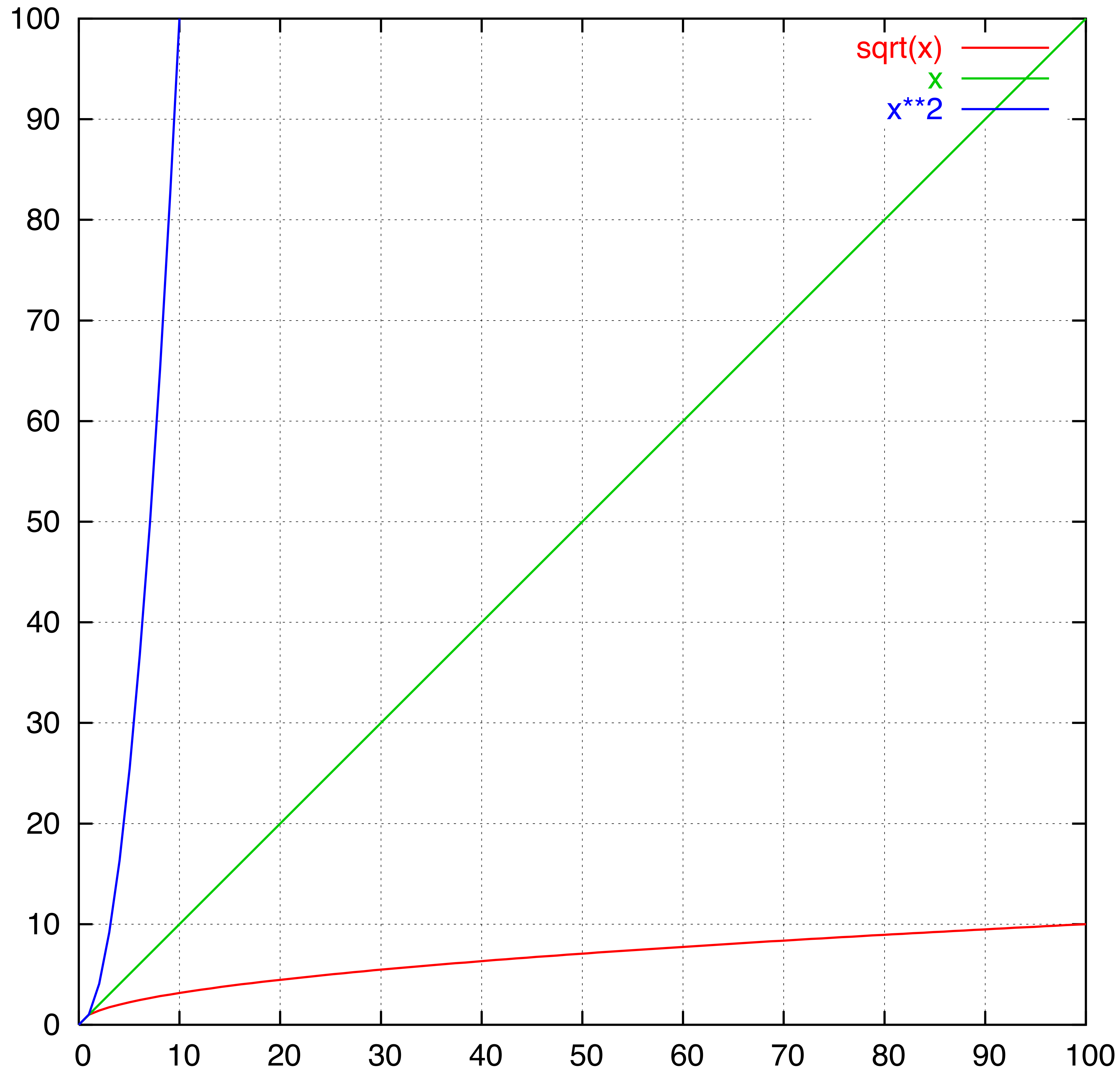
# of in-links (in-degree) averages 8–15, not randomly distributed (Poissonian), instead a power law:

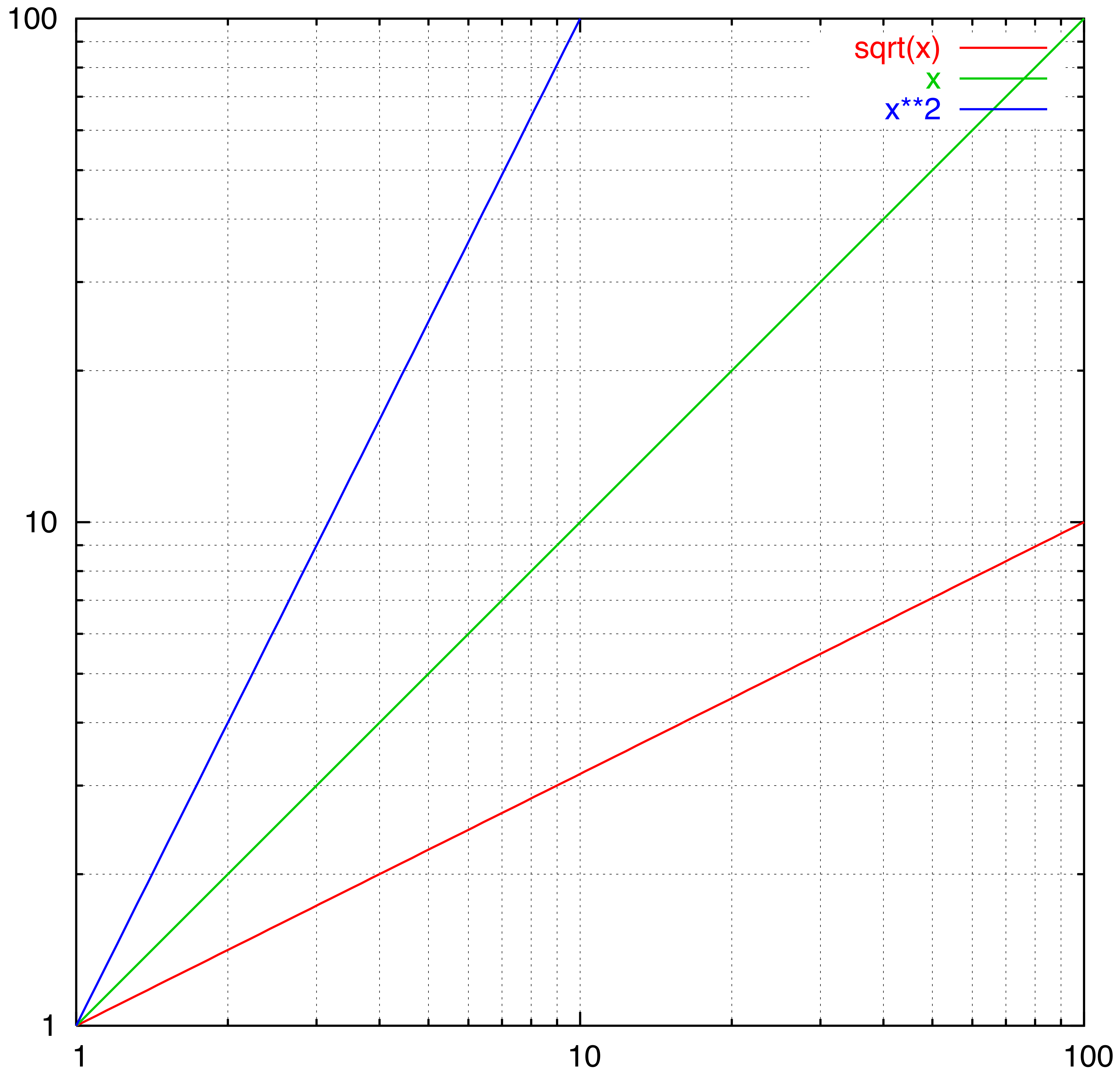
pages with in-degree i is $\propto 1/i^{\alpha}$, $\alpha \approx 2.1$

Power Laws in log-log space

$y = cx^k$ (k=1/2,1,2)

$\log_{10} y = k * \log_{10} x + \log_{10} c$





Power Laws in log-log space

$y = cx^{-k}$ (k=1/2,1,2)

$\log_{10} y = -k * \log_{10} x + \log_{10} c$