Info 2950, Lecture 14

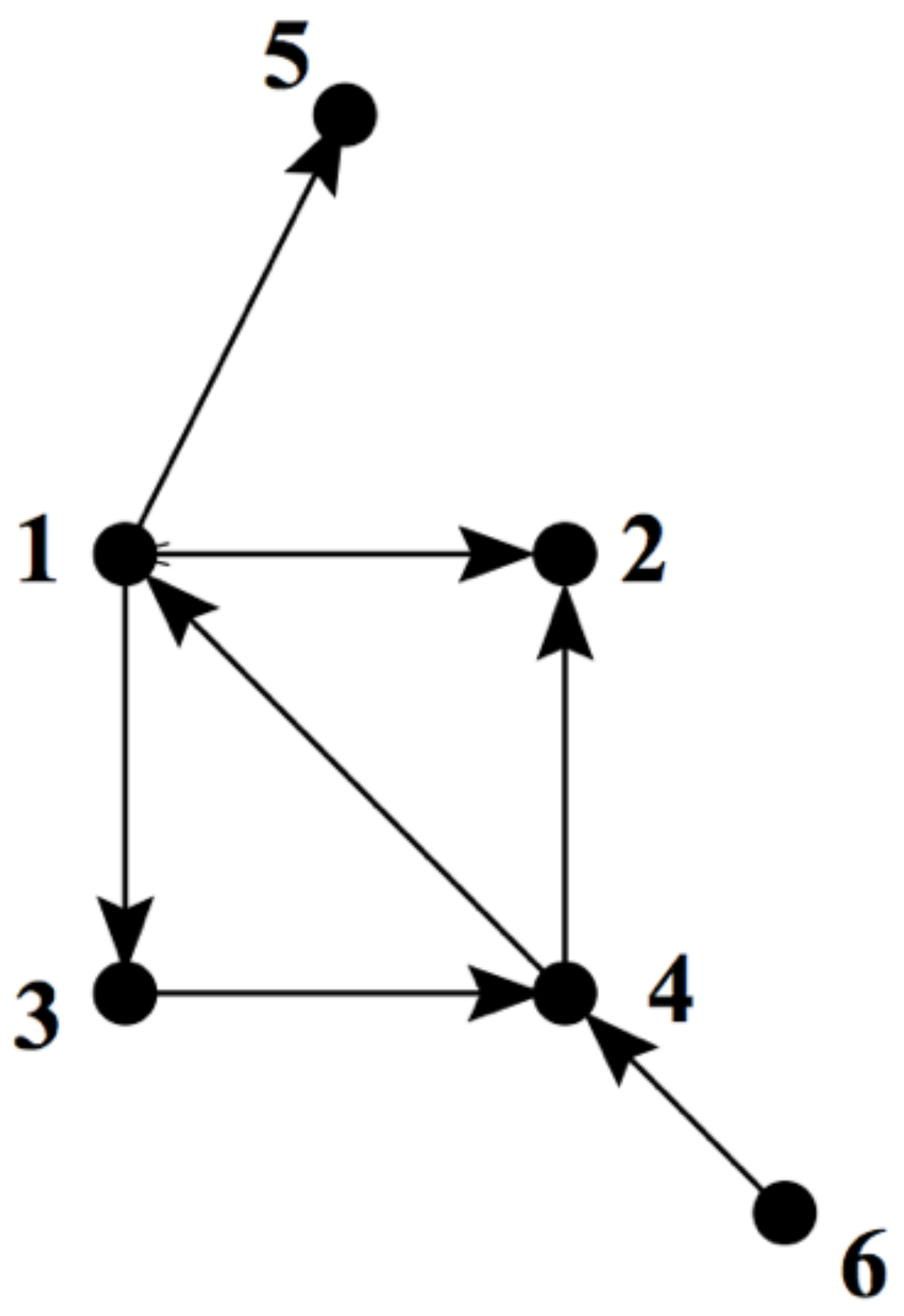
16 Mar 2017

Prob Set 4: due Mon night 20 Mar

Midterm: Thu 23 Mar (one week from today)

In a directed graph, the outdegree of a vertex v is the number of edges starting at v and the indegree is the number of edges ending at v.

Example: outdeg(1) = 3, indeg(1) = 1,
outdeg(2) = 0, indeg(2) = 2,
outdeg(3) = 1, indeg(3) = 1,
outdeg(4) = 2, indeg(4) = 2,
outdeg(5) = 0, indeg(5) = 1,
outdeg(6) = 1, indeg(6) = 0.



Why your friends . . .

Definitions:

Consider sampling N values X_i of some variable X.

Then the *expectation value* is the average: $\mathrm{E}[X] = \frac{1}{N}\sum_i X_i$.

The *variance* is defined as $\mathrm{Var}[X] = \frac{1}{N}\sum_i (X_i - \mathrm{E}[X])^2$,

and satisfies $\mathrm{Var}[X] = \mathrm{E}[X^2] - (\mathrm{E}[X])^2$.

The *standard deviation* is the square root: $\mathrm{std}[X] = \sqrt{\mathrm{Var}[X]}$

Feld 1991:

Node i has degree d_i, i.e., d_i friends.

$$total\_fof = \sum_{\text{nodes } i} \sum_{\text{friends f of i}} d_f = \sum_i d_i^2$$

(since each d_f occurs d_f times in the first double sum).

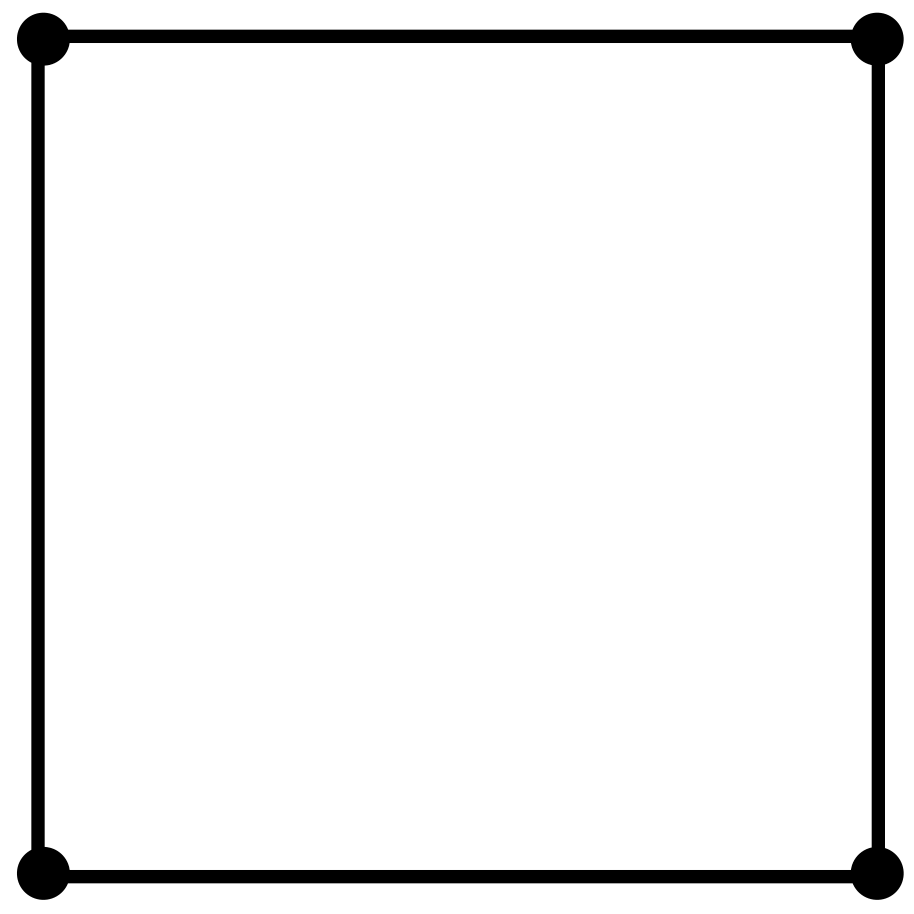
Average fof per person $= \frac{1}{N} \sum_i d_i^2 = \mathrm{E}[d^2] = \mathrm{Var}[d] + (\mathrm{E}[d])^2$

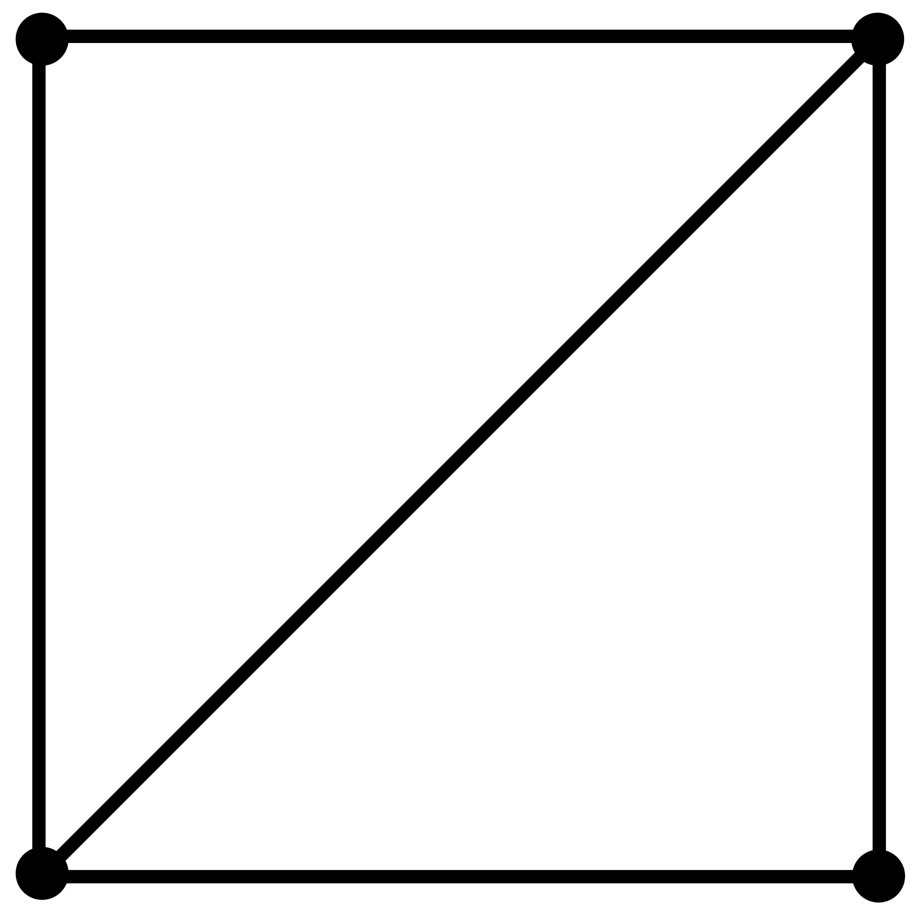
The average fof per friend $= \mathrm{E}[d^2]/\mathrm{E}[d] = \mathrm{E}[d] + \mathrm{Var}[d]/\mathrm{E}[d]$

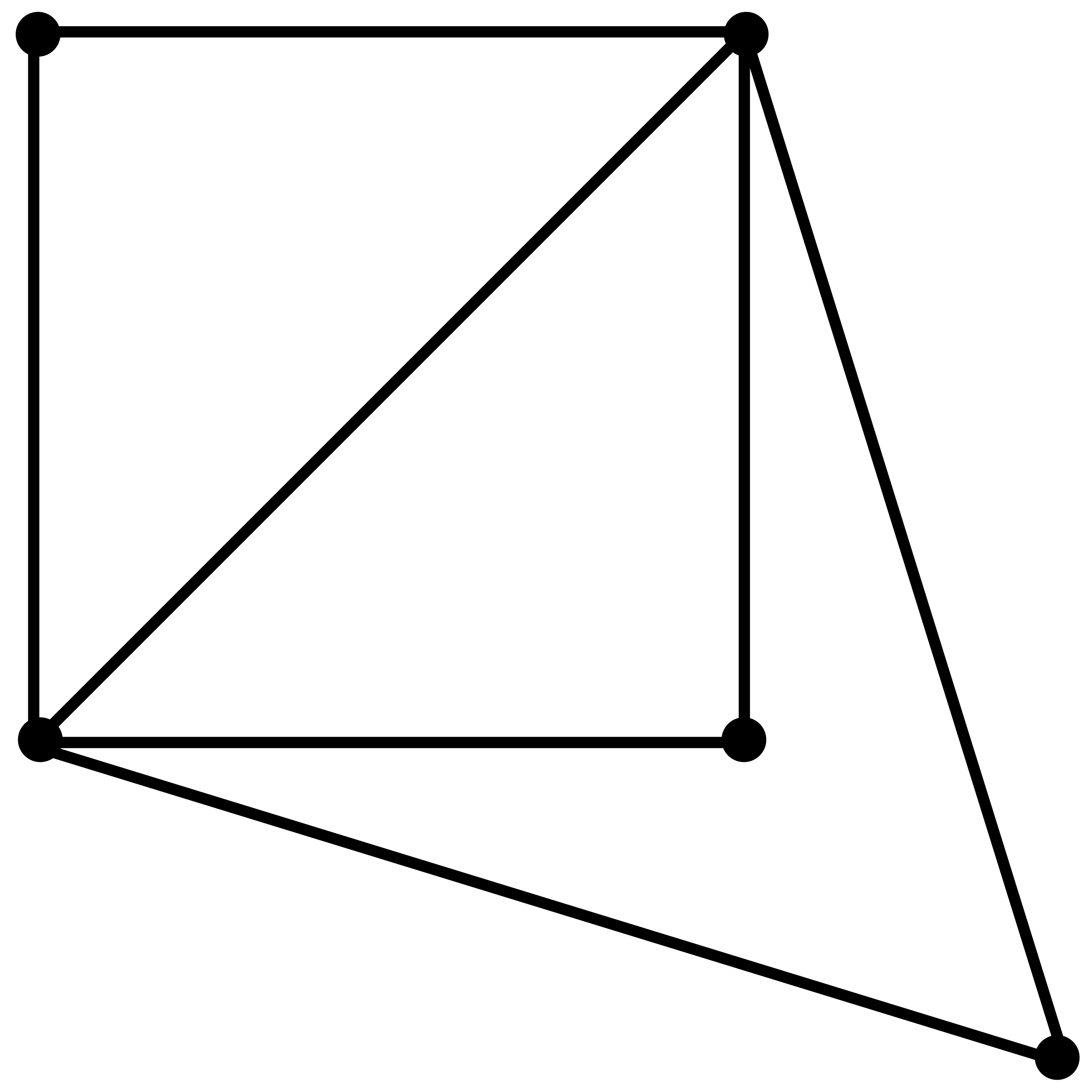
The variance is positive, so the above is always greater than E[d].

Used: detecting flu, disease innoculation, administrative propaganda

Eulerian circuit of a graph G is a cycle which contains each edge of G exactly once

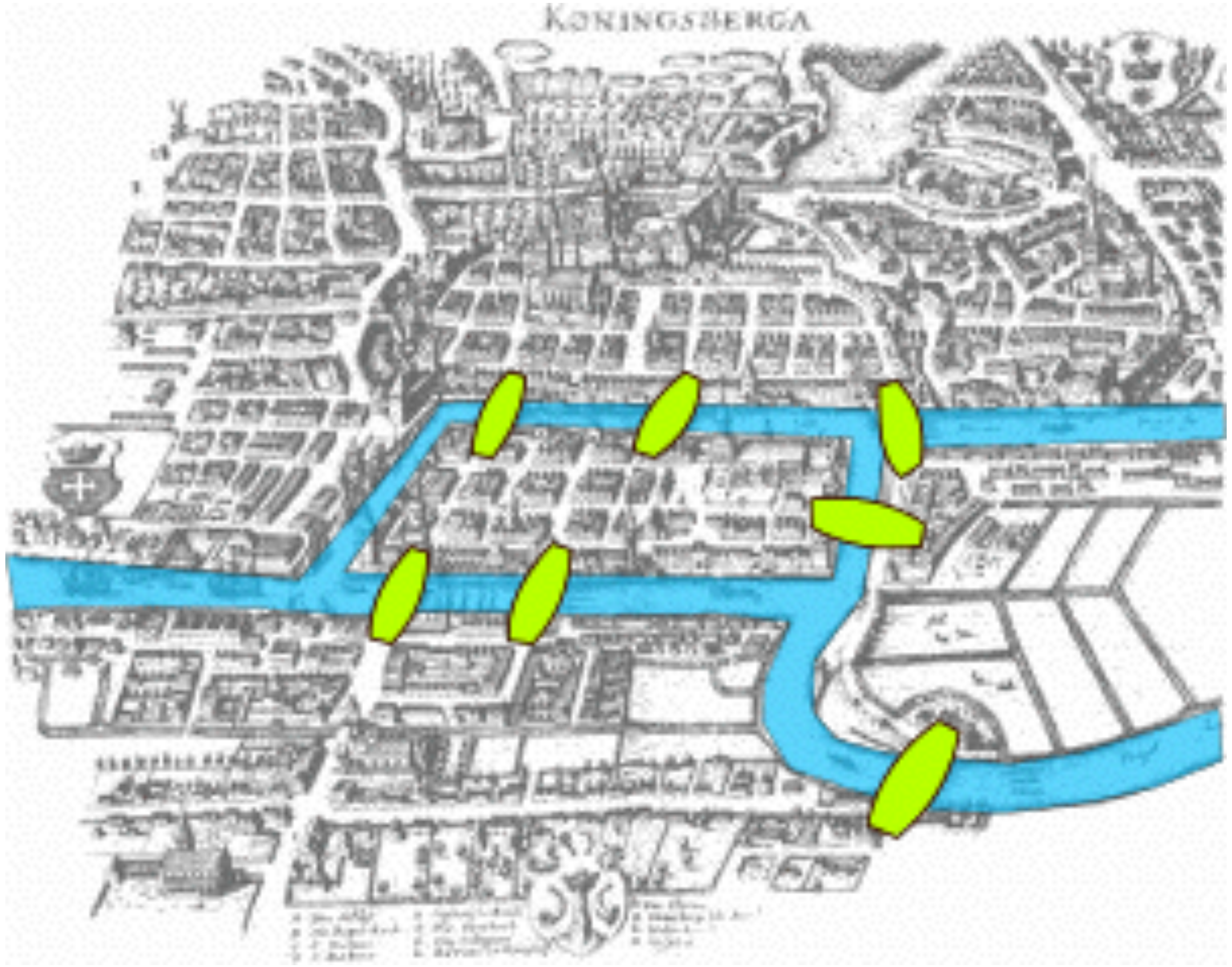






Seven Bridges of Königsberg
Euler (1736)

https://en.wikipedia.org/wiki/Seven_Bridges_of_Königsberg

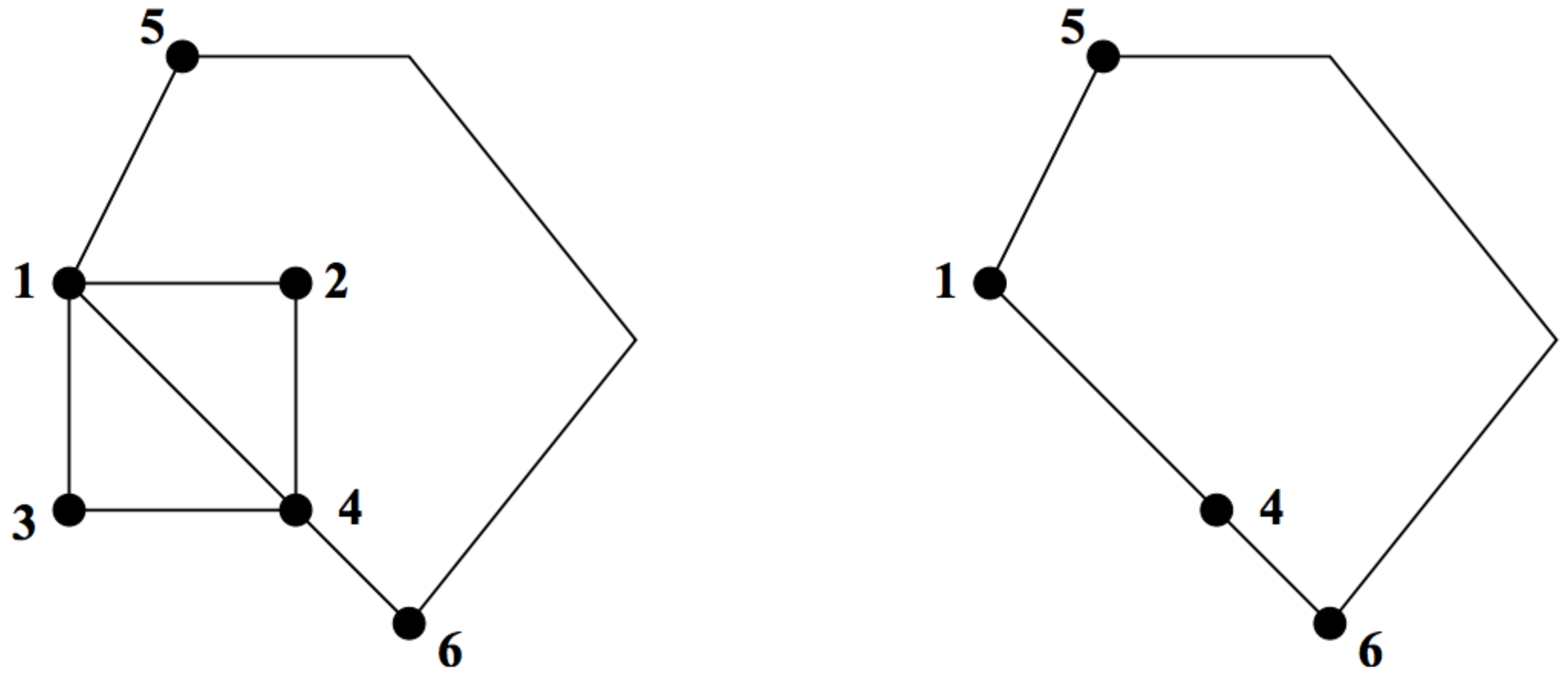


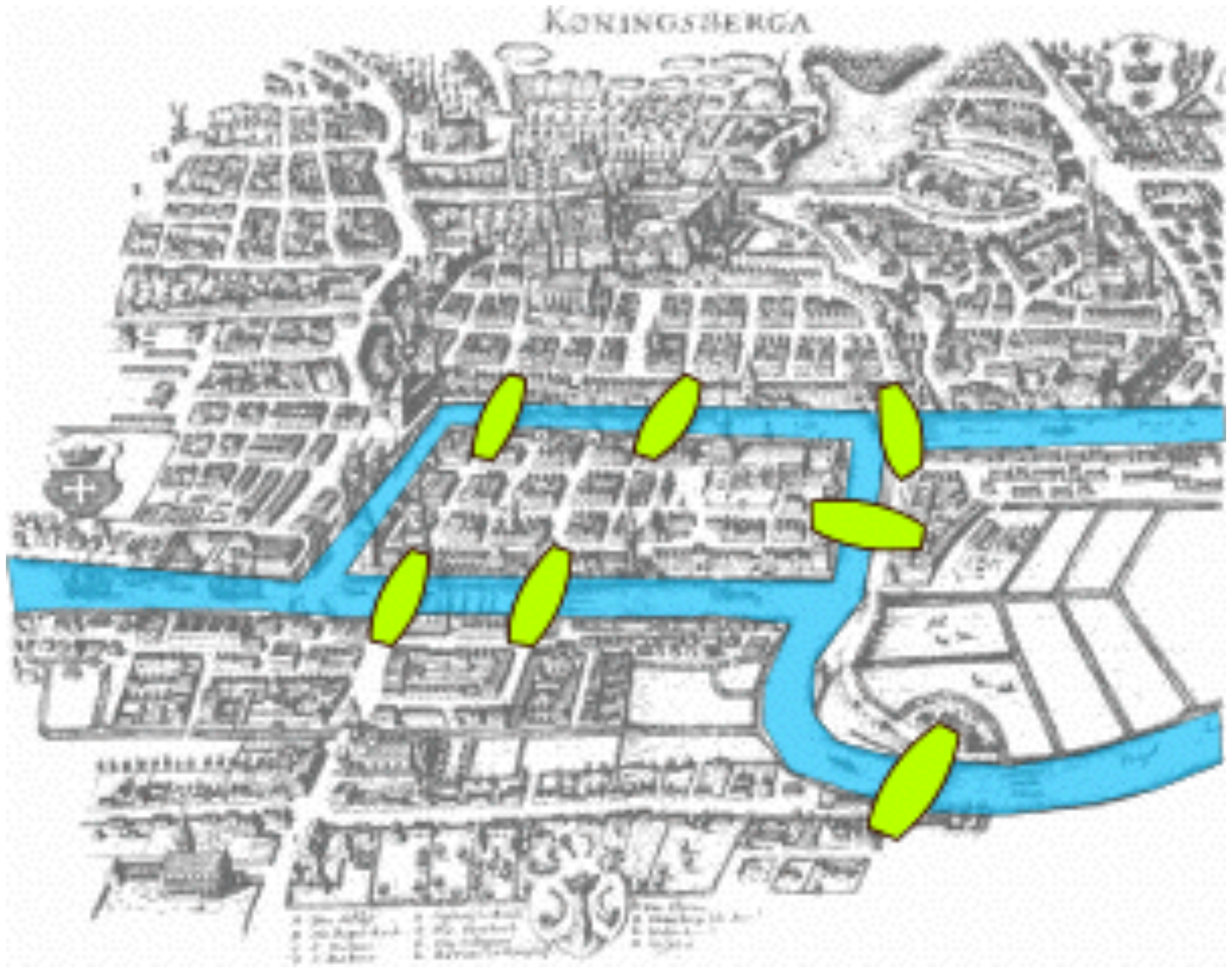
Theorem: A connected graph has an Eulerian circuit if and only if for all vertices v, deg(v) is even.

Well if it has an Eulerian circuit, means we leave the first vertex,
every time we enter any other vertex we leave by a new edge,
and return to the start vertex.
That means every vertex has to have even degree.

And conversely if every vertex has even degree, then start at any vertex,
go as long as possible without repeating edges.
If result contains all edges, then done.
Otherwise must be back at start (why?)
Any remaining graph still has all even degree vertices,
so repeat as many times as necessary,
and glue result together into single circuit (example on next slide).

Example:

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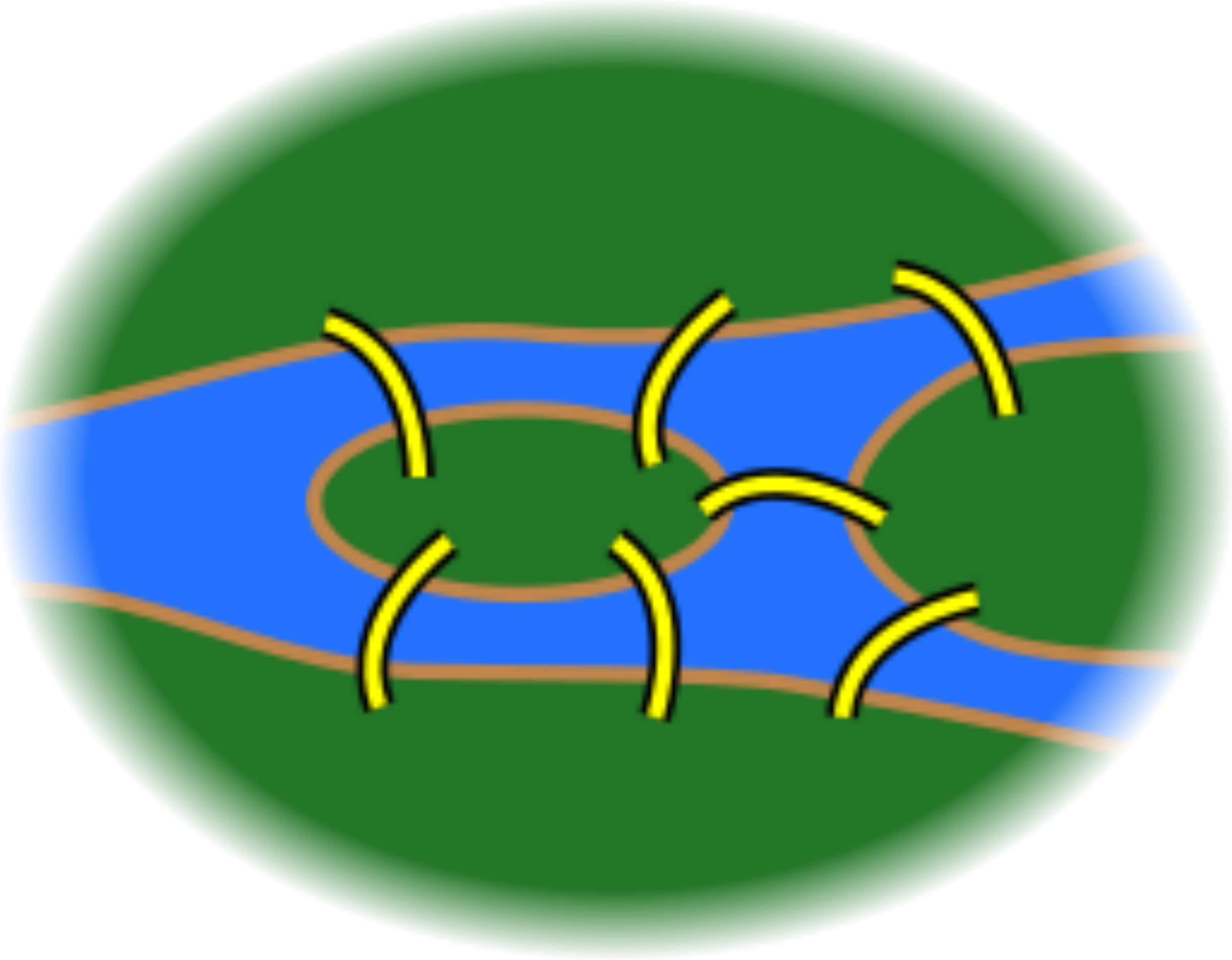

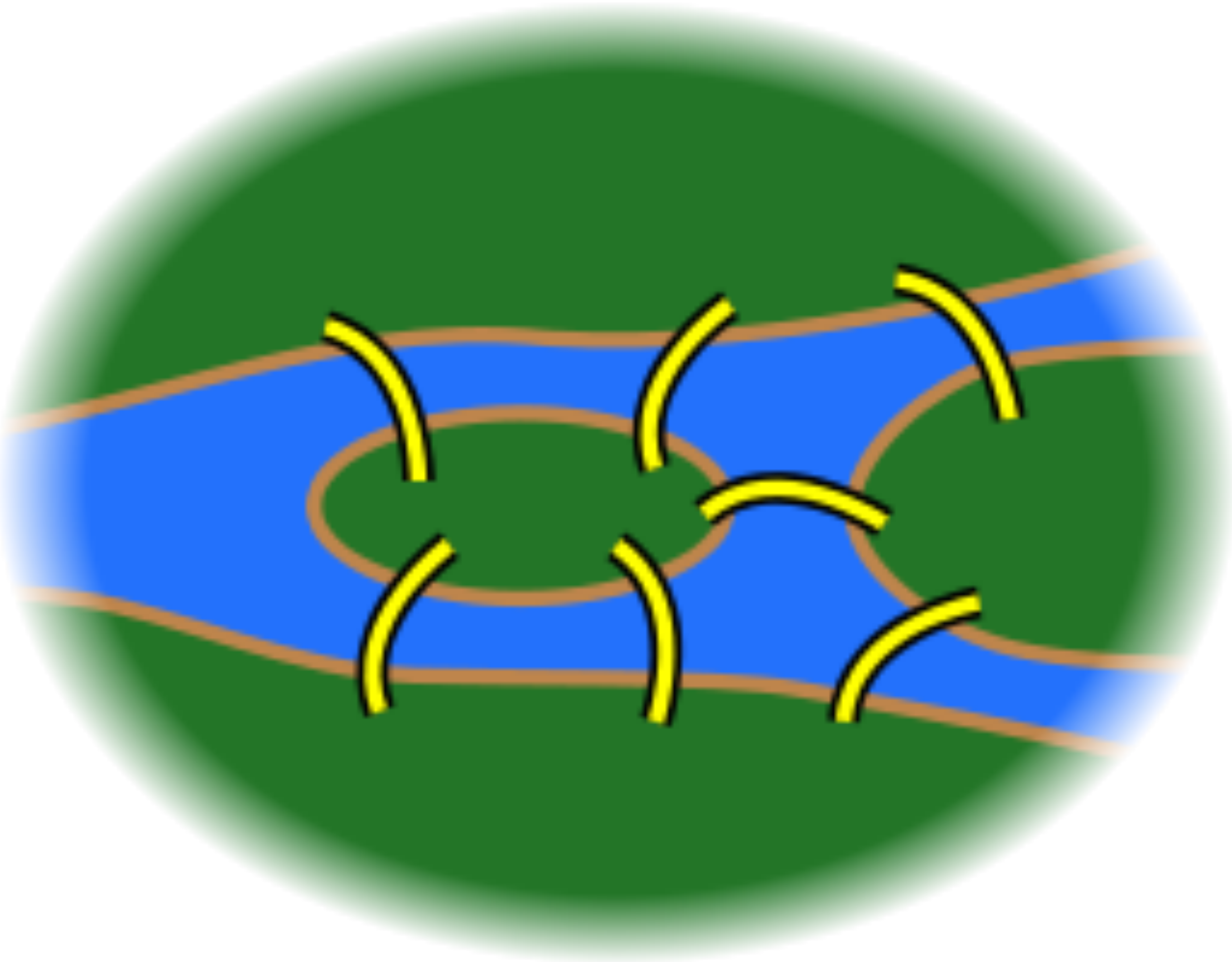

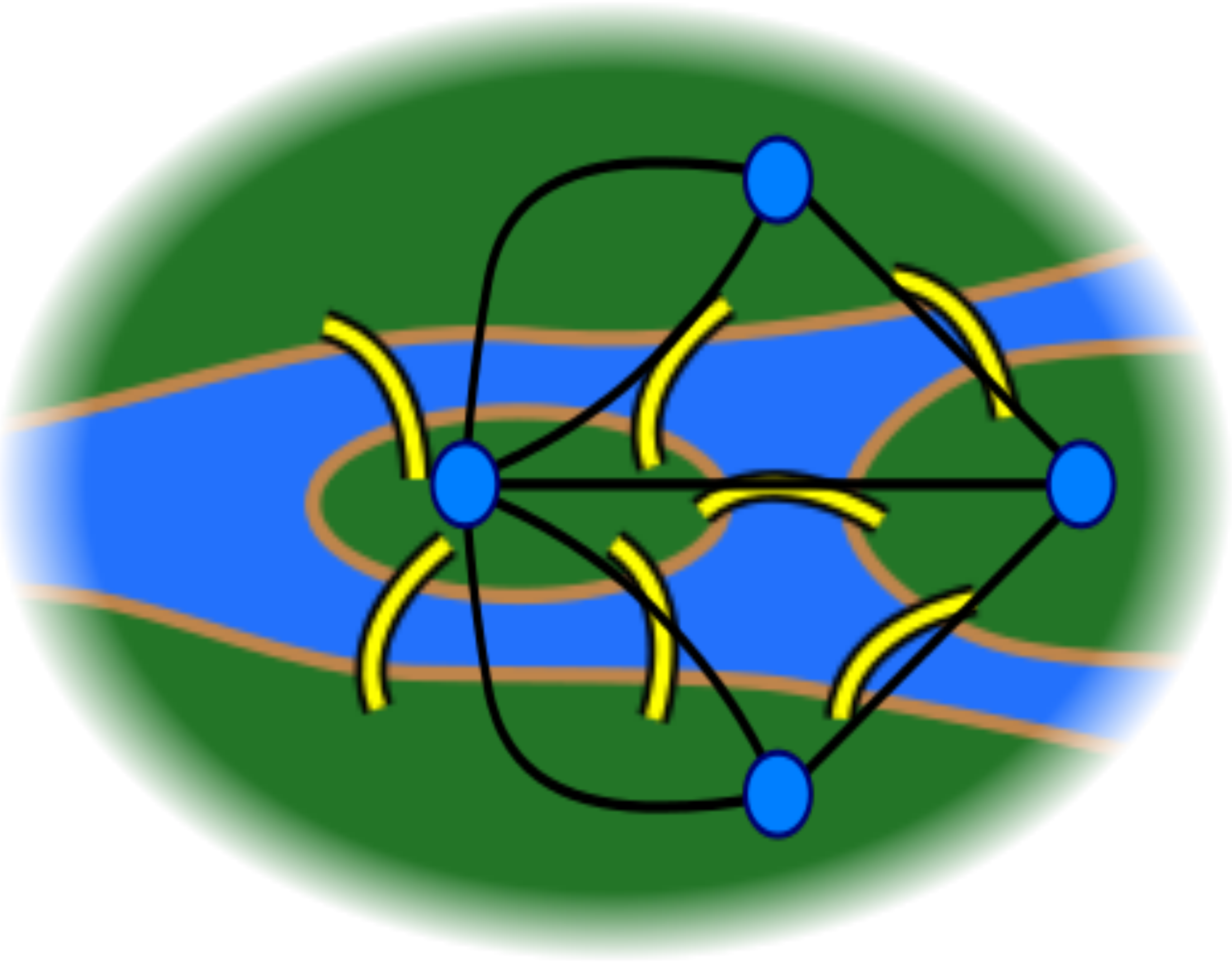

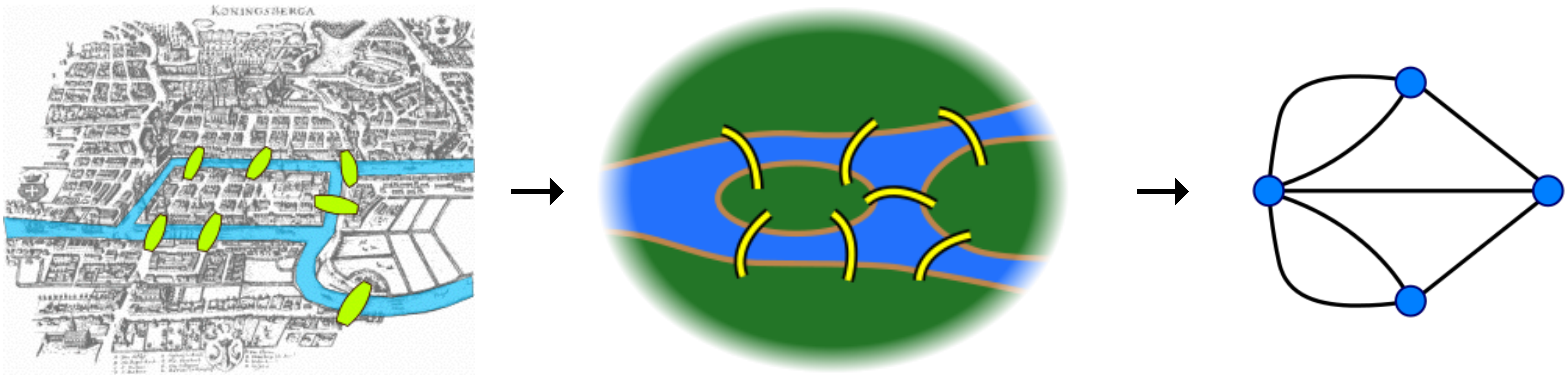
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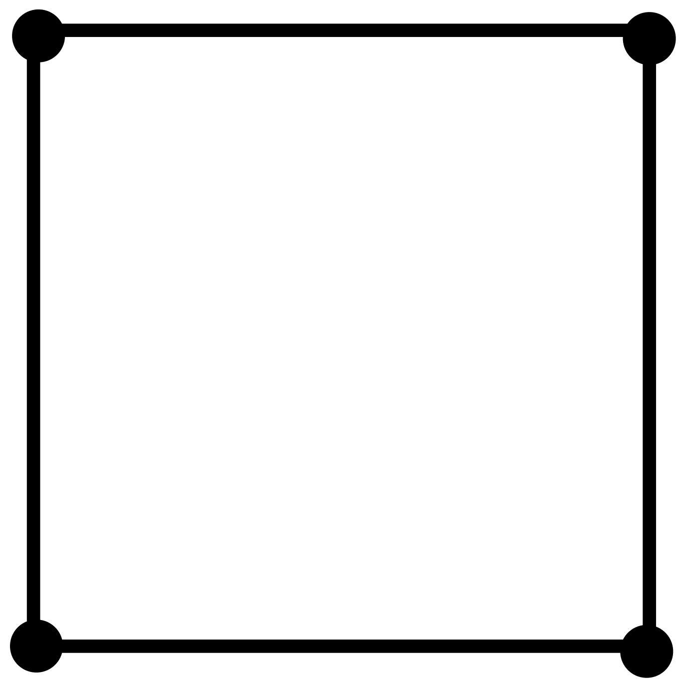
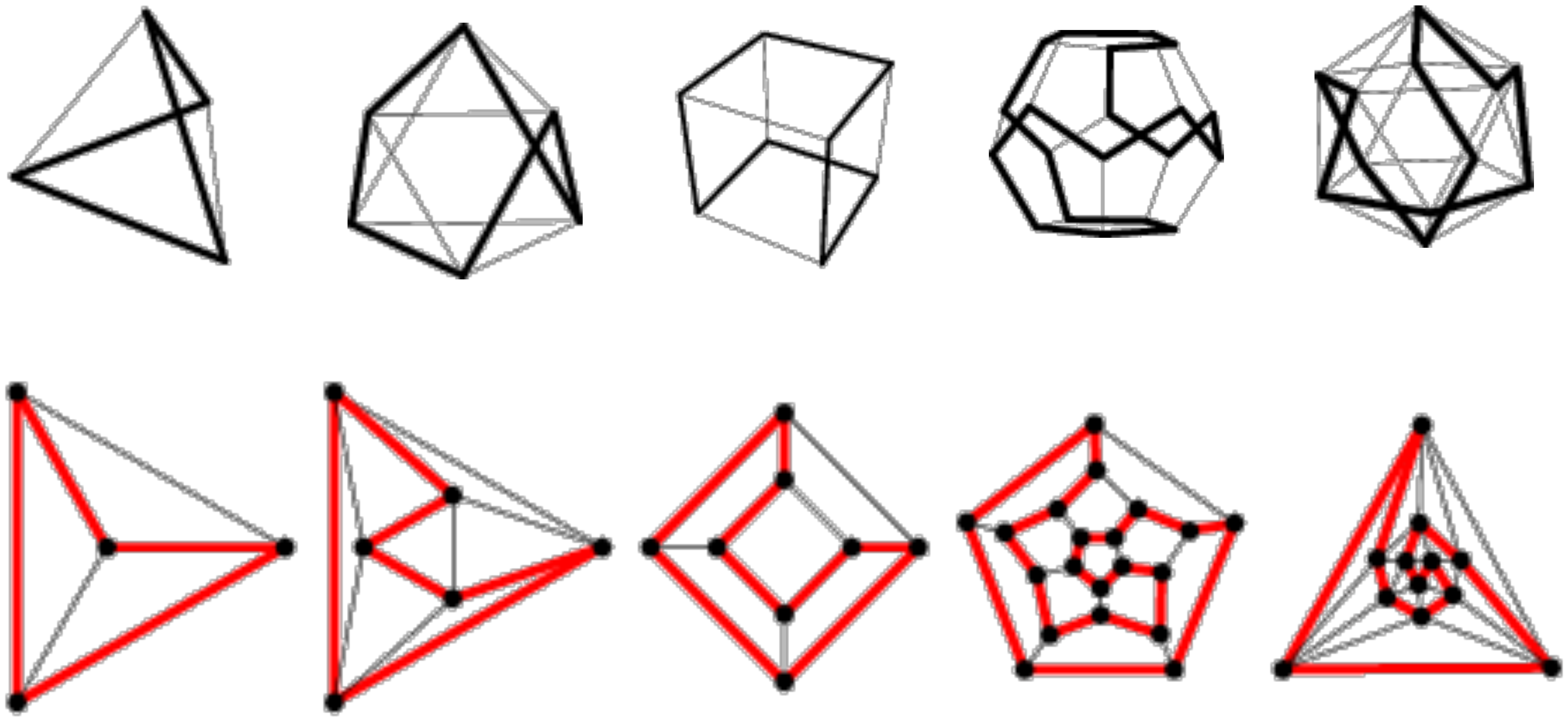
But it’s not always so easy …

A Hamiltonian circuit of a graph G is a cycle which passes through each vertex exactly once.





http://mathworld.wolfram.com/HamiltonianCycle.html