Info 2950, Lecture 13

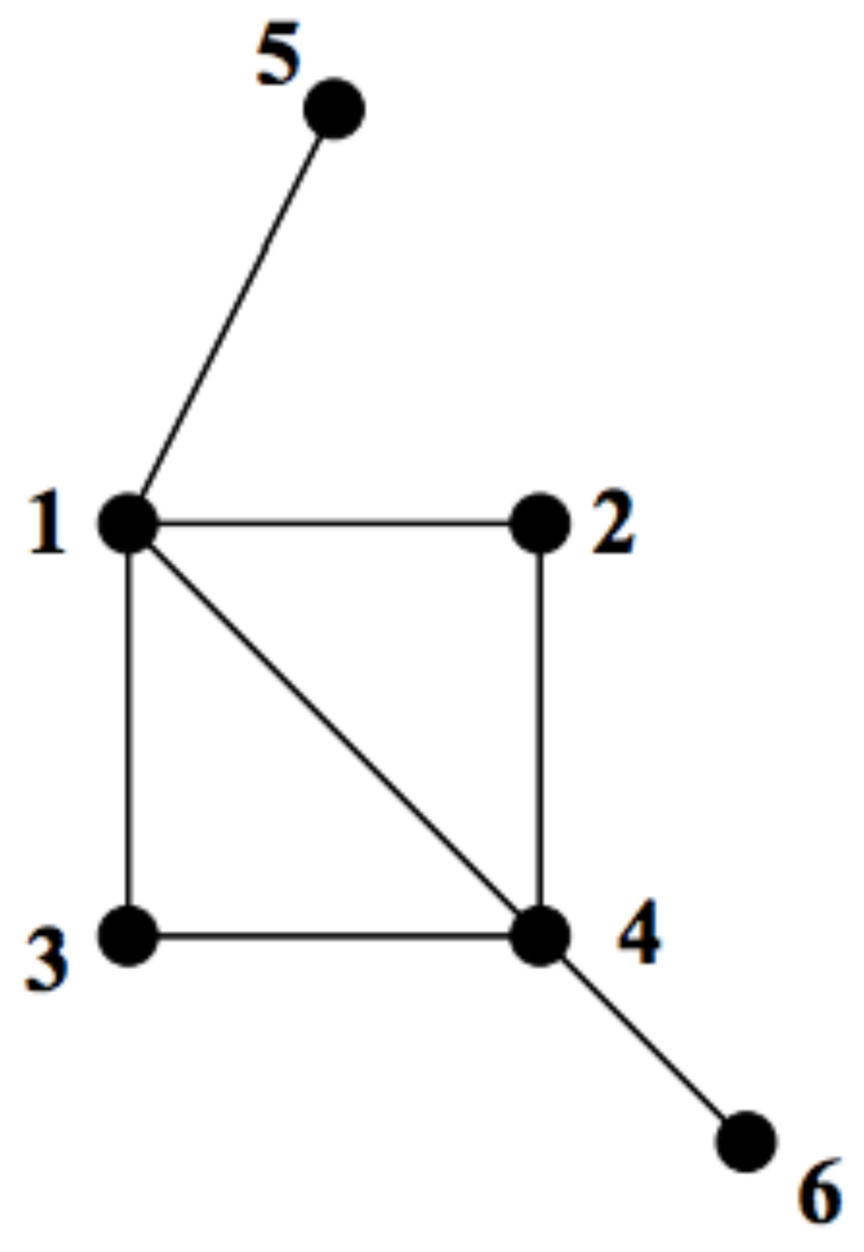
14 Mar 2017

Prob Set 4: due Mon night 20 Mar

Graphs

Definition: A graph G is a pair (V, E), where V is a finite set and E is a collection of unordered pairs in V × V .

The elements of V are referred to as the vertices of the graph and the elements of E are referred to as the edges.



Example: Define a graph G_1 with V = {1, 2, 3, 4, 5, 6} and E = {(1, 2), (1, 3), (1, 4), (2, 4), (3, 4), (1, 5), (4, 6)}

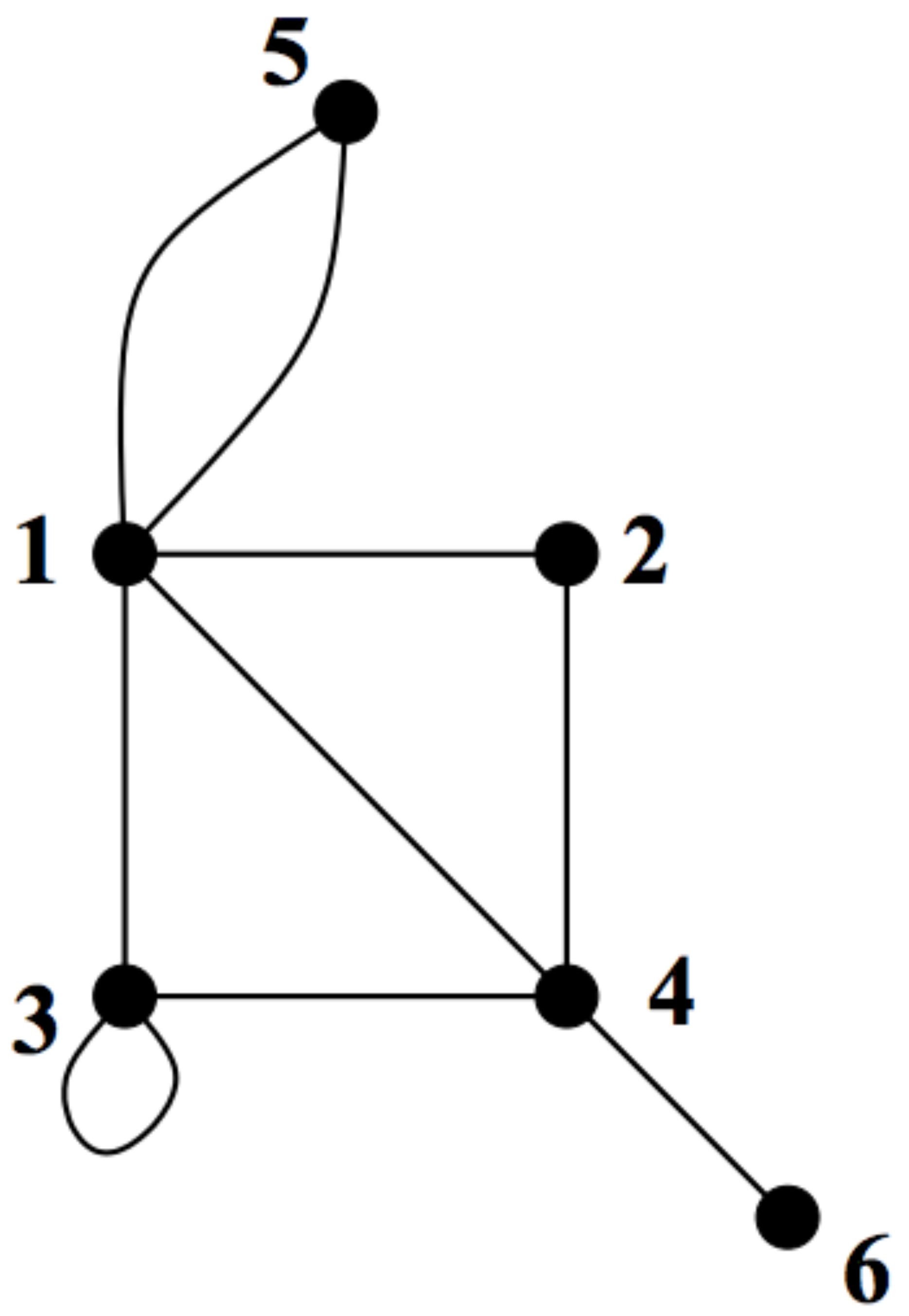
We could have repeated elements in E.

For example, we could include an edge twice.

In this case, we say that G has multiple edges.

We could also have an edge between just one vertex.

In this case, we call the edge a loop



Example: (3, 3) is an edge and (1, 5) has been included twice.

A simple graph is a graph with no loops and no multiple edges.

In our definition of a graph, edges are unordered pairs.

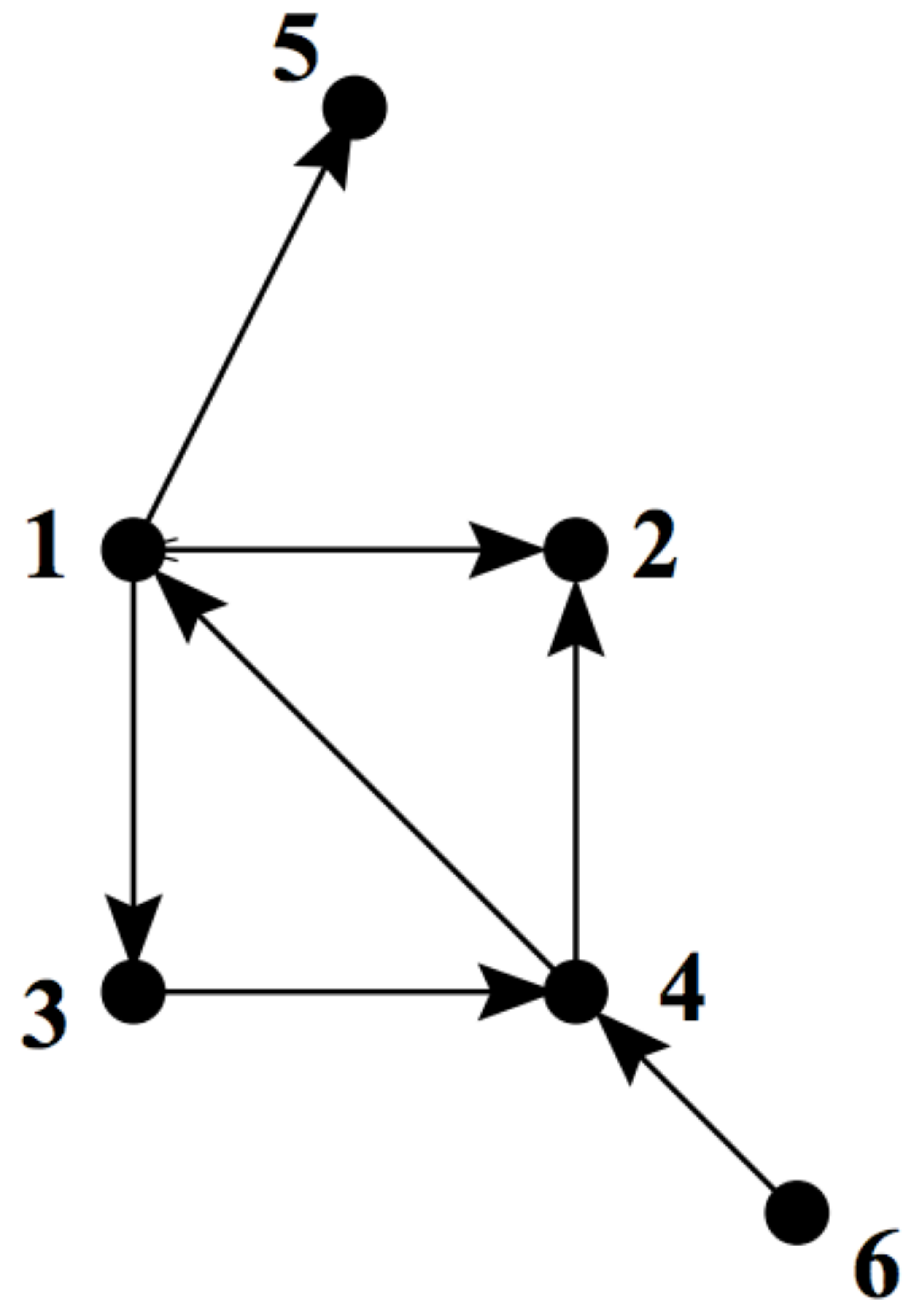
So the edge (1, 4) is the same thing as the edge (4, 1).

This kind of graph is referred to as undirected.

A directed graph is a graph whose edges are ordered pairs (shown by arrows in the points/lines representation)

For edge (v_1, v_2), draw an arrow from v_1 to v_2.

Example: a directed graph with $V = \{1, 2, 3, 4, 5, 6\}$ and $E = \{(1, 2), (1, 3), (4, 1), (4, 2), (3, 4), (1, 5), (6, 4)\}$.

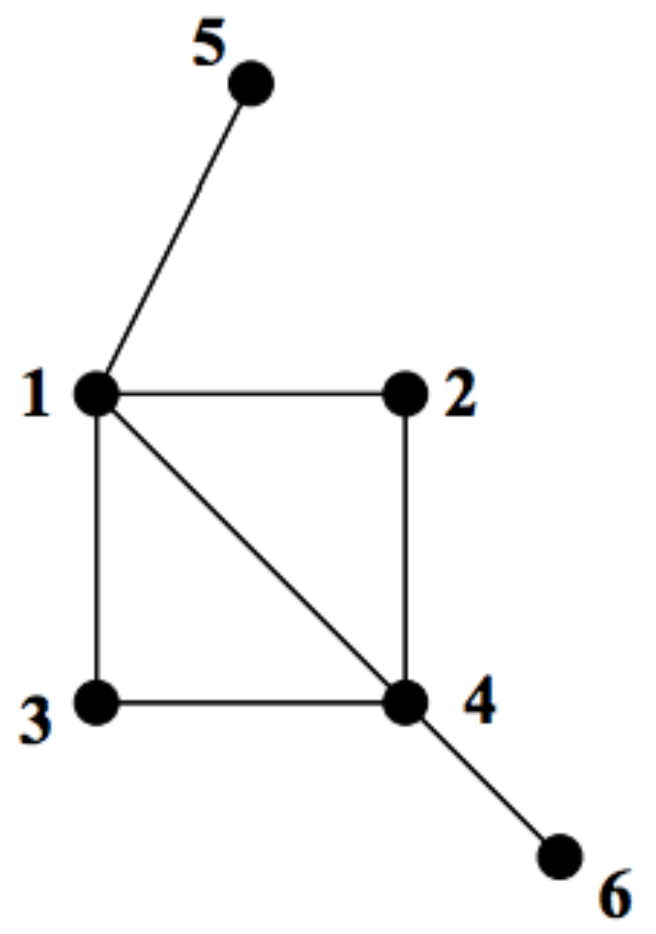


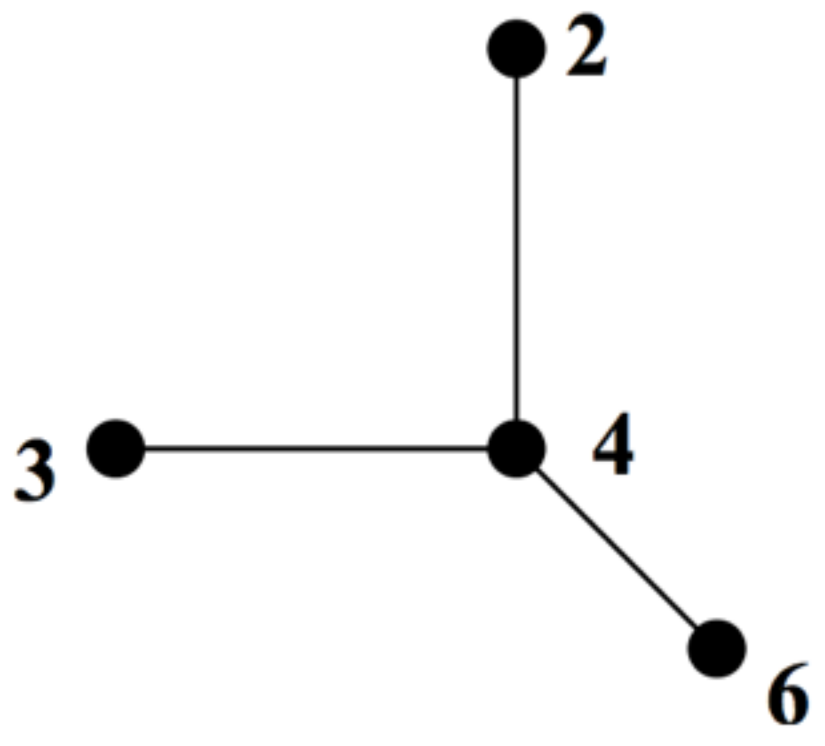
A subgraph $H = (V', E')$ of a graph $G = (V, E)$ is a pair $V' \subseteq V$ and $E' \subseteq E$.

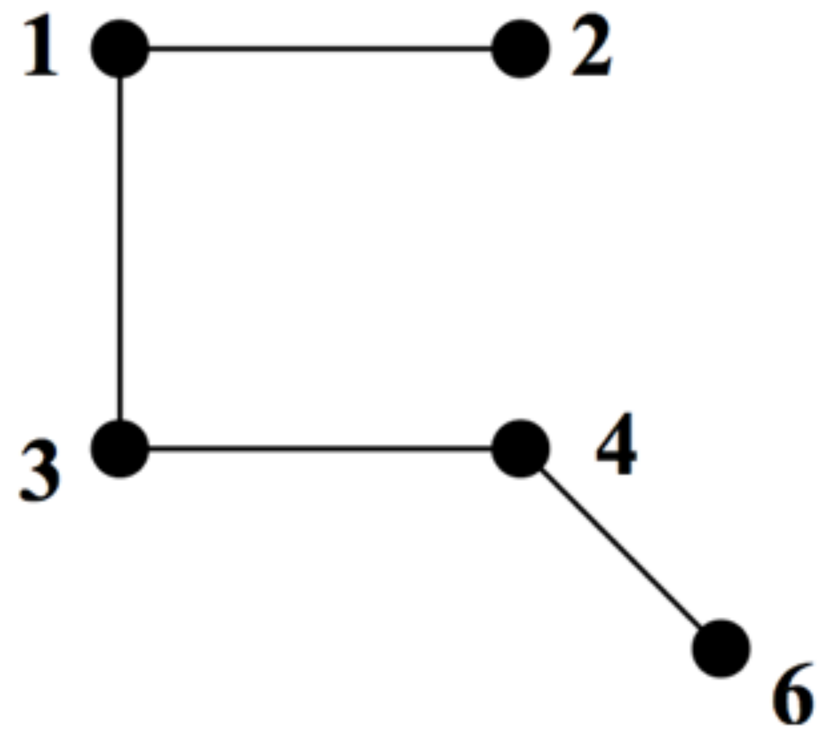
We say that H is an induced subgraph of G if all the edges between the vertices in V' from E are in E'

Example: Two subgraphs of G_1. The first is an induced subgraph. All edges between the vertices 2, 3, 4, and 6 that are in G_1 are also in this graph. The second subgraph is not an induced subgraph because the edges (2, 4) and (1, 4) are missing.

G1=

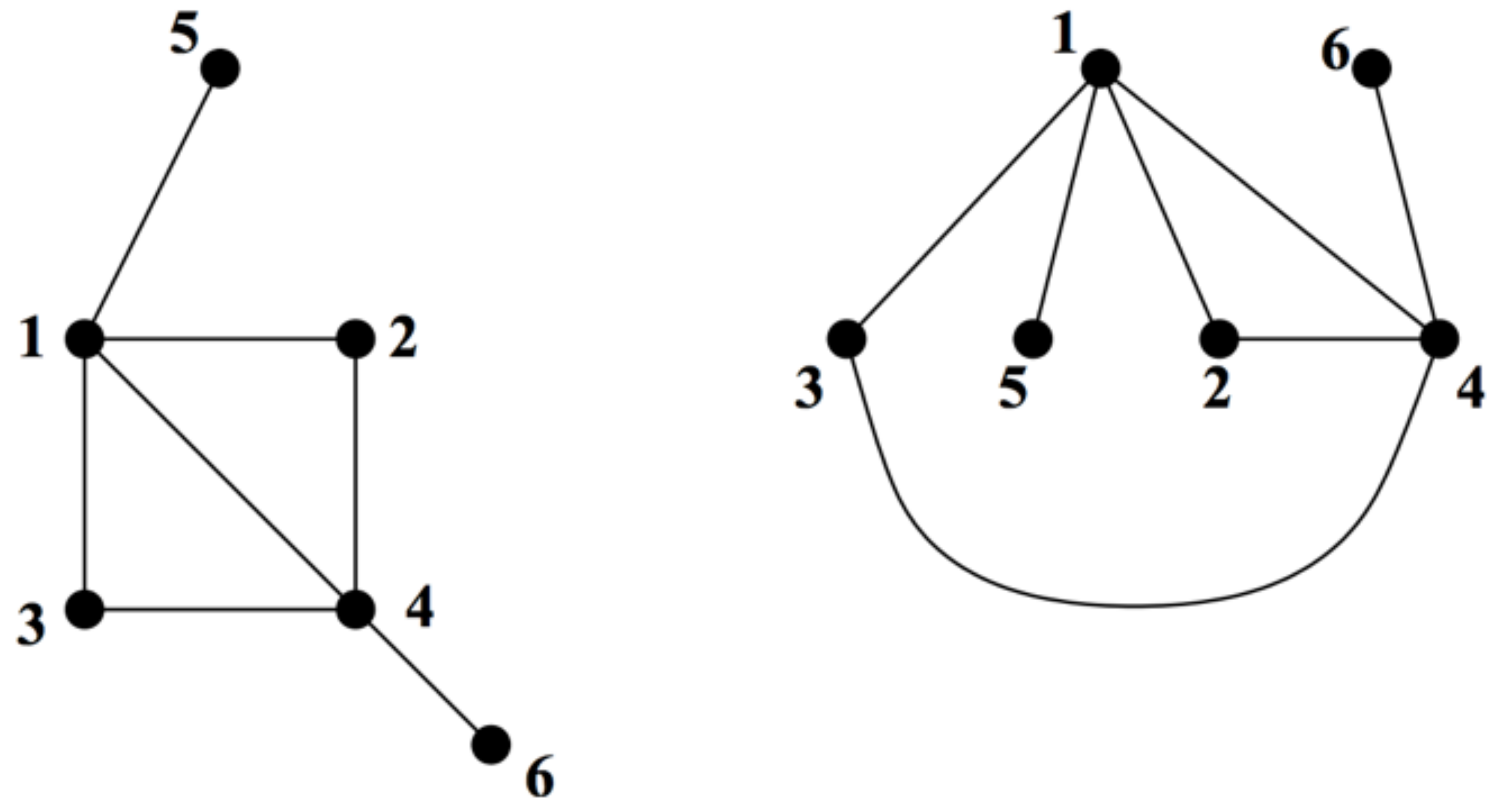






We say that two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are the same or isomorphic if there is a one to one and onto map f from V_1 to V_2 such that $(f(v_1), f(v_2))$ is an edge in G_2 if and only if (v_1, v_2) is an edge in G_1.

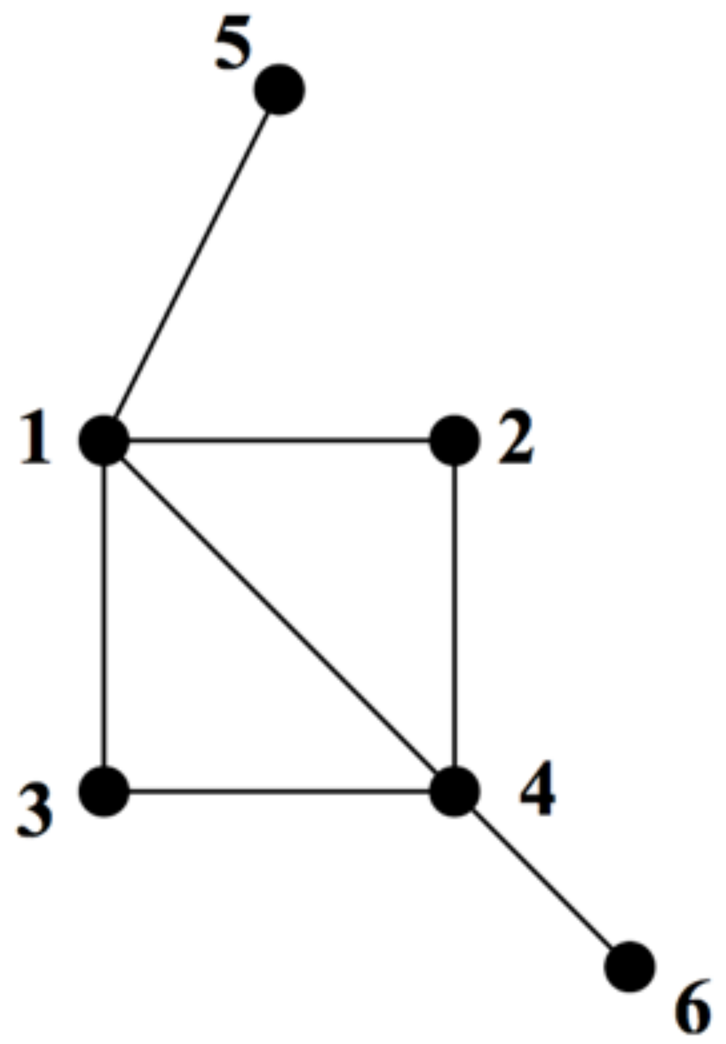
Example: The two graphs are isomorphic.
In this example, we can simply take the map
$f(1) = 1, f(2) = 2, \dots, f(6) = 6$.

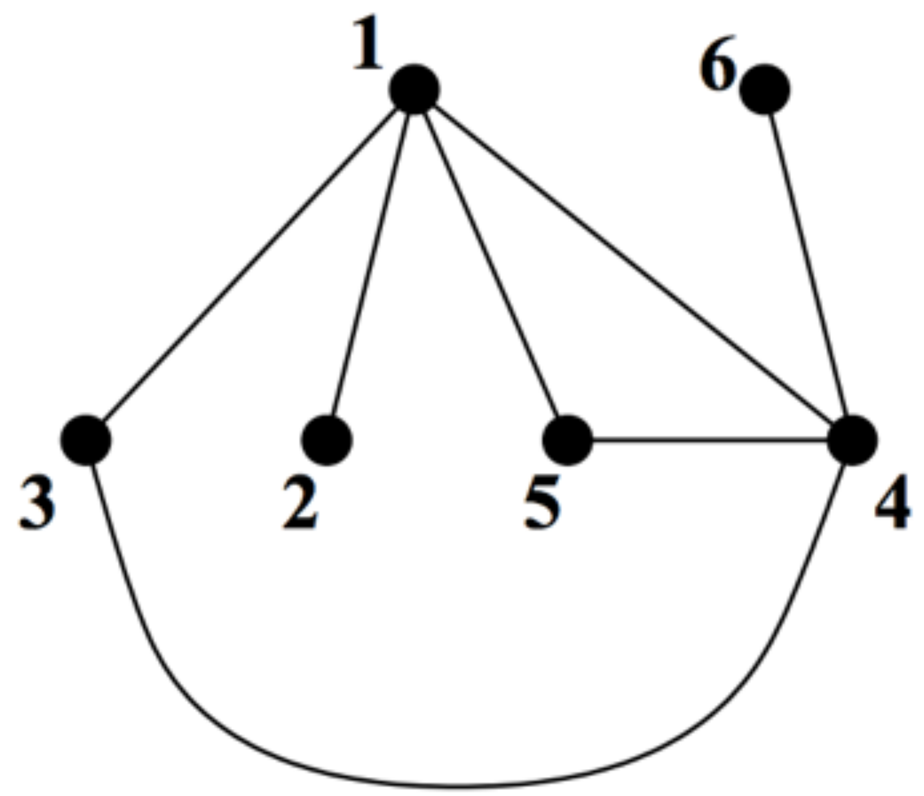


In general, if graph has labeled vertices
and we can find a map such that $f(i) = i$
then the graphs are isomorphic as labeled graphs.

We can change the labeling on one of the graphs so they're no longer isomorphic as labeled graphs (they are still isomorphic as unlabeled graphs).

Example: In the labels 2 and 5 have been changed, but the graphs are still isomorphic.

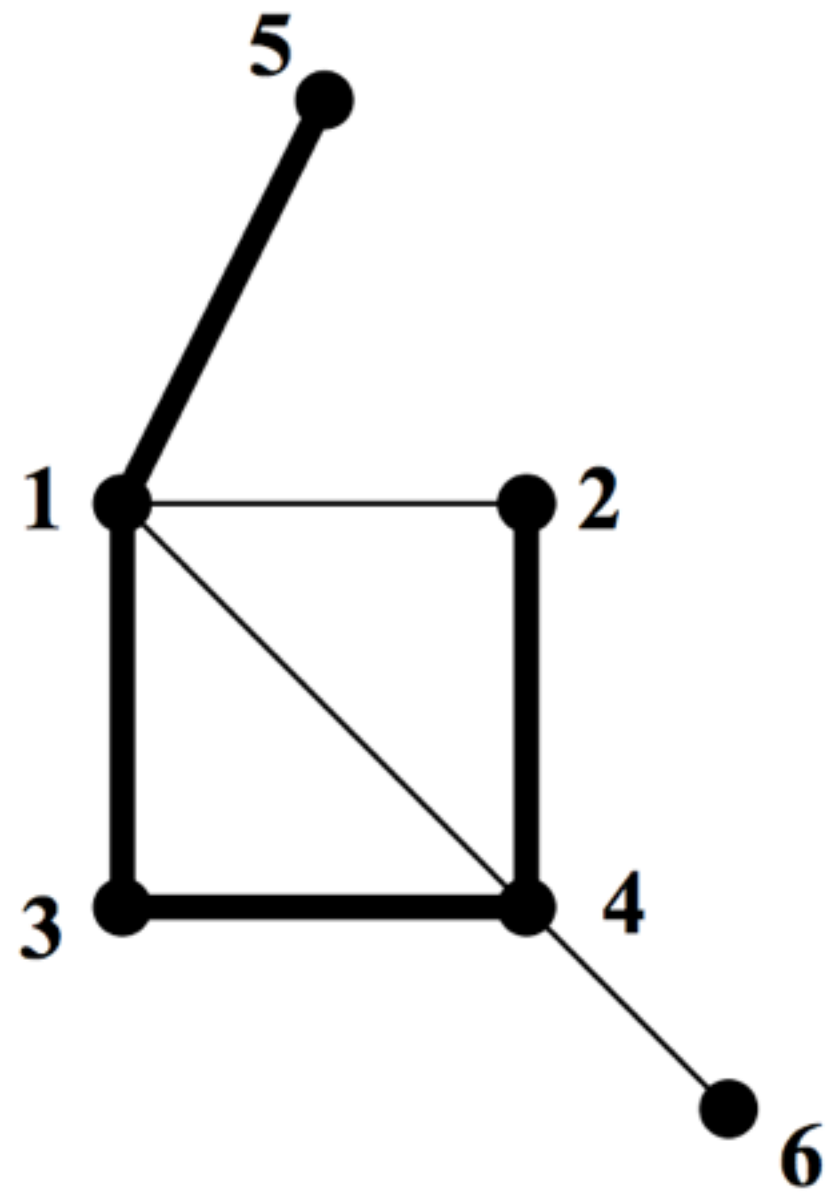




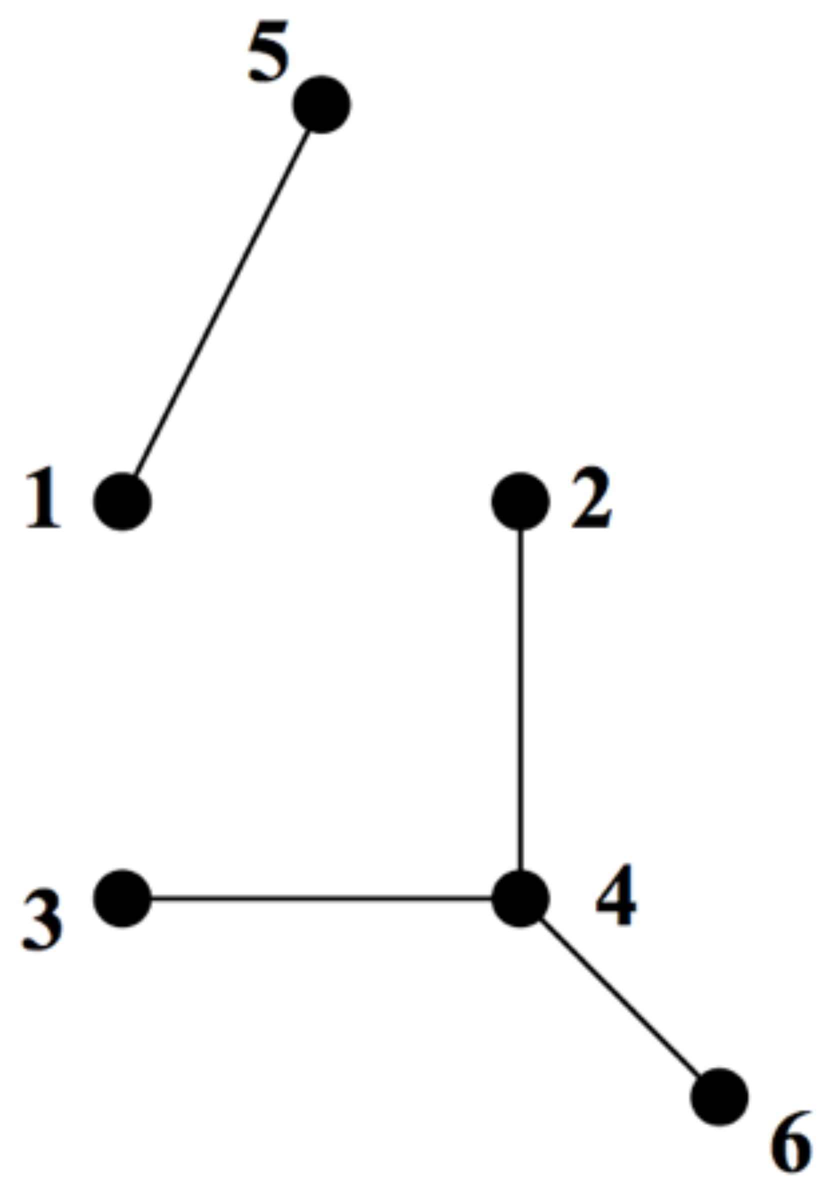
Two vertices v_1, v_2 are adjacent or neighbors if $(v_1,v_2) \in E$.
Two edges are adjacent if they share a common vertex.

A path from v_1 to v_n in a graph is a sequence of adjacent edges such that v_1 is in the first edge of the sequence and v_2 is in the last edge of the sequence.

Example: the path (5, 1)(1, 3)(3, 4)(4, 2) from 5 to 2 is highlighted.



We say a graph is connected if for every pair of vertices there exists a path between them. All the graphs so far have been connected.
The figure below shows a graph that is not connected.



A disconnected graph consists of multiple connected pieces called components.

We call a vertex v incident to an edge e if $v \in e$.

The degree of a vertex v, written deg(v),
is the number of edges to which it's incident.

Example: In G_1, deg(1) = 4, deg(2) = 2, deg(3) = 2, deg(4) = 4, deg(5) = 1, and deg(6) = 1.