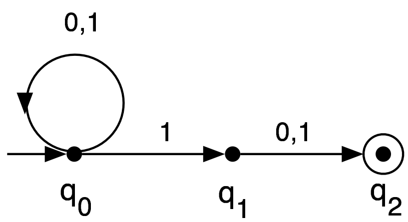
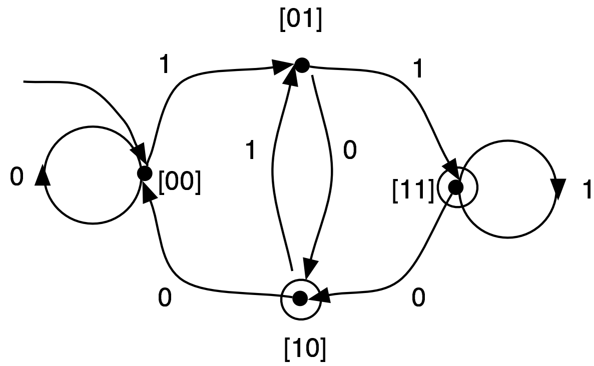
Example of subset construction (from Kozen, *Automata and Computation*):
Consider the set $A = \{x \in \{0,1\}^* |\ \text{2nd symbol from right is 1}\}$.



(a)



(b)

A 3-state non-deterministic machine (NFA) with the set A of accepted strings is given in the figure above (a), with the states labelled q_0, q_1, q_2. The transition function for an NFA returns sets of possible states rather than particular states. Each such subset can be regarded as a particular state of an associated DFA. The "subset construction" thus associates to the above NFA a deterministic machine (DFA) whose states are the subsets $\phi, \{q_0\}, \{q_1\}, \{q_2\}, \{q_0, q_1\}, \{q_0, q_2\}, \{q_1, q_2\}, \{q_0, q_1, q_2\}$ of the set $\{q_0, q_1, q_2\}$ of states of the NFA. The state transitions of the NFA are given at left below:

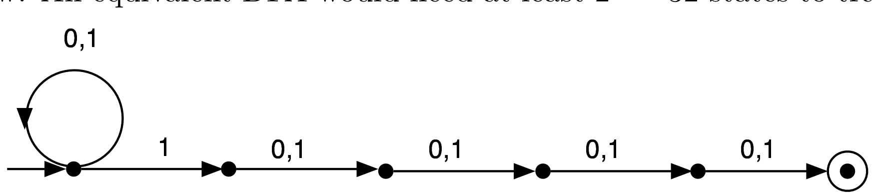
	0	1
ϕ	ϕ	ϕ
$\{q_0\}$	$\{q_0\}$	$\{q_0, q_1\}$
$\{q_1\}$	$\{q_2\}$	$\{q_2\}$
$\{q_2\}$F	ϕ	ϕ
$\{q_0, q_1\}$	$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$
$\{q_0, q_2\}$F	$\{q_0\}$	$\{q_0, q_1\}$
$\{q_1, q_2\}$F	$\{q_2\}$	$\{q_2\}$
$\{q_0, q_1, q_2\}$F	$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$

$\Longrightarrow$

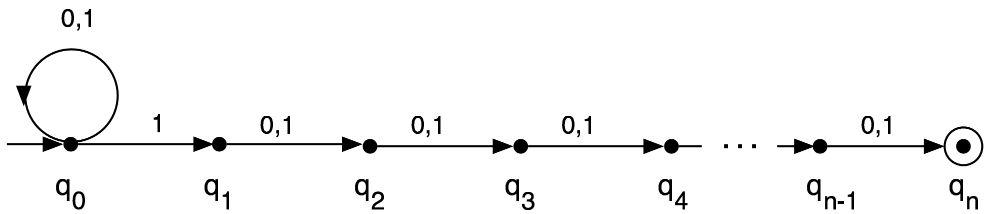
	0	1
$\{q_0\}$	$\{q_0\}$	$\{q_0, q_1\}$
$\{q_0, q_1\}$	$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$
$\{q_0, q_2\}$F	$\{q_0\}$	$\{q_0, q_1\}$
$\{q_0, q_1, q_2\}$F	$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$

where F designates accepted final states, those containing q_2. Note however that the states ϕ, $\{q_1\}$,$\{q_2\}$,$\{q_1, q_2\}$ are inaccessible, i.e., cannot be reached from the start state q_0 via any string of transitions. Excluding those four leaves the state transitions at right above, defining a 4-state DFA. This is identical to a deterministic model that remembers the last two bits seen and accepts the string iff the next to last is a 1, as shown at right in the above figure (b), with states labelled by the last two bits seen. The transitions of the two models agree under the identifications $\{q_0\}$=[00], $\{q_0, q_1\}$=[01], $\{q_0, q_2\}$=[10], $\{q_0, q_1, q_2\}$=[11].

A 6-state NFA that accepts $A = \{x \in \{0,1\}^* |\ \text{5th symbol from right is 1}\}$ is shown below. An equivalent DFA would need at least $2^5 = 32$ states to track the last 5 bits seen.



More generally, we can consider an $n+1$ state NFA that accepts the set of strings $A = \{x \in \{0,1\}^* |\ n^{\text{th}}$ symbol from right is $1\}$:



with the states now labelled $q_0 \ldots q_n$. This NFA is exponentially smaller than the equivalent DFA, which requires at least 2^n states to keep track of the last n bits.

To see how the subset construction of a DFA works in this case, note first of all that since we start at state q_0, and since there is always a path from q_0 to itself for an arbitrary string of 0's and 1's, then accessible states must always contain q_0 (just as the accessible states always contained the state q_0 in the 3 state NFA on the previous page). This reduces the states of the DFA from the 2^{n+1} possible to 2^n accessible. The transition table takes the form:

	0	1
$\{q_0\}$	$\{q_0\}$	$\{q_0, q_1\}$
$\{q_0, q_i\}\quad (i < n)$	$\{q_0, q_{i+1}\}$	$\{q_0, q_1, q_{i+1}\}$
$\{q_0, q_n\}$F	$\{q_0\}$	$\{q_0, q_1\}$
$\{q_0, q_i, q_j\}\quad (i < j < n)$	$\{q_0, q_{i+1}, q_{j+1}\}$	$\{q_0, q_1, q_{i+1}, q_{j+1}\}$
$\{q_0, q_i, q_n\}$F	$\{q_0, q_{i+1}\}$	$\{q_0, q_1, q_{i+1}\}$
$\vdots$	$\vdots$	$\vdots$
$\{q_0, q_{i_1}, \ldots, q_{i_{k-1}}, q_{i_k}\}$	$\{q_0, q_{i_1+1}, \ldots, q_{i_{k-1}+1}, q_{i_k+1}\}$	$\{q_0, q_1, q_{i_1+1}, \ldots, q_{i_{k-1}+1}, q_{i_k+1}\}$
$\{q_0, q_{i_1}, \ldots, q_{i_{k-1}}, q_n\}$F	$\{q_0, q_{i_1+1} \ldots, q_{i_{k-1}+1}\}$	$\{q_0, q_1, q_{i_1+1}, \ldots, q_{i_{k-1}+1}\}$

(where in the next to last line we have $i_1 < \ldots < i_{k-1} < i_k < n$).

We see that the mapping of states from the NFA to DFA via the subset construction is such that the q_i for $i = 1, \ldots, n$ store the state of the previous n bits, with the absence or presence of a given q_i indicating whether or not the i^{th} member of the previous n bits (counting from the right) is 0 or 1. For example

$\{q_0\} = [0 \ldots 0]$,
$\{q_0, q_1\} = [0 \ldots 01]$,
$\{q_0, q_i\} = 0 \ldots 1 \ldots 0]$ (1 in i^{th} position from right),
$\{q_0, q_i, q_j\} = [0 \ldots 1 \ldots 1 \ldots 0]$ (1s in $i^{\text{th}}, j^{\text{th}}$ positions from right),
$\{q_0, q_i, q_j, q_k\} = [0 \ldots 1 \ldots 1 \ldots 1 \ldots 0]$ (1s in $i^{\text{th}}, j^{\text{th}}, k^{\text{th}}$ positions from right),
etc.

and the accepted states are the 2^{n-1} that contain q_n, i.e., with a 1 in the n^{th} position from the right.