1) Show that $\log Y = \alpha + \beta \log X + u$ gives the same least squares estimator of $\beta$ whether the logs are taken to base 10 or to base e. Is this true for the estimator of $\alpha$? Do your conclusions change if log X is replaced by t (time)?

2) The following exercise is a remark on bivariate normal distributions and normality of its component:

   Evaluate the following statement: if true, prove it; if false, show a counterexample: if X and Y are two standard normal random variables, then the joint distribution of (X,Y) is bivariate normal.

   Remark: note that we are not saying anything about X and Y being independent. If they were, the statement is true. You have to consider the case of X and Y not necessarily being independent, and do not make them perfectly (positively or negatively) correlated. Also note that if (X,Y) is bivariate normal, then X will be a normal random variable and Y as well (but not necessarily standard normal).

   Exercise 2 is difficult and you are allowed to skip it if you cannot come with the proof or a counterexample.

3) (Midterm 2001) Consider the simple regression model with no intercept

   $$ y_i = \beta x_i + \varepsilon_i \quad i = 1, 2 $$

   and suppose that the true value of $\beta$ is 1 and the values of x realized in your sample are $x_1 = 1$ and $x_2 = 2$. The distribution of $\varepsilon$ is given by $P(\varepsilon = -1) = P(\varepsilon = 1) = \frac{1}{2}$, and the $\varepsilon_i$ are independent.

   a) Does this model satisfy the requirements for OLS (ordinary least squares) to be BLUE? (You do not need to provide a proof here)

   b) Calculate the exact distribution of the OLS estimator.

   c) Consider the alternative estimator $\beta^* = \frac{\sum y_i}{\sum x_i}$ and calculate its exact distribution. Is it unbiased?

   d) compare the exact variances of the OLS estimator and $\beta^*$. Which one is smaller?

4) Suppose that the have the following information:
\[ n = 22, \quad \sum_i x_i = 220, \quad \sum_i x_i^2 = 2260, \quad \sum_i y_i = 440, \quad \sum_i x_i y_i = 4430 \]

a) Compute the OLS estimators of \( \alpha \) and \( \beta \) in the model

\[ y_i = \alpha + \beta x_i + \varepsilon_i \]

\[ E(\varepsilon_i) = 0, \quad V(\varepsilon_i) = \sigma^2 \quad \text{and} \quad E(\varepsilon_i \varepsilon_j) = 0 \quad \text{when} \quad i \neq j. \]

b) Compute the \( R^2 \)

c) Assume now that the errors are normally distributed. Test the following hypothesis at the 5% significance level:

\[ H_0 : \beta = 0 \quad \quad H_A : \beta \neq 0 \]

d) Test the following hypothesis at the 10% significance level:

\[ H_0 : \alpha - \beta = 10 \quad \quad H_A : \alpha - \beta \neq 10 \]

5) (This question is mechanic but worth trying)

Show that the OLS estimator of the intercept in the model

\[ y_i = \alpha + \beta x_i + \varepsilon_i \]

\[ E(\varepsilon_i) = 0, \quad V(\varepsilon_i) = \sigma^2 \quad \text{and} \quad E(\varepsilon_i \varepsilon_j) = 0 \quad \text{when} \quad i \neq j \] is BLUE.

Hint: Write \( \hat{\alpha} = \bar{y} - \hat{\beta} \bar{x} = \frac{1}{n} \sum_i y_i - \bar{x} \sum_i (x_i - \bar{x})(y_i - \bar{y}) \sum_i (x_i - \bar{x})^2 = \sum_i \left[ \frac{1}{n} - \frac{1}{\sum_i (x_i - \bar{x})^2} \right] y_i = \sum_i m_i y_i \) where \( m_i = \frac{1}{n} - \frac{1}{\sum_i (x_i - \bar{x})^2} \).