1. (Warmup)
   a.) Prove that for Nx1 real vectors x and y, x’x = 1 and y’y=1 together imply 1 >=|x’y|. Hint: consider the vector x-y … (>= means greater than or equal to).
   b.) Use your result from a. to prove the Cauchy-Schwartz inequality: for arbitrary real vectors x and y, (x’x)(y’y) >=(x’y)^2.

2. Suppose you are interested in testing the simple hypothesis H0: p=1/4 against the alternative H1: p=3/4 where the observations are independent binomial draws (f(x) = px(1-p)(1-x) for x in {0,1}). Let \( \alpha = \Pr(H_0 \text{ rejected} | H_0 \text{ true}) \) and \( \beta = \Pr(H_0 \text{ not rejected} | H_1 \text{ true}) \). Suppose you have one observation.
   a.) What is the best test with \( \alpha = \frac{1}{4} \)?
   b.) What is the \( \beta \) associated with this test?
   c.) Graph the set of obtainable \( \alpha, \beta \) pairs for tests based on one observation.
   d.) (harder) Graph the set of obtainable \( \alpha, \beta \) pairs for tests based on two observations. (even harder) Interpret briefly but accurately.

3. You have N pairs of points \((y_i,x_i)\) and model their relationship linearly as \( E y_i = \alpha + \beta x_i \). Having forgotten the OLS formulas (now useless on the allowed formula sheet), you reason that a line can be fit by two points, so you arrange the \( x_i \) in order and calculate the estimators \( \alpha^* \) and \( \beta^* \) by solving the 2 equations \( y_1 = \alpha + \beta x_1 \) and \( y_N = \alpha + \beta x_N \) for values of \( (\alpha, \beta) \).
   a.) Are these estimators unbiased? b.) What is their variance? c.) Are they consistent?
   In view of your results, you decide to calculate sample means for the first and second halves of the sample (luckily N is even). Let \( y(1) = (2/N) \sum_{i=1}^{N/2} y_i \), \( y(2) = (2/N) \sum_{i=N/2+1}^{N} y_i \), and similarly for the x. You calculate estimators \( \alpha^{**} \) and \( \beta^{**} \) by solving \( y(1) = \alpha + \beta x(1) \) and \( y(2) = \alpha + \beta x(2) \).
   d.) Are these estimators unbiased? e.) What is their variance? f.) Are they consistent? g.) Are they BLUE?