Midterm

You may use books, notes, calculators intuition and math, but not collusion. Good luck!

1. (Warmup) You wish to fit the model \( y = X\beta + \varepsilon \), but you do not have the full data set \([y \ X] = Z\). Instead you have only \( Z'Z \)

\[
Z'Z = \begin{bmatrix} 100 & 10 & 25 \\ 10 & 20 & 0 \\ 25 & 0 & 75 \end{bmatrix}
\]

Calculate \( \hat{\beta} \), \( \sigma^2 \) and \( R^2 \). Is there anything to be gained by seeing the full dataset?

2. You are interested in fitting the regression model

\[ y_i = x_i'\beta + \varepsilon_i \]

with \( x_i' = (1 \times 2) = (1 \ 1) \) in \( N/2 \) observations (Group 1) \( x_i' = (1 - 1) \) in the other \( N/2 \) observations (Group 2). Although ols will work you design an alternative estimator \( \hat{\beta} \) by noting that

\[
\bar{y}(1) = \beta_1 + \beta_2 + \bar{\varepsilon}(1)
\]

\[
\bar{y}(2) = \beta_1 - \beta_2 + \bar{\varepsilon}(2)
\]

where \( \bar{y}(j) \) is the sample mean of the \( y_i \) in group \( j \), etc. Since \( \bar{\varepsilon}(j) \), has expectation zero, you solve the 2 linear equations * with the \( \varepsilon \) set to zero to obtain \( \hat{\beta} \). Give a formula for \( \hat{\beta} \). Is \( \hat{\beta} \) unbiased? What is its sampling variance? (Assume \( \varepsilon_i \) are independent, mean zero, variance \( \sigma^2 \)). Is \( \hat{\beta} \) consistent? What is the variance of the ols estimator? Explain.

OVER PLEASE
3. In another simple regression

\[ y_t = \alpha + x_t \beta + \epsilon_t \]

with \( x_t \) scalar, you do not observe \( x_t \) but rather only a badly measured version \( z_t = x_t + \nu_t \) (a "proxy"). So you regress \( y_t \) on \( z_t \) and obtain a slope parameter \( \tilde{\beta} \). Assume that \( \nu_t \) is independent of \( x_t \) and \( \epsilon_t \). Show that

\[ \text{plim } \tilde{\beta} < \beta. \]

Explain (briefly).

Now you observe \( w_t \), which is correlated with \( x_t \) but not \( \nu_t \) or \( \epsilon_t \). You multiply through and regress \( w_t y_t \) on \( w_t \) and \( w_t z_t \). Find the plim of the coefficient of \( w_t z_t \). Explain.