1. Suppose there are 2 firms with the following joint demand function and individual cost functions:

**Demand:** \( P = 12 - 2X \) where \( X = x_1 + x_2 \)

**Total costs:**
\[
c(x_1) = 1 + 5x_1^2 \text{ and } c(x_2) = 2 + 1x_2^2
\]

a) Find the Cournot-Nash reaction functions for firms 1 and 2. Graph them.
b) Find the Cournot-Nash equilibrium solution, i.e., price, quantities, and profits in the Cournot-Nash equilibrium.
c) Find the "multi-plant monopoly" solution.
d) Find the "competitive" solution.

2. Suppose that Abe is a true believer in Cournot’s model of oligopoly while Betty believes that Bertrand’s oligopoly model is more accurate. How would Abe’s and Betty’s analyses of the Coke/Pepsi “cola-duopoly” differ? Specifically, how would Abe and Betty compare the prevailing market prices and quantities of colas, as well as firm profitability, to what the prices, quantities, and profits would be if the cola market were perfectly competitive?

3. Suppose the airline industry consists of only two firms: Firm1=American and Firm2 =Texas Air Corp. Let the two firms have identical cost functions: \( t_c(x_i) = 40x_i \). Assume the demand curve for the industry is given by \( P = 100 - X \) where \( X = x_1+x_2 \), and that each firm expects the other to behave as a Cournot competitor.

a) Calculate the Cournot Nash equilibrium (price, quantities, and profits) for each firm assuming that each chooses the output level that maximizes its profits taking its rival’s output as given.
b) Given your answers to (a), how much should Texas Air be willing to invest to lower its marginal cost from 40 to 25, assuming that American will not follow suit?
c) What would be the equilibrium price and quantities if Texas Air had constant marginal and average costs of 25, and American had constant marginal and average costs of 40?

4. Suppose Tops and Wegmans are the only two big grocery stores in a small town. They open their grocery store from 8 a.m. to midnight and have revenues of $4 million/year and costs of $2.5 million/year each. Now Tops and Wegmans are individually considering extending their operating hours to 24 hours/day. If one store extends its hours and its rival store doesn't, the store extending its hours will increase its revenues by 40% and its costs by 50%, while its rival store will lose 20% of its revenues and 20% of its costs. If they both extend their hours, both will see revenues increase by 10% and costs increase by 20%.

a) Construct the payoff matrix for this game and find the Nash equilibrium. Make Tops the "row player" and Wegmans the "column player."
b) Which outcome maximizes the joint profit of these two stores?
c) Suppose Wegmans promises to give Tops $0.32 million/year if Tops doesn't extend its hours. Should Tops accept this offer?

5. Suppose that Nick and Bill are the only two farmers supplying eggs to Ithaca. The market demand for eggs is \( Q = 50 - 2P \), where \( Q \) is dozens/week and \( P \) is $/dozen. Consumers will buy eggs from the producer who offers a lower price for eggs. The cost of producing 1 dozen of eggs is $5 for both Nick and Bill.
a) Assume that Nick and Bill enter an agreement where they decide to maximize joint profit by charging the same price, acting like a two-plant monopolist and each taking half of all the customers. What price will they agree on and how many eggs are supplied in equilibrium? What are their individual profits?

b) Suppose that Nick adheres to the agreement and charges the price they agreed upon in (a). However, Bill secretly communicates to everyone in the market that he will beat Nick’s price by $1. If Nick does not find out about this scheme, what are Bill's profits? (Assume that they do not have fixed costs.)

c) Suppose Nick finds out that Bill cheats and starts to undercut Bill's prices. What do you expect to happen in the long-run?