

## How the Electoral College Influences Campaigns and Policy: The Probability of Being Florida

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*This paper analyzes how US presidential candidates should allocate resources across states to maximize the probability of winning the election, by developing and estimating a probabilistic-voting model of political competition under the Electoral College system. Actual campaigns act in close agreement with the model. There is a 0.9 correlation between equilibrium and actual presidential campaign visits across states, both in 2000 and 2004. The paper shows how presidential candidate attention is affected by the states' number of electoral votes, forecasted state-election outcomes, and forecast uncertainty. It also analyzes the effects of a direct national popular vote for president. (JEL D72)*

The president of the United States is arguably the world's most powerful political leader, and the incentives created by the electoral process are important. It is not surprising, therefore, that the Electoral College system of electing the president<sup>1</sup> has been under constant debate. According to federal historians, over 700 proposals have been introduced in Congress in the last 200 years to reform or eliminate the Electoral College system. Indeed, there have been more proposals for constitutional amendments to alter or abolish the Electoral College than on any other subject.

The drive toward reform is understandable given the perceived impact on the democratic process and economic policy. For example, voters in states like Utah, Massachusetts, and Idaho did not see much of Bush and Kerry in person or in advertisements, while more than three of every five candidate visits in 2004 were in Florida, Ohio, Wisconsin, Iowa, and Pennsylvania. This leaves people in the former states feeling neglected, and with weaker incentives to be informed and to vote. Equally worrying are the fears that the Electoral College might induce distorted and inefficient government policies.<sup>2</sup> This debate is as old as

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<sup>1</sup> When US citizens vote for president and vice president, ballots show the names of the presidential and vice presidential candidates, although they are actually electing a slate of "electors" who represent them in each state. After the election, the party that wins the most votes in each state appoints *all* of the electors for that state (except in Maine and Nebraska). The number of electors equals the state's number of US senators (always two) plus the number of its US representatives (increasing in the state's population as determined in the Census). The electors from every state combine to form the Electoral College. The electors cast their votes and send them to the president of the Senate. On January 6, the president of the Senate opens all of the sealed envelopes containing the electoral votes and reads them aloud. To be elected president or vice president, a candidate must have an absolute majority of the electoral votes for that position.

<sup>2</sup> See for example, Susan Page, "Bush policies follow politics of states needed in 2004," *USA Today*, June 16, 2002; John Mercurio, "Bush Stumps in Rust Belt. On the Verge of Repealing Steel Tariffs," CNN, December 2, 2003; or Carl Hulse and Matthew L. Wald, "President's Response to Hurricane Carries Reminders of Political Fallout for Past Candidates," *New York Times*, August 15, 2004.

the Constitution.<sup>3</sup> Still, the overwhelming part of previous work on redistributive politics in the United States has been concerned with Congress, not the role of the president. An important exception is a set of early game-theoretic analyses of the Electoral College system, following Steven J. Brams and Morton D. Davis (1974).

The goal of the current paper is to provide answers to a set of questions regarding the impact of the Electoral College. The developed model of electoral competition gives a precise formula for how presidential candidates should allocate campaign resources across states to win the presidential election. The model explains why candidate resources are so concentrated and, in general, what determines how concentrated they will be. It also identifies the states—like Florida, Ohio, and Iowa in 2004—that should receive the most attention in each election, by deriving an analytic formula determining how this is related to the number of electoral votes, opinion polls, and other variables available in September of the election year. The number of times the candidate should visit each state, based on this formula, is then compared to the actual campaign visits in 2000 and 2004. The simple correlation is 0.9; see Figure 1 (p. 782). This provides some evidence that the model and the actual campaigns agree on which states are important for the election outcome. Additional evidence will be available after the next election.<sup>4</sup>

Since the Electoral College creates very sharp incentives to target a selected group of states, it is natural to analyze alternative ways of electing the president. This paper analyzes the effects of a change to a direct national popular vote system (Direct Vote). This particular reform is studied since it has come closest to being accepted. The paper identifies states that would gain or lose candidate attention after reform, and explains why Direct Vote would create less sharp incentives to favor certain states.<sup>5</sup>

The incentives created by the Electoral College are similar for campaigning and policy, in the sense that politicians with election concerns should grant possible policy favors to the same states (important for the election outcome) that they target when campaigning. Consequently, the theoretical results regarding what states should be targeted also apply to campaign promises about policy, and with some modification to policies implemented by incumbent presidents. This paper does not empirically investigate effects on policy, however, since policy depends on a multitude of factors and discussing these is beyond the scope of this paper.

This paper relates to the parallel literatures investigating political competition in campaigns and policy. It fits well into the burgeoning literature on comparative political economy, a literature that links political institutions to outcomes in politics and economic policy; for an overview, see Torsten Persson and Guido Tabellini (2000). It is particularly closely related to Alessandro Lizzeri and Nicola Persico (2001), Persson and Tabellini (1999), and Gian Maria Milesi-Ferretti, Roberto Perotti, and Massimo Rostagno (2002), who compare taxes, spending, and transfers in politics with majoritarian and proportionally representative electoral rules. For an overview of results regarding the impact of US political institutions on policy, see Timothy Besley and Anne Case (2003).

The current paper also relates to the literature on campaigning, in particular to Brams and Davis (1974). These authors analyze how presidential candidates should allocate campaign resources across states to maximize their expected electoral vote. They find that resources should be allocated disproportionately in favor of large states if each candidate wins every state with equal probability and the number of undecided voters is proportional to the number of electoral votes.

<sup>3</sup> For example, in the *Federalist Papers* (#64), John Jay discusses the “fears and apprehensions of some, that the President and Senate may make treaties without an equal eye to the interests of all the States.”

<sup>4</sup> In September 2008, the model’s recommended shares of campaign visits to all states will be posted on the AER Web site (<http://www.aeaweb.org/articles.php?doi=10.1257/aer.98.3.770>).

<sup>5</sup> For a less technical description of the results in this paper, see Rebecca B. Morton (2006, chap. 13).

This result is disputed by Claude S. Colantoni, Terrence J. Levesque, and Peter C. Ordeshook (1975) who, instead, argue that a proportional rule, modified to take into account the ex post closeness of the state election, predicts actual campaign allocations better. Also closely related is James M. Snyder's (1989) model of two-party competition for legislative seats that allows for variation in vote shares and where candidates maximize the chance of winning.

A major difference between this paper and earlier work is the complete integration of theory and empirics, tying together theoretical insights with empirical results on actual campaigns. This can be done since the constructed probabilistic-voting model is sufficiently general to be directly applied to the real world problem of a US presidential candidate, yet can be explicitly solved, and directly estimated. This framework allows for new questions to be asked and answered.

More specifically, the model of this paper differs from that of Brams and Davis (1974) in allowing states to have different partisan leanings, in allowing for uncertainty regarding the election outcome at both state and national levels, and in having candidates maximizing the probability of winning the election instead of the expected number of electoral votes that they win. It differs from the model of Snyder (1989) in allowing for different state sizes, uncertainty at both state and national levels, and in providing closed-form solutions. While Persson and Tabellini (1999) use a similar probabilistic-voting framework, they use only three states, each with one vote, and no state-level uncertainty. Perhaps the setup of Lizzeri and Persico's (2001) model is the most different. They use a continuum of states, each with one electoral vote, and let votes be entirely decided by campaign efforts (no uncertainty). However, the models of Persson and Tabellini (1999) and Lizzeri and Persico (2001) allow the candidates to use richer strategies, setting taxes and public spending in addition to targeted spending.

The model finally demonstrates that there is a link between all the literature above and the literature concerning "voting power," that is, the probability that a vote is decisive in an election. The statistical properties of voting power have been analyzed extensively; see John F. Banzhaf (1968), Gary Chamberlain and Michael Rothschild (1988), Andrew Gelman and Jonathan N. Katz (2001), Gelman, Gary King, and W. John Boscardin (1998), and Samuel Merrill III (1978). The "voting power" analyzed in this paper is slightly different, since it is conditional on the candidates' equilibrium strategies.

The theoretical model of presidential campaigning is developed in Section I. The model is estimated in Section II, using data from presidential elections from 1948 to 2004. This answers the question of how candidates should have allocated campaign resources across states to maximize the probability of winning. Section III studies whether presidential campaign visits in the 2000 and 2004 elections resemble those recommended by theory. Section IV shows that the rather complex equilibrium condition has a very intuitive meaning. It also answers questions such as: Do large or small states benefit from the electoral college system? What would happen if opinion polls were not allowed? Should 50-50 states receive most attention? Section V discusses the impact of electoral reform. Section VI concludes.

## I. Model

### A. Setup

We begin by assuming that there are two presidential candidates, indexed by superscript  $R$  and  $D$ ; for now, we ignore the impact of minority-party candidates. Both candidates have to make campaign plans for the last  $I$  days before the election.<sup>6</sup> The plans specify how many of the  $I$  days

<sup>6</sup> Historically, presidential campaigns have begun around Labor Day and ended on election day, although candidates often start campaigning as soon as nomination is secured.

to use to visit each of  $S$  states (in 2004,  $S$  equaled 51, including the 50 states and Washington, DC). Formally, let  $d_s^J$  represent the number of days candidate  $J = R, D$  visits state  $s$ . Each candidate  $J$  chooses  $d_s^J$ , subject to the constraint

$$(1) \quad \sum_{s=1}^S d_s^J \leq I.$$

We assume that the objective of the candidates is to maximize their probability of winning the election. Of course, candidates may have other goals in campaigning, such as helping out congressional or senatorial candidates or candidates in state and local races. But, at least in close races, maximizing the probability of winning is likely to be the main concern.

How the election result is affected by campaign visits depends on how voters respond, and the formal electoral rules. Starting with the latter, each state  $s$  is allocated a number of electoral votes,  $e_s$ . In practice,  $e_s$  equals the state's number of US senators (always two) plus the number of its US representatives (increasing in the state's population as determined in the Census). In each state, there is a popular election. The candidate who receives a majority of the votes in a state  $s$  gets all the  $e_s$  electoral votes of that state.<sup>7</sup> After elections have been held in all states, the electoral votes are counted, and the candidate who gets more than half those votes wins the election. The model ignores that in two states, Maine and Nebraska, two electoral votes are chosen by statewide popular vote and the remainder by the popular vote within each congressional district.<sup>8</sup>

Turning to voter behavior, we assume that campaign visits by presidential candidates matter—they affect whom voters choose on election day.<sup>9</sup> Formally, the increasing popularity of candidate  $J$ , campaigning  $d_s^J$  days in state  $s$ , is captured by the increasing and concave function  $u(d_s^J)$ . This implies that the effect of visits decreases with the number of visits.

Vote choices, of course, depend on things other than candidate visits. Voters are affected by the candidates' divergent policy positions, and perhaps candidate appearance. We will call all these other factors ideological preferences and treat them as exogenous. Key to the model is that at the time campaign strategies are chosen, the candidates are uncertain about the voters' ideological preferences on election day. This uncertainty arises both because the true preferences of the voters at the time campaign plans are made are not known, and because of the realization of stochastic events after plans are made, which shift voter preferences. Some of this uncertainty affects all voters in a similar way and creates national swings. For example, in 1980 a US government attempt to rescue hostages in Iran ended in a helicopter crash that killed eight servicemen. Voter evaluations of President Carter's performance fell across the nation. Other uncertainty is state-specific and generates swings in only one state. For example, the unexpected downturn in a state's economy may shift voter preferences for government spending and unemployment insurance.

<sup>7</sup> The model ignores that the current 538 individuals (one for each electoral vote) who actually vote for the president in the Electoral College are not formally bound to vote for the candidate for which they have pledged support. There have been eight "faithless electors" in this century, and as recently as 2004 a Democrat elector in the State of Minnesota cast his votes for John Edwards instead of presidential candidate John Kerry. However, faithless electors have never changed the outcome of an election; most often it seems that their purpose is to make a statement rather than make a difference.

<sup>8</sup> This method was used in Maine since 1972 and Nebraska since 1996, though since both states have adopted this modification, the statewide winners have consistently swept all of the state's districts as well. Consequently, neither state has ever split its electoral votes.

<sup>9</sup> This paper does not address the question of why campaigning matters. This is an interesting question in its own right, with many similarities to the question of why advertisements affect consumer choice. For a discussion of how visits matter and empirical evidence that they do, see Morton (2006).

These features are modeled by writing the ideological preferences of the voters as a sum of three parts— $R_i$ ,  $\eta_s$ , and  $\eta$ —where  $R_i$  is predictable at the time the campaign plans are made, and  $\eta$  and  $\eta_s$  are the unpredictable national and state swings. The predictable part of voters’ ideological preferences within each state is normally distributed, with a mean that may shift over time but with a constant variance, specifically,

$$(2) \quad R_i \sim F_s = N(\mu_s, \sigma_{fs}^2).$$

The  $S$  state-level popularity parameters,  $\eta_s$ , and the national popularity parameter,  $\eta$ , are independently drawn from the distributions

$$\eta_s \sim G_s = N(0, \sigma_s^2)$$

and

$$\eta \sim H = N(0, \sigma^2).$$

The voters may vote for candidate  $R$  or candidate  $D$ . Considering campaigning and ideological preferences, a voter  $i$  in state  $s$  will vote for candidate  $D$  if

$$(3) \quad u(d_s^D) - u(d_s^R) = \Delta u_s \geq R_i + \eta_s + \eta,$$

where the first equality defines  $\Delta u_s$ . Consequently, the share of votes,  $y_s$ , that candidate  $D$  receives in state  $s$  on election day, when the candidates have chosen strategies resulting in  $\Delta u_s$  and after the shocks  $\eta_s$  and  $\eta$  have been realized, is

$$(4) \quad y_s = F_s(\Delta u_s - \eta_s - \eta).$$

### B. The Approximate Probability of Winning

We now turn to computing the approximate probability of winning the presidential election, given any set of campaign strategies chosen by the two candidates. Candidate  $D$  wins a particular state  $s$  if

$$y_s \geq \frac{1}{2},$$

or, equivalently, if

$$\eta_s \leq \Delta u_s - \mu_s - \eta.$$

The probability of this event, conditional on the aggregate popularity  $\eta$ , and the campaign visits,  $d_s^D$  and  $d_s^R$ , is

$$(5) \quad G_s(\Delta u_s - \mu_s - \eta).$$

Let  $e_s$  be the number of votes of state  $s$  in the Electoral College. Define the stochastic variables,  $D_s$ , which indicate whether  $D$  wins state  $s$ , as

$$D_s = 1, \text{ with probability } G_s(\cdot),$$

$$D_s = 0, \text{ with probability } 1 - G_s(\cdot).$$

Let  $\Delta u = (\Delta u_1, \Delta u_2, \dots, \Delta u_s)$  be the utility differentials resulting from any allocation of campaign resources across states of the two campaigns. The probability that  $D$  wins the election for any national popularity shock,  $\eta$ , and utility differentials from campaigning is then

$$(6) \quad \tilde{P}^D(\Delta u, \eta) = \Pr \left[ \sum_s D_s e_s > \frac{1}{2} \sum_s e_s \right].$$

It is difficult, however, to find strategies that maximize the expectation of the probability of winning (above). The reason is that it is a sum of the probabilities of all possible combinations of state election outcomes that would result in  $D$  winning. The number of such combinations is of the order of  $2^s$ , for each of the infinitely many realizations of  $\eta$ .

A way to cut this Gordian knot, and to get a simple analytical solution to this problem, is to assume that the candidates are considering their approximate probabilities of winning.<sup>10</sup> Since, given the national shock  $\eta$ , the  $\eta_s$  are independent, so are the  $D_s$ . Therefore, by the Central Limit Theorem of Liapounov,<sup>11</sup>

$$\frac{\sum_s D_s e_s - \mu}{\sigma_E},$$

where

$$(7) \quad \mu = \mu(\Delta u, \eta) = \sum_s e_s G_s(\Delta u_s - \mu_s - \eta),$$

and

$$(8) \quad \sigma_E^2 = \sigma_E^2(\Delta u, \eta) = \sum_s e_s^2 G_s(\cdot)(1 - G_s(\cdot))$$

is asymptotically distributed as a standard normal. The mean,  $\mu$ , is the expected number of electoral votes, that is, the sum of the electoral votes of each state, multiplied by the probability of winning that state. The variance,  $\sigma_E^2$ , is the sum of the variances of the state outcomes, which is the  $e_s^2$  multiplied by the usual expression for the variance of a Bernoulli variable. Using the asymptotic distribution, the approximate probability of  $D$  winning the election, for any given national popularity shock and utility differentials from campaigning, is

$$(9) \quad \tilde{P}^D(\Delta u, \eta) = 1 - \Phi \left[ \frac{\frac{1}{2} \sum_s e_s - \mu(\Delta u, \eta)}{\sigma_E(\Delta u, \eta)} \right],$$

<sup>10</sup> Assar Lindbeck and Jörgen W. Weibull (1987) use this trick in a different setting.

<sup>11</sup> See, for example, Ramu Ramanathan (1993, 157). The formal requirements for convergence are  $E[|D_s e_s - \mu_s|^{2+\delta}] = \rho_s$  for some  $\delta > 0$  and

$$\lim_{S \rightarrow \infty} \frac{(\sum_{s=1}^S \rho_s)^2}{(\sum_{s=1}^S \sigma_s^2)^{2+\delta}} = 0.$$

For our purposes, it is more interesting to know the approximation error using our sample (51 states in the elections from 1948 to 2004) than the limiting behavior. The approximate distributions of elections outcomes, computed using the Central Limit theorem approximation, are plotted together with the distributions of election outcomes, not using the approximation, in Appendix 6.8 of Strömberg (2002). From the presidential candidates' point of view, the relevant statistic is the error that they make by maximizing the approximate probability of winning. This error is discussed in Section IV.

where  $\Phi[\cdot]$  is the standard normal cumulative density function. The approximate probability of winning as a function of campaigning only is found by taking the expectation over the national popularity shocks,

$$(10) \quad P^D(\Delta u) = \int \tilde{P}^D(\Delta u, \eta) h(\eta) d\eta.$$

C. Equilibrium

Having derived the approximate probability of winning, we now characterize the Nash Equilibrium. The equilibrium strategies  $(d^D, d^R)$  are characterized by

$$P^D(d^D, d^R) \geq P^D(d^D, d^{R*}) \geq P^D(d^D, d^{R*})$$

for all  $d^D, d^R$  in  $X$ , the set of allowable campaign visits,

$$X = \left\{ d \in \mathbb{R}_+^S : \sum_{s=1}^S d_s \leq I \right\}.$$

This game has a unique interior pure-strategy equilibrium characterized by the proposition below.

**PROPOSITION 1:** *The unique pair of strategies for the candidates  $(d^D, d^R)$  that constitute an interior NE in the game of maximizing the expected probability of winning the election must satisfy  $d^D = d^R = d^*$ , and, for all  $s$  and for some  $\lambda > 0$ ,*

$$(11) \quad Q_s u'(d_s^*) = \lambda,$$

where

$$(12) \quad Q_s = \frac{\partial P^D}{\partial \Delta u_s}.$$

**PROOF:**

See Appendix A.

Proposition 1 says that presidential candidates should spend more time in states with high values of  $Q_s$ . This follows since  $u'(d_s^*)$  is decreasing in  $d_s^*$ . In the empirical section,  $u'(0)$  will be assumed to be sufficiently high that the equilibrium is interior.

Note that  $Q_s$  consists of two additively separable parts:

$$(13) \quad \begin{aligned} Q_s &= Q_{s\mu} + Q_{s\sigma} \\ &= e_s \int \frac{1}{\sigma_E} \varphi(x(\eta)) g_s(-\mu_s - \eta) h(\eta) d\eta \\ &\quad + \frac{e_s^2}{\sigma_E^2} \int \varphi(x(\eta)) x(\eta) \left( \frac{1}{2} - G_s(-\mu_s - \eta) \right) g_s(-\mu_s - \eta) h(\eta) d\eta, \end{aligned}$$

where

$$x(\eta) = \frac{\frac{1}{2} \sum_s e_s - \mu}{\sigma_E},$$

and  $\varphi(\cdot)$  is the standard normal probability density function. The first arises because the candidates have an incentive to influence the expected number of electoral votes won by  $D$ , that is, the mean of the normal distribution. The second arises because the candidates have an incentive to influence the variance in the number of electoral votes. This variance matters since they maximize the probability of winning, which is a locally convex (concave) function for the trailing (leading) candidate; for further discussion, see Section IVD.

The value of  $Q_s$  depends only on the parameters  $e_s$ ,  $\mu_s$ , and  $\sigma_s$ ; for all states  $s = 1, 2, \dots, S$ , and  $\sigma$ ,

$$(14) \quad Q_s = Q_s(e_1, e_2, \dots, e_S, \mu_1, \mu_2, \dots, \mu_S, \sigma_1, \sigma_2, \dots, \sigma_S, \sigma).$$

This can be seen by inserting the definitions of  $\sigma_E^2$ ,  $\mu$ ,  $g_s(\cdot)$ , and  $h(\cdot)$  in equation (13). The distribution of electoral votes,  $e_s$ , is known. To determine the value of  $Q_s$  for each state in each election, we now estimate the remaining parameters.

## II. Estimation

Estimation of  $\mu_s$ ,  $\sigma_s$ , and  $\sigma$ , provides the link between the theoretical probabilistic-voting model above and the empirical applications discussed below. Intuitively, estimation amounts to predicting state-level Democratic vote shares 1948–2004 ( $\mu_s$ ) using a set of observables, and then estimating the national, and independent state-level, uncertainty in this prediction ( $\sigma$  and  $\sigma_s$ ). Since we are using a panel, let subscript  $t$  denote time, and subscript  $s$  denote state.

Formally, both candidates choose the same allocation in equilibrium, so that  $\Delta u_{st} = 0$  in all states. The Democratic vote-share in state  $s$  at time  $t$  equals, using equations (4) and (2),

$$(15) \quad y_{st} = F_{st}(-\eta_{st} - \eta_t) = \Phi\left(\frac{-\mu_{st} - \eta_{st} - \eta_t}{\sigma_{fs}}\right),$$

where  $\Phi(\cdot)$  is the standard normal distribution or, equivalently,

$$(16) \quad \Phi^{-1}(y_{st}) = \gamma_{st} = -\frac{1}{\sigma_{fs}}(\mu_{st} + \eta_{st} + \eta_t).$$

For now, assume that all states have the same variance of preferences,  $\sigma_{fs}^2 = 1$ , and the same variance in state-specific shocks,  $\sigma_s^2$ .<sup>12</sup> Further assume that the predicted mean of the ideological preference distribution,  $\mu_{st}$ , depends linearly on a set of observable variables

$$\mu_{st} = x_{1st}\beta_1 + x_{2st}\beta_2 + \dots + x_{Kst}\beta_K = X_{st}\beta.$$

We then get the following estimable equation:

$$(17) \quad \gamma_{st} = -(X_{st}\beta + \eta_{st} + \eta_t).$$

<sup>12</sup> These assumptions will be removed in Section III. The estimates become imprecise, however, if separate values of  $\mu_{st}$ ,  $\sigma_{fs}$ , and  $\sigma_s$  are estimated for each state using only 15 observations per state. Therefore, the more restrictive specification will be used for most of the paper.

The parameters  $\beta$ ,  $\sigma_s$ , and  $\sigma$  are estimated using a standard maximum-likelihood estimation of the time random-effects model above.<sup>13</sup>

The variables in  $X_{st}$  are basically those used in Campbell (1992). Table A1 in the Appendix lists all variable definitions and sources, and Table 1 displays the summary statistics. The nationwide variables are: the Democratic vote share of the two-party vote share in trial-heat polls from mid-September (all vote-share variables  $x$  used in the transformed form,  $\Phi^{-1}(x)$ ); the lagged Democratic vote share of the two-party vote share; second-quarter economic growth; incumbency; and incumbent president running for reelection. The statewide variables for 1948–1984 are: lagged and twice lagged difference from the national mean of the Democratic two-party vote share; the first quarter state economic growth; the average ADA-scores of each state’s members of Congress the year before the election; the Democratic vote-share of the two-party vote in the midterm state legislative election; the home state of the president; the home state of the vice president; and dummy variables described in Campbell (1992). After 1984, state-level opinion polls were available. For this period, the statewide variables are: lagged difference from the national mean of the Democratic vote share of the two-party vote share; the average ADA-scores of each state’s members of Congress the year before the election; and the difference between the state and national polls. Other statewide variables were insignificant when state polls were included. The coefficients  $\beta$  and the variance of the state-level popularity shocks,  $\sigma_s^2$ , are allowed to differ when opinion polls were available and when they were not. Estimates of equation (17) yield forecasts by mid-September of the election year.

The dataset contains state elections for the 50 states for 1948–2004, except Hawaii and Alaska, which began voting in the 1960 election; see Table A2. During this period there were a total of 744 state-level presidential election results. Four elections in Alaska and Hawaii were excluded because there were no lagged vote returns. Nine elections were omitted because of idiosyncrasies in presidential voting in Alabama in 1948 and 1964, and in Mississippi in 1960; see Campbell (1992). This leaves a total of 731 observations.

The estimation results are shown in Table 2. The statistics of main interest to us are:

$$(18) \quad \begin{aligned} \hat{\sigma}_{s, \text{state polls}} &= 0.073, \\ \hat{\sigma} &= 0.035, \end{aligned}$$

and

$$(19) \quad \hat{\mu}_{st} = X_{st} \hat{\beta},$$

where  $\hat{\beta}$  is the vector of estimated coefficients in Table 2, and  $\hat{\sigma}_s$  is the estimated standard deviation of the state level shocks after 1984. Although the prediction errors are correlated across states, the correlation is far from perfect. The standard error of the independent state shocks corresponds to around 2.9 percent in terms of vote shares, while the standard error in the national swings corresponds to around 1.7 percent.<sup>14</sup>

<sup>13</sup> The model has been extended to include regional swings; see Appendix 6.4 of Strömberg (2002). In this specification, the democratic vote-share in state  $s$  equals

$$y_{st} = F_{st}(\eta_{st} + \eta_{rt} + \eta_t),$$

where  $\eta_{rt}$  denotes independent popularity shocks in the Northeast, Midwest, West, and South. However, taking into account the information of September state-level opinion polls, there are no significant regional swings. Therefore, the simpler specification without regional swings is used below.

<sup>14</sup> Computed at  $\hat{\mu}_{st} = 0$ , we get  $\Phi(0.073) - \Phi(0) = 0.029$ .

TABLE 1—SUMMARY STATISTICS

Variable	Obs.	Mean	Std. dev.	Min.	Max.
Democratic two-party state vote	731	47.22	9.51	20.00	86.40
Democratic two-party national vote	731	48.51	5.71	38.20	61.30
National trial-heat polls	731	49.36	8.64	37.10	69.20
State trial-heat polls	212	51.51	8.83	21.98	75.00
2nd-qr. GNP growth $\times$ incumbent party	731	-0.27	1.28	-2.35	1.78
1st-qr. state growth $\times$ incumbent party	731	0.00	0.99	-4.88	5.66
Incumbent President	731	-0.08	1.00	-1.00	1.00
Elected incumbent seeking re-election	731	-0.07	0.77	-1.00	1.00
Presidential home state advantage	731	0.00	0.18	-1.00	1.00
Vice-presidential home state advantage	731	-0.00	0.20	-1.00	1.00
State liberalism, deviation from mean	731	43.77	21.05	0.00	97.50
State legislature party division, inv. normal	731	57.20	22.35	0.88	100.00
Presidential home region (southern advantage)	731	0.06	0.24	0.00	1.00
Southern state (1964)	731	0.01	0.11	0.00	1.00
Deep South state (1964)	731	0.00	0.06	0.00	1.00
New England state (1960 & 1964)	731	0.02	0.13	0.00	1.00
Rocky Mountain West state (1976 & 1980)	731	0.03	0.16	0.00	1.00
North Central state (1972)	731	0.01	0.11	0.00	1.00
TV-advertisements	75	2,311	2,346	0	7,192
TV-advertisement costs	75	1.56e+06	2.15e+06	0	1.10e+07

The model does well in predicting the vote-share outcomes. The average absolute error in state-election vote-forecasts,  $|\hat{y}_{st} - y_{st}|$ , where

$$(20) \quad \hat{y}_{st} = \Phi(-\hat{\mu}_{st}),$$

is 3.0 percent and the wrong winner is predicted in 13 percent of the state elections. This is comparable to the best state-level election-forecast models (Campbell 1992; Gelman and King 1993; Thomas M. Holbrook and Jay A. DeSart 1999; Steven J. Rosenstone 1983). Table 3 shows the forecasted vote outcomes,  $\hat{y}_{st}$ , for the last two elections. Given  $\hat{\mu}_{st}$ ,  $\hat{\sigma}_s$ , and  $\hat{\sigma}$ ,  $Q_s$  can be calculated using equation (14). Table 3 also reports the computed values of  $Q_s$  for the last two elections, multiplied by the constant  $2\sqrt{2\pi}$ , for reasons explained below.

### III. Relation between $Q_s$ and Actual Campaigns

This section discusses how the actual allocation of presidential candidate visits to states in the two last presidential elections corresponds to the theoretical equilibrium campaigns. Both sides' actual post-convention campaign visits in 2000 and 2004 are reported in Table 3. The visit data were collected by Daron Shaw and are described in Shaw (2007). I have excluded the early September visits, since I use polling information from mid-September. Visits by vice-presidential candidates are counted as 0.5 presidential candidate visits. For example, in 2000, Bush visited Florida seven times while Cheney visited six times. Consequently, the weighted number of Republican visits to Florida in 2000 is ten. The correlation between the Republican and Democratic campaigns' visits is 0.9 in both elections.

If one assumes that the impact of visits,  $u(d_s)$ , is of log form, then the equilibrium number of days spent in each state should be proportional to  $Q_s$ ,

$$(21) \quad \text{state } s \text{ share of equilibrium visits} = \frac{d_s^*}{\sum d_s^*} = \frac{Q_s}{\sum Q_s},$$

TABLE 2—DEPENDENT VARIABLE:  $\gamma_{st}$ ,  $\Phi^{-1}$  (DEMOCRATIC SHARE OF TWO-PARTY VOTE)

<i>National variables</i>	
$\Phi^{-1}$ (national trial-heat poll)	0.508 (0.060)***
$\Phi^{-1}$ (Lagged Democratic vote share)	0.379 (0.165)**
2nd-qtr. GNP growth $\times$ incumbent party	0.057 (0.011)***
Incumbent president	-0.102 (0.034)***
Elected incumbent seeking reelection	0.08 (0.029)***
Constant	-0.034 (0.011)***
<i>State variables when state polls not available</i>	
$\Phi^{-1}$ (Lagged state vote)	0.267 (0.032)***
$\Phi^{-1}$ (Twice lagged state vote)	0.22 (0.025)***
Presidential home state advantage	0.175 (0.028)***
Vice-presidential home state advantage	0.056 (0.023)**
1st-qtr. state growth $\times$ incumbent party	0.017 (0.005)***
State liberalism, deviation from year mean	0.002 (0.000)***
$\Phi^{-1}$ (State legislature party division)	0.019 (0.007)**
Presidential home region (southern advantage)	0.179 (0.026)***
Southern state (1964)	-0.219 (0.047)***
Deep South state (1964)	-0.426 (0.073)***
New England state (1960 and 1964)	0.187 (0.032)***
Rocky Mountain West state (1976 and 1980)	-0.155 (0.027)***
North Central state (1972)	0.099 (0.038)***
<i>State variables when state polls available</i>	
$\Phi^{-1}$ (Lagged state vote)	0.531 (0.056)***
State liberalism, deviation from mean	0.0009 (0.0003)***
State trial-heat polls	0.386 (0.045)***
<i>Variance</i>	
$\sigma$	0.035 (0.007)***
$\sigma_s$ , state polls	0.073 (0.004)***
$\sigma_s$	0.102 (0.003)***
Observations	731

Note: Standard errors in parentheses.  $\Phi^{-1}$  denotes the inverse of the standard normal cumulative distribution function,  $\Phi$ .

\*\*\* Significant at the 1 percent level.

\*\* Significant at the 5 percent level.

\* Significant at the 10 percent level.

TABLE 3

State	2000					2004				
	<i>Forc. vote</i>	$\frac{Q_s^*}{2\sqrt{2\pi}}$	<i>Rep. visits</i>	<i>Dem. visits</i>	<i>Decisive swing st.</i>	<i>Forc. vote</i>	$\frac{Q_s^*}{2\sqrt{2\pi}}$	<i>Rep. visits</i>	<i>Dem. visits</i>	<i>Decisive swing st.</i>
Florida	49.5	4.65	10	15.5	4.53	45.3	0.57	27.5	24.5	0.52
Michigan	51.4	3.57	14.5	9	3.61	49.3	0.26	11.5	4	0.18
California	55.3	3.31	13	5.5	2.72	52.1	0.00	0	1	0.07
Pennsylvania	53.3	3.16	14.5	9	3.18	47.0	0.48	19	5.5	0.34
Ohio	48.3	2.89	9.5	5.5	2.79	43.9	0.26	23.5	17.5	0.24
Missouri	49.5	2.01	11	11	1.95	44.3	0.14	6	1.5	0.13
Tennessee	49.5	2.00	6.5	4	1.92	42.0	0.05	0	0	0.05
Wisconsin	52.5	1.81	9	9	1.77	46.9	0.21	16.5	12.5	0.16
Washington	52.7	1.72	7	5	1.71	50.0	0.15	0	0	0.10
Oregon	51.5	1.32	5.5	5	1.26	46.8	0.14	2.5	1.5	0.11
Louisiana	48.2	1.25	3.5	3	1.21	40.5	0.01	0.5	0	0.01
Iowa	52.1	1.23	7.5	9	1.13	46.0	0.13	14	13	0.11
Illinois	55.4	1.20	12	8	1.19	53.0	0.04	0.5	1	0.03
Arkansas	51.4	1.14	4.5	2.5	1.08	44.2	0.07	0.5	0	0.05
Kentucky	48.1	1.08	3.5	3	1.03	38.3	0.00	0	0	0.00
New Mexico	51.2	0.96	5	4	0.86	48.0	0.10	5	5	0.08
North Carolina	46.0	0.77	2.5	1	0.77	42.5	0.09	1	0.5	0.09
New Hampshire	49.4	0.72	3	2	0.63	46.6	0.08	4.5	2.5	0.05
Georgia	45.9	0.66	0.5	2	0.65	40.7	0.03	0.5	0	0.03
Nevada	48.5	0.60	1.5	1.5	0.54	43.9	0.05	5	2	0.04
Arizona	46.6	0.60	1	0	0.58	42.5	0.06	2	3	0.05
Colorado	46.5	0.58	0.5	0	0.55	42.6	0.05	5.5	2	0.05
Minnesota	55.3	0.58	2	1.5	0.58	48.2	0.20	9.5	4	0.15
Delaware	50.1	0.58	0.5	0.5	0.49	52.7	0.02	0	0	0.01
Connecticut	55.4	0.43	0	0	0.43	55.4	0.01	0	0	0.00
Mississippi	45.1	0.40	0	0	0.38	37.0	0.00	0	0	0.00
New Jersey	56.7	0.37	1	1.5	0.37	53.6	0.03	1.5	1	0.02
Virginia	44.8	0.34	0	0	0.34	42.6	0.08	0	0	0.08
West Virginia	55.1	0.32	3	1	0.29	46.5	0.10	3.5	0	0.07
Indiana	44.2	0.21	0	0	0.22	39.7	0.01	0	0	0.01
Maryland	57.6	0.12	0	0	0.13	54.1	0.02	0	0.5	0.01
Maine	58.0	0.04	3	2.5	0.04	49.8	0.06	1	0.5	0.04
North Dakota	43.3	0.03	0	0	0.03	34.0	0.00	0	0	0.00
Alaska	41.0	0.03	0	0	0.03	30.2	0.00	0	0	0.00
Montana	42.9	0.02	0	0	0.02	33.2	0.00	0	0	0.00
South Dakota	42.7	0.02	0	0	0.02	37.1	0.00	0	0	0.00
Alabama	41.4	0.01	0	0	0.02	37.4	0.00	1	0	0.00
Wyoming	39.0	0.01	0	0	0.01	27.0	0.00	0.5	0	0.00
South Carolina	40.7	0.01	0	0	0.01	39.7	0.01	0	0	0.01
Vermont	60.4	0.00	0	0	0.00	53.2	0.01	0	0	0.01
New York	63.6	0.00	0	0	0.00	56.8	0.00	4	0	0.00
Hawaii	63.6	0.00	0	0	0.00	55.6	0.00	0.5	0	0.00
Oklahoma	37.0	0.00	0	0	0.00	34.6	0.00	0.5	0	0.00
Nebraska	36.9	0.00	0	0	0.00	31.0	0.00	0	0	0.00
Rhode Island	64.4	0.00	0	0	0.00	62.7	0.00	0	0	0.00
Kansas	36.6	0.00	0	0	0.00	38.5	0.01	0	0	0.01
Idaho	35.6	0.00	0	0	0.00	28.3	0.00	0	0	0.00
Massachusetts	66.6	0.00	0	0	0.00	64.4	0.00	0	0	0.00
Texas	34.8	0.00	0	0	0.00	35.7	0.00	4	0	0.00
Utah	33.9	0.00	0	0	0.00	26.7	0.00	0	0	0.00
Washington, DC	87.0	0.00	0	0	0.00	87.0	0.00	1.5	0	0.00

Notes: *Forc. vote* reports the forecasted vote,  $\Phi(-X\beta)$ ; see equation (20).  $Q_s$  is computed using equation (13). To be comparable to *Decisive swing state*,  $Q_s$  has been multiplied by  $2\sqrt{2\pi} \approx 5.013$ , as explained in footnote 32. *Decisive swing state* is the percent of one million simulated elections that the state was decisive in the Electoral College and the state election win margin was less than 2 percent. *Dem. (Rep.) visits* is the number of presidential candidate visits +  $0.5 \times$  the number of vice presidential visits by the Democrats (Republicans).

using equation (11). For the elections in 2000 and 2004, each state's equilibrium share of visits was computed using the equation above and the estimated  $Q_s$ .

The actual and equilibrium shares are shown in Figure 1, where both sides' campaign visits have been added together. For example, the actual campaigns increased the share of their visits spent in Florida from 9 percent in 2000 to 19 percent in 2004. In comparison,  $Q_{Florida}/\sum Q_s$  increased from 11 percent in 2000 to 16 percent in 2004.

The actual campaigns closely resemble the model's equilibrium campaigns based on September opinion polls. The model and the candidates' actual campaigns agree on eight of the ten states that should receive most attention in both 2000 and 2004. The model and the data are also, largely, in agreement on which states should receive basically no attention. The simple correlation between equilibrium and actual visit shares is 0.9 in both years. A tougher comparison is that of campaign visits per electoral vote. The correlation between  $d_s/e_s$  and  $Q_s/e_s$  was 0.8 in both years.

A noticeable discrepancy is that the model underpredicts the number of visits to some mid-western states: Illinois, Iowa, Wisconsin, and Missouri in 2000, and Iowa, Wisconsin, and Ohio in 2004. Two plausible explanations are that my model does not include travel costs and that the campaigns use polling data of better quality and of later date than mid-September. Other factors the model does not account for are the geographic location of the campaign headquarters and the perceived effectiveness of campaigns in different states, depending, for example, on the support from governors in the campaign. (I talked extensively about this with campaigners, in particular, Daron Shaw who worked on the Republican side in the 2000 election, and Samuel Popkin who worked on the Democratic side in the 1992, 1996, and 2000 elections.)

There is, of course, considerable persistence in candidate attention to states from one election to the next. Florida, Pennsylvania, Michigan, and Ohio are at the top in both elections. Still, while California and Tennessee, for example, were high on the list in 2000, they had fallen considerably in 2004. The model also explains these changes reasonably well. The correlation between the changes in equilibrium and actual visit shares between these two years is 0.6.

Obviously, candidates would like to visit large states more frequently. However, size is not everything, as is apparent from the many large states found at the bottom of Figure 1. An obvious reason for this is that some states, like Texas and New York, are clearly in one candidate's column and that campaigning is not likely to affect this; see Colanti, Levesque, and Ordeshook (1975). Candidates should concentrate on close races. Consistent with this idea, Florida was forecasted to have a 49.5 percent Democratic vote share in 2000 and received the most attention. However, although the forecasted Democratic vote share in California was closer to 50 percent in 2004 than in 2000 (52 percent compared to 55 percent), much less attention should be (and was) given to California in 2004. The next section explains why. Finally, the distribution of visits seems more concentrated in 2004 than in 2000. The five top states (in terms of visits) received 41 percent of all visits in 2000 and 63 percent in 2004. The next section discusses what determines the skewness of the electoral incentives.

The working paper version of this paper also analyzes the allocation of TV advertisements in the 2000 election. The dataset records a total of 174,851 advertisements that were aired across the 75 largest media markets at a total cost of \$118 million. Analyzing this problem adds two complications: advertisement prices differ and media markets cross state boundaries. The model shows that equilibrium advertisements should be proportional to the population-weighted  $Q_s$ , divided by the price of advertising,  $Q_m/p_m$ . Figure 2 plots the equilibrium and actual advertisements. The model and the data agree on the two media markets where most ads should be aired (Albuquerque–Santa Fe and Portland, Oregon). These two markets have the highest effect on the win probability per initial advertising dollars. The correlation between actual campaign advertisement and equilibrium advertisement is 0.8. Finally, since the correlation between price and

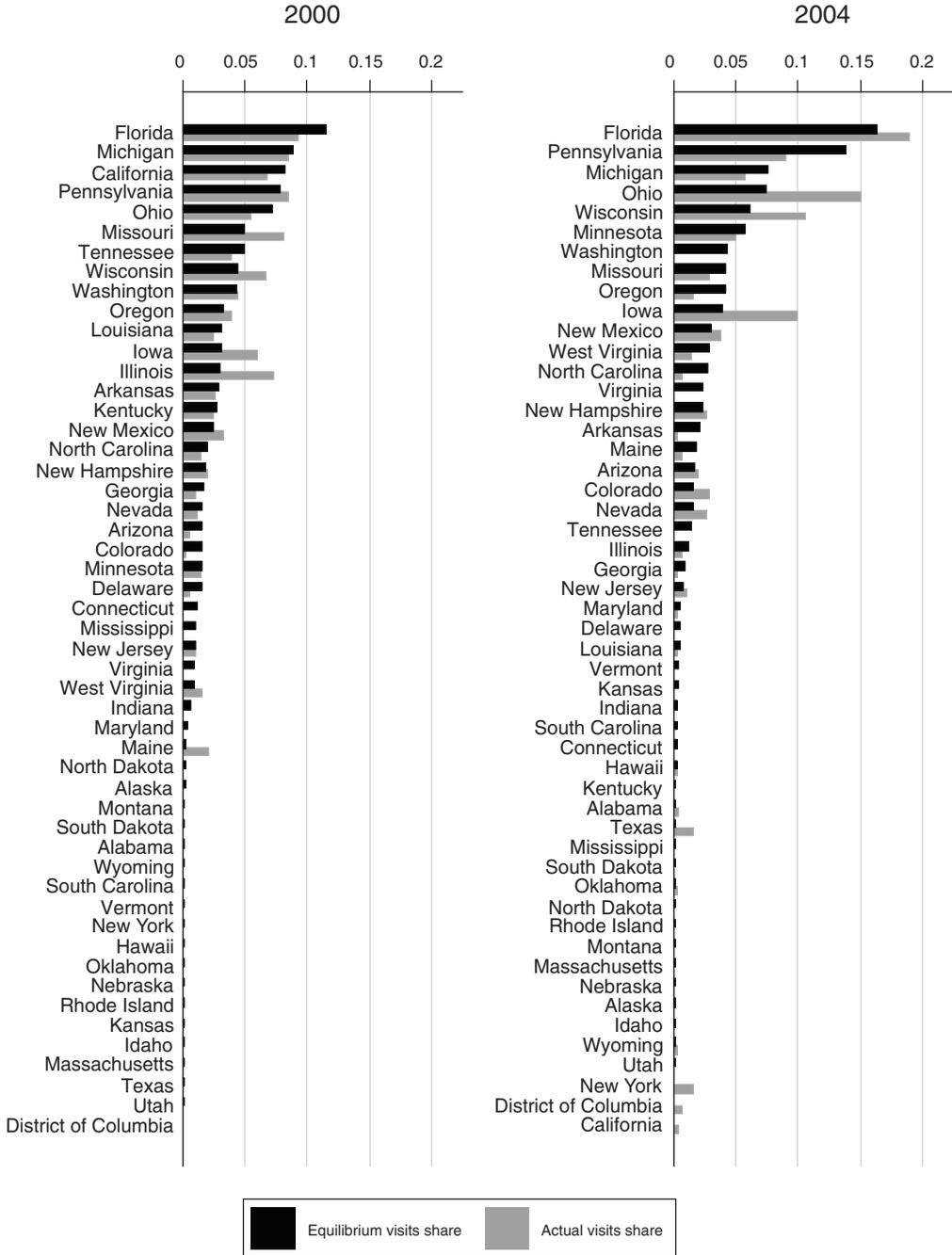


FIGURE 1. ACTUAL AND EQUILIBRIUM SHARES OF CAMPAIGN VISITS

market size is close to one (0.92), there is no significant relationship between market size and the number of ads. Market size instead affects the costs, since prices are proportional to this size. Assuming log utility, equilibrium expenditures are proportional to  $Q_m$ . Empirically, the simple correlation between equilibrium and actual advertisement expenditures is 0.9.

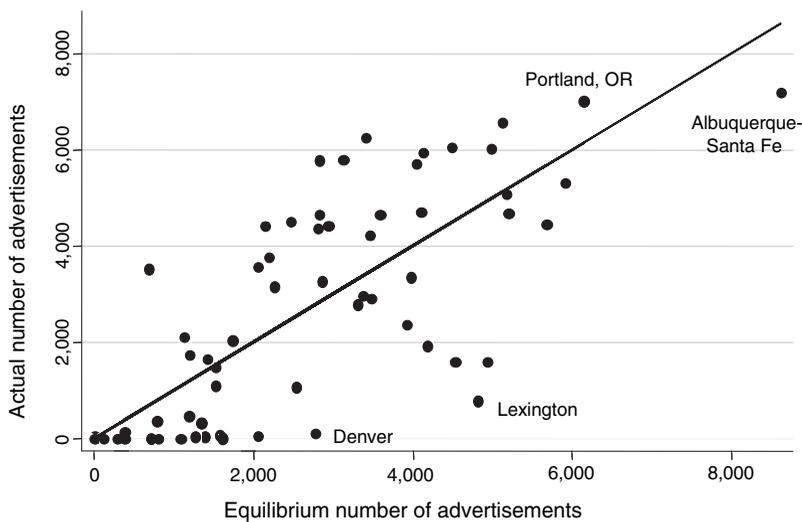


FIGURE 2. EQUILIBRIUM AND ACTUAL NUMBER OF ADVERTISEMENTS (SEPTEMBER 1 TO ELECTION DAY) FOR THE 75 LARGEST MEDIA MARKETS IN 2000

A stricter comparison of the equilibrium and actual visit shares is shown in Table 4. The first column contains the results from an OLS regression of actual visit shares in 2000 and 2004 on equilibrium visit shares, that is, estimations of the equation

$$(22) \quad \frac{d_s}{\sum d_s} = \beta \frac{Q_s}{\sum Q_s} + \varepsilon.$$

As predicted by equation (21), the coefficient on equilibrium visits is significantly different from zero, but not significantly different from one. The second column includes a number of variables that seem intuitively likely to be correlated with candidate visits: the number of electoral votes, the closeness of the presidential elections at the state level (measured as  $50 - |\% \text{ Democratic votes} - 50|$ ), and the Democratic vote share in the election.<sup>15</sup> The number of electoral votes and the closeness of the elections are both significantly positively correlated with candidate visits. However, adding the equilibrium shares completely removes the significance of the electoral votes and the closeness of the elections, and the  $R$ -squared is 0.77 with or without them; see columns 3 and 1 of Table 4. The variables are neither individually nor jointly significant; the  $F$ -statistic for joint significance is 0.21 with three degrees of freedom. In other words, electoral votes and closeness are correlated with the actual visit shares only through their correlation with the equilibrium shares. The next three columns show that this is also true when explaining changes in visits, 2000–2004. However, the number of electoral votes and the closeness of the elections do not explain much of the change in visits, even without controlling for equilibrium visits (the number of electoral votes is different in 2000 and 2004 because of adjustments due to the new

<sup>15</sup> A problem here is that the Democratic vote share is endogenous to campaigning. I use the actual vote share, since it is a frequently used measure of the closeness of elections; see, for example, Colantoni, Levesque, and Ordeshook (1975). Results are similar when using the expected Democratic vote share. I also tried including the interaction between the number of electoral votes with the closeness of the election. This makes little difference; the  $R$ -squared in column 3 remains at 0.77.

TABLE 4—OLS REGRESSIONS ON CANDIDATE VISIT SHARES, 2000 AND 2004

	Levels			Changes 2004–2000		
	(1)	(2)	(3)	(4)	(5)	(6)
$Q_s$ share	1.00 (0.05)**		0.97 (0.07)**	0.89 (0.17)**		0.93 (0.17)**
No. electoral votes		0.50 (0.17)**	0.02 (0.10)		2.08 (2.69)	3.36 (2.14)
Close state election		3.97 (0.87)**	0.37 (0.55)		2.15 (3.46)	2.75 (2.74)
Dem. vote share		0.76 (0.67)	0.14 (0.38)		2.21 (4.25)	−1.46 (3.43)
Observations	102	102	102	51	51	51
$R$ -squared	0.77	0.29	0.77	0.37	0.03	0.41

Notes: All variables have been rescaled, dividing by the sum of the variable in that year. The variable “close state election” equals  $50 \cdot [ \% Democratic\ votes - 50 ]$ . Standard errors in parentheses.

\*\*\* Significant at the 1 percent level.

\*\* Significant at the 5 percent level.

population estimates in the 2000 Census). The  $R$ -squared in column 5 is 0.03 and the included variables are jointly and individually insignificant.

To sum up, equilibrium visit shares are strongly correlated with actual visits shares, with an estimated coefficient of around one. This is true for explaining both cross-sectional variation and changes over time. Once  $Q_s$  is added to the regression, the other variables do not contribute to explaining visits at all; in other words,  $Q_s$  seems to be a sufficient statistic in explaining candidate visits.

Is there any reason to believe that the estimated correlation between actual and equilibrium visits is spurious or biased? This cannot be ruled out, as  $Q_s$  was not randomly assigned to states. Still, it is not easy to find alternative explanations for the fact that changes in equilibrium visits between 2000 and 2004 are correlated with changes in actual visits, controlling for changes in the closeness of the state election and the number of electoral votes. A subtle bias could arise if the campaigns use better information and, consequently, face less uncertainty regarding vote outcomes (smaller  $\sigma_s$  and  $\sigma$ ) than our measures. This will affect the whole distribution of equilibrium visits; Section IV explains how. This implies that our measure of  $Q_s$  contains a classical measurement error, presumably producing a downward attenuation bias.

Finally, I will discuss three issues in matching the theory to reality: candidates may have unequal budgets; the influence of campaigning on votes may be better described by a function that is not logarithmic; and campaigning effects may be heterogeneous across states. Starting with the first, the model can also be solved using unequal candidate budgets. Suppose that the candidates have unequal budgets,  $I^D \neq I^R$ , and that  $u(d_s)$  is logarithmic,  $u(d_s) = \theta \log d_s$ . The first-order conditions for maximizing the probability of winning are:

$$d_s^J = \frac{Q_s}{\sum_{s'=1}^S Q_{s'}} I^J, \quad J = D, R.$$

Both candidates spend the same share of their budget in each state, but the party with the larger budget spends proportionally more everywhere. This implies that  $Q_s$  will now be evaluated at  $\Delta u_s = \theta \log(I^D/I^R)$ , rather than at  $\Delta u_s = 0$ , as was previously the case. In the estimation,  $\Delta u_s$  will end up in the constant in the national variables in Table 2 if it is constant over time. If it is time varying and not measurable, it will be part of the national popularity shock.

Second, one can use a more flexible functional form for the influence of campaign visits on votes, such as

$$u(d) = \frac{\theta}{1 - \alpha} d_s^{1-\alpha},$$

where  $\alpha, \theta > 0$ . Ideally, one would like to estimate the parameters of this function based on the observed impact of visits on votes; and the working paper version of this paper discusses how. In practice, however, estimates will be very imprecise, since the difference between the two candidates' number of visits to states is typically not large. Another approach is to estimate  $\alpha$  based on the observed relationship between  $Q_s$  and candidate visits. Equilibrium visits are now

$$\frac{d_s^*}{\sum_{s'=1}^S d_{s'}^*} = \frac{Q_s^\alpha}{\sum_{s'=1}^S Q_{s'}^\alpha}.$$

Taking logarithms and adding an error term, one may estimate the equation

$$(23) \quad \log d_s = \text{const} + \alpha \log Q_s + \varepsilon.$$

Regressing log candidate visits in 2000 and 2004 on log  $Q_s$  and including a year fixed effect, the estimated coefficient  $\alpha$  is 1.1 with a standard error of 0.2.<sup>16</sup> This is clearly consistent with our baseline assumption of a logarithmic vote influence function, implying  $\alpha = 1$ .

Third, note that the model implies heterogeneous effects of campaigning on vote shares. The impact of campaign visits on vote shares is largest in states where the election is close (like Florida in 2000), than in states with lop-sided races (like Utah, Nebraska, and the Dakotas). This is a direct consequence of the assumption that voter preferences are normally distributed within states, since the normal density is highest at 50-50.

#### IV. Interpretation

Because of the very high correlation between the two, understanding why  $Q_s$  varies goes a long way toward understanding why resource allocation varies across states. This section first analyzes what  $Q_s$  represents. It goes on to describe in detail why some states have large values of  $Q_s$ , and what determines whether the distribution of  $Q_s$  is very equal or unequal. These questions can be answered precisely, since we have an analytical expression for  $Q_s$ .

##### A. What is $Q_s$ ?

A qualified guess is that  $Q_s$  approximately measures the joint probability (strictly speaking, the likelihood) of two events: that a state is, ex post facto, (i) *decisive* in the Electoral College and (ii) a *swing state* in the sense of having a very close state-level election,

$$(24) \quad Q_s \approx \Pr(\text{decisive swing state}).$$

<sup>16</sup> An issue here is that the sample includes only states where  $d_s$  is greater than zero, creating a sample selection problem. For this reason, a truncated regression was used.

Candidates want to reach voters whose votes can be pivotal in the final outcome. For this reason, they will concentrate on states where the outcome is close (*swing states*). This is a reason why Florida, where the race was close, received a lot of attention in 2000, while New York and Texas received very little. But closeness is not the only aspect that determines whether a vote will be pivotal in the national election. The electoral votes of the state must also be *decisive* in the Electoral College in the sense that, ex post facto, moving a state from one candidate's column to the other's changes the national outcome. For example, in 2000, Bush won by 271 to 266 electoral votes. The margin was so close that all (and only) the 28 states that voted for Bush were *decisive* in the Electoral College. Had Utah, or Florida, or any other of the Bush states, voted for Gore, then Gore would have won. Of these 28 states, however, only Florida satisfied criterion (ii) of being an ex post facto swing state with a very close state election. On the other hand, New Mexico was a swing state in 2000 as Gore won only 50.03 percent of the two-party vote, but not decisive in the Electoral College. Ex post facto, Bush would have received a majority in the Electoral College, irrespective of the outcome in New Mexico. Of course, we make these categorizations after knowing the outcome, something that was not possible for Bush and Gore when they planned their campaigns in the fall of 2000. Instead, candidates must make their plans based on the probability of states being *decisive swing states* based on information available in the fall. We will show that these probabilities are what the  $Q_s$  measure.

The conjecture above regarding the interpretation of  $Q_s$  is based on the fact that the probability of being a decisive swing state replaces  $Q_s$  in the equilibrium condition of the model without the Central Limit approximation. It is also intuitive in that only in decisive swing states will a marginal change in voter support alter the outcome of the national election. Further, Appendix C shows analytically why  $Q_s$  approximates the probability of being a decisive swing state.

To investigate this guess, one million electoral vote outcomes were simulated for the elections in 2000 and 2004 by using the predicted state-preference means, and by drawing state and national popularity-shocks from their estimated distributions.<sup>17</sup> Then, the share of elections where a state was decisive in the Electoral College and at the same time had a state election outcome between 49 and 51 percent was recorded. The results are reported in Table 3, in the columns labeled *Decisive swing st.* These simulated probabilities should roughly be equal to  $Q_s$ , multiplied by  $2\sqrt{2\pi}$  (Appendix C explains the scaling factor). The simple correlation between these simulated probabilities and  $Q_s$  is 0.998. This correlation does not arise simply because large states get more visits. The correlation between  $Q_s$  per electoral vote and the simulated probabilities per electoral vote is still 0.998. So  $Q_s$  and the probability of being a decisive swing state are interchangeable, for practical purposes. The 0.002 difference could result on the  $Q_s$ -side from using the approximate probability of winning the election, and on the simulation-side from using a finite number of simulations and recording state election results between 49 and 51 percent, whereas theoretically it should be exactly 50 percent.

Another interpretation of  $Q_s$  provides a link to the literature concerning "voting power," that is, the probability that one vote decides the election.<sup>18</sup> In Appendix C, it is shown that

$$(25) \quad Q_s \approx (\# \text{ votes cast})_s \times (\text{"voting power"})_s \\ \times (\text{marginal voter density, conditional on tied state election})_s.$$

<sup>17</sup> Replace  $\mu_{st}$  by the estimated  $\hat{\mu}_{st}$  in equation (15), and draw  $\eta_{st}$  and  $\eta_t$  from their estimated distributions  $N(0, \hat{\sigma}_t^2)$  and  $N(0, \hat{\sigma}^2)$ , respectively, to generate election outcomes  $y_{st}$ .

<sup>18</sup> See, for example, Banzhaf (1968), Merrill (1978), Chamberlain and Rothschild (1988), Gelman, King, and Boscardin (1998), and Gelman and Katz (2001).

This shows that candidates in a probabilistic-voting model who act strategically to win the election will allocate more resources to places with higher voting power. In contrast, the previous literature on probabilistic voting has focused on the marginal voter density in explaining government resource allocations (see Lindbeck and Weibull 1987, or Persson and Tabellini 2000). This is natural, since that literature dealt primarily with direct national elections. (See equation (29) for the equivalent interpretation of  $Q_s$  under direct elections.)<sup>19</sup>

### B. The Effect of Electoral Votes on Influence

The analytic form of  $Q_s$  helps us understand why some states get more resources. A first important observation is that  $Q_s$  is roughly proportional to the number of electoral votes.<sup>20</sup> This is evident from the definition of  $Q_s$  in equation (13), in combination with the fact that the first term, labeled  $Q_{s\mu}$ , is considerably larger than the second term,  $Q_{s\sigma}$  (see below). Since campaign visits seem to be allocated proportionally to  $Q_s$ , this implies that campaign resources are increasing in proportion to the number of electoral votes.

This implies that small states are advantaged. Since all states have at least three electoral votes, small states have many electoral votes per capita. At the extremes, in 2000, Wyoming had 6.1 electoral votes per million, and California only 1.6. For this reason, small states should get more resources per capita. Small states are, however, disadvantaged for another reason. To see this, one must look at the distribution of  $Q_s$  per electoral vote. The following two subsections will, in turn, discuss the two additively separable parts of  $Q_s/e_s$  defined in equation (13):

$$Q_s/e_s = Q_{s\mu}/e_s + Q_{s\sigma}/e_s.$$

### C. Influence per Electoral Vote and Forecasted Vote Shares ( $Q_{s\mu}/e_s$ )

The attention that states with similar numbers of electoral votes get depends crucially on the forecasted Democratic vote shares, but in a nonobvious way. This is illustrated in Figure 3. The dots show the estimated probabilities that a state is decisive in the Electoral College and simultaneously has a state-election outcome between 49 and 51 percent, per electoral vote. The graph also includes a solid line plotting the values of  $Q_{s\mu}$  per electoral vote, defined in equation (13). Recall that  $Q_{s\mu}$  is the part of  $Q_s$  that arises because the candidates try to affect the mean number of electoral votes. As Figure 3 shows, the curve accounts for most of the variation across states. Since states like New York and Texas are far out in the tails of this curve, in a million simulated elections, they were never decisive in the Electoral College and simultaneously had a state-election outcome between 49 and 51 percent. Consequently, the candidates can safely ignore them. States like Florida, Michigan, Pennsylvania, and Ohio are close to the center of this distribution, and candidates should pay them frequent visits. The solid line is, in fact, closely approximated by a normal distribution, multiplied by a constant (see Appendix D). It can therefore be characterized by three features: its *amplitude*, its *mean*, and its *variance*.

<sup>19</sup> In an extension, however, Lindbeck and Weibull (1987) show that candidates who maximize the probability of winning in a direct national election should promise more spending targeted to voters with high “voting power” if this is heterogeneous across voters.

<sup>20</sup> This result is consistent with “voting power” being proportional to the number of electoral votes. This has been argued theoretically by Chamberlain and Rothschild (1981) and empirically by Gelman and Katz (2001). It is inconsistent with “voting power” being more than proportional to size. Banzhaf (1967) and Brams and Davis (1974) find this result in the special case that each state votes for either candidate with probability ½.

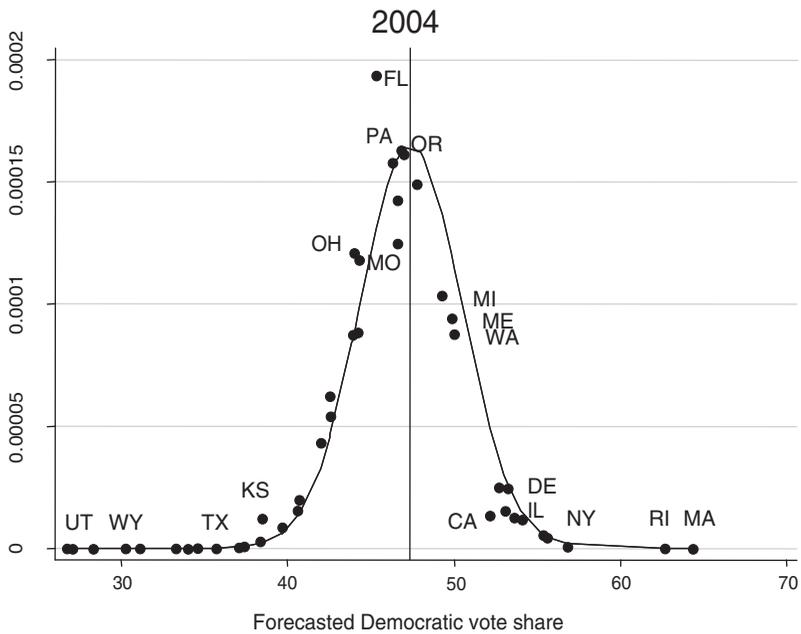
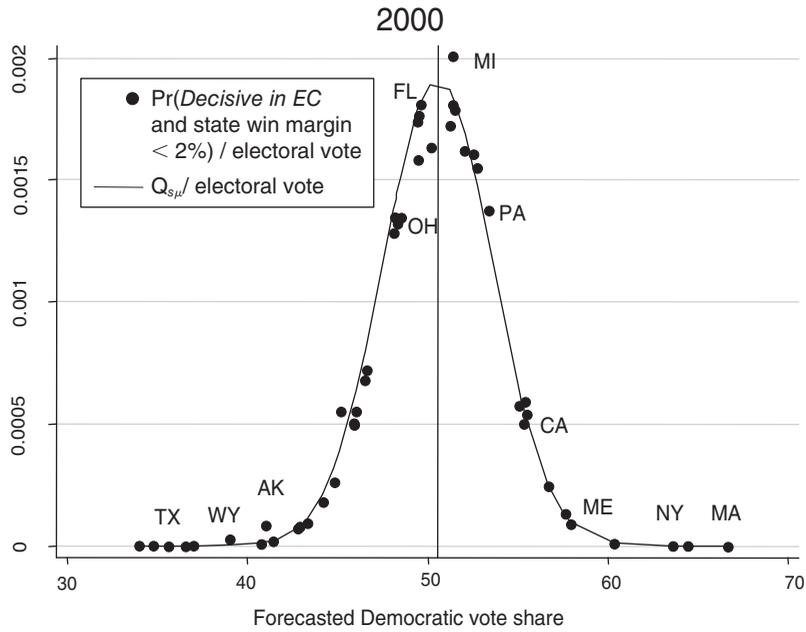


FIGURE 3. EQUILIBRIUM RESOURCE ALLOCATION PER ELECTORAL VOTE AS A FUNCTION OF FORECASTED DEMOCRATIC VOTE SHARES ( $\hat{y}_{i,t}$ )

*Amplitude.*—The amplitude denotes the general height of the distribution and is increasing in the expected closeness of the election. It affects all states in a single election in the same way, and explains why the average probability of being a decisive swing state varies between elections. For example,  $Q_{s\mu}$  reaches ten times higher values in 2000 than in 2004, since the former election was expected to be closer. Since differences in amplitude affect all states in the same way, they do not affect the relative distribution of resources between states in any given election. The amplitude is important, however, because it affects the competitiveness of elections.

*Mean.*—The mean of the distribution is obviously important in characterizing which states get a lot of resources. As can be seen in Figure 3, this mean shifted in a pro-Republican direction between 2000 and 2004. This is one reason why California received less attention in 2004. Although the forecasted Democratic vote share in California was closer to 50 percent in 2004, it was slightly farther from this mean.

The position of this mean is the result of a trade-off between average, and timely, influence on state election outcomes. To get the intuition, consider the 1996 election. In September, Clinton was ahead by 60 to 40 nationally, and in Pennsylvania. Yet, in Texas the race was close. A visit to Texas was therefore more likely to affect the state outcome than a visit to Pennsylvania. This is clearly one reason for Clinton to go there rather than to Pennsylvania.

The existence of common national preference shocks, however, makes it likely that the outcome in Pennsylvania will be close exactly when the national election is close. Suppose, for example, that there is a sudden downturn of the national economy and Clinton is blamed for this, dropping his polls to 50-50 nationally. Then, the situation in Pennsylvania is also likely to be 50-50 since all states are affected by the shock, and Texas would go to Dole for sure. In general, if Texas is still a 50-50 state on election day, then chances are that Clinton is winning by a landslide, while if Pennsylvania is a 50-50 state on election day, then it is more likely that the national election is close. Therefore, although a visit influences the outcome in Texas more frequently, it may influence the outcome in Pennsylvania more when it matters.

More formally, note that

$$Q_s \approx \Pr[\textit{decisive swing state}] = \Pr[\textit{swing state}] \Pr[\textit{decisive} | \textit{swing state}].$$

The probability of being a swing state measures the average influence on the state elections, which is high for 50-50 states like Texas. The probability of a state's electoral votes being decisive, conditional on that state having a 50-50 election, is high for states like Pennsylvania. An extra visit matters only if the state is 50-50 on election day, and the candidates must condition their visits on this circumstance.<sup>21</sup>

The model shows how to strike a balance between average and timely influence. The less correlated the state election outcomes, the more time should be spent in 50-50 states like Texas.<sup>22</sup> The reason is that without national swings, the state outcomes are not correlated, and a state being a swing state on election day carries no information about the outcomes in the other states. In my estimates, maximum attention should typically be given to states approximately halfway between 50-50 and the forecasted national election outcome. In September of 2004, Bush was ahead by 4 percentage points. The maximum  $Q_{s\mu}$  per electoral vote was obtained for states where Bush was expected to win by 2.5 percentage points, as illustrated in Figure 3. We now end the

<sup>21</sup> Timothy J. Feddersen and Wolfgang Pesendorfer (1996) put forward similar arguments for voters in "The Swing Voter's Curse."

<sup>22</sup> Appendix C shows this analytically.

discussion of the position of the mean, which is helpful in identifying which states will get most attention, and look at the *variance* which affects the concentration of attention.

*Variance.*—Voters in states like Utah, Massachusetts, and Idaho did not see much of Bush and Kerry in person, while more than three of every five candidate visits in 2004 went to Florida, Ohio, Wisconsin, Iowa, and Pennsylvania. This concentration in visits is a consequence of the small variance of  $Q_{s\mu}$  per electoral vote. The smaller this variance is, the sharper the incentives are to target only a few states. This variance, in turn, depends on the variance in the state- and national-level popularity shocks, and the variance in the total electoral vote outcome.<sup>23</sup> For example, the increasing availability of state-level opinion polls has created sharper incentives for a more targeted resource allocation. This effect is illustrated in the left graph in Figure 4, which shows the distribution of  $Q_{s\mu}$  with and without opinion polls for the year 2000. States with forecasted vote shares close to the mean of  $Q_{s\mu}$  per electoral vote distribution gain from opinion polls, while states far from the center would lose.

#### D. Incentives to Affect the Variance in the Electoral Vote Outcome ( $Q_{s\sigma}/e_s$ )

There is another reason why California should receive less attention in 2004 than in 2000. By rescheduling visits from California to Florida, Kerry could increase the variance in the electoral vote outcome, and we will now explain why this was desirable to him. In general, candidates have incentives to influence the variance of the electoral vote distribution, even if this means decreasing the expected number of electoral votes. This is captured by the term  $Q_{s\sigma}$  in equation (13). This term explains why, in Figure 3, the simulated values for California lie slightly above the solid line in 2000 and considerably below the line in 2004. (In general, this term explains why states just to the right of the mean lie above the curve in 2000, while states just to the left of the mean lie below. In 2004, the situation is the reverse.)

To get the intuition of why such behavior is rational, consider the following example from the world of ice hockey. One team is trailing by one goal and there is only one minute left in the game. To increase the probability of scoring an equalizer, the trailing team pulls out the goalie and puts in an extra offensive player. Most frequently, the result is that the leading team scores. But the trailing team does not care about this, since they are losing the game anyway; they care only about increasing the probability that they score an equalizing goal, which is higher with an extra offensive player. Therefore, it is better to increase the variance in goals, even though this decreases net expected goals. In contrast, if they were allowed, the leading team would like to pull out an offensive player and put in an extra goalie.

Similarly, a presidential candidate who is behind should try to increase variance in the election outcome. He can do this by spending more time in large states where he is behind (putting in an extra offensive player), and less time in states where he is ahead (pulling the goalie). A candidate who is ahead should, instead, try to decrease variance in electoral votes, thus securing his lead, by spending more time in large states where he is ahead (putting in an extra goalie), and less time in states where he is behind (pulling out an offensive player). Both candidates thus spend more time in large states where the expected winner is leading.

To see why a trailing candidate increases the variance by spending more time in states with many electoral votes where he is behind, consider equation (8), which shows the variance, conditional on a national shock. The variance in the number of electoral votes from a state is proportional to these votes squared. Therefore, the effect on the total variance, per electoral

<sup>23</sup> See Appendix C for the analytical expression for the variance.

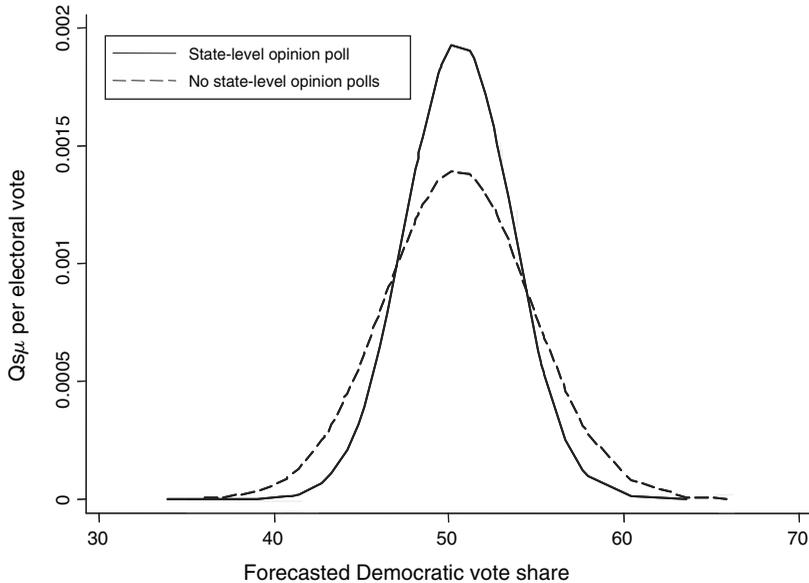


FIGURE 4. INFORMATION AND TARGETING OF RESOURCES, 2000 ELECTION

vote, is larger in large states. Further, the variance in a state outcome is higher the closer the expected result is to a tie. By visiting a state where the leading candidate is ahead, the trailing candidate moves the expected result closer to a tie, and increases the variance in election outcome. Similarly, decreasing the number of visits to a state where the lagging candidate is leading increases the variance.

Figure 5 illustrates this effect by plotting the values of the analytical expression for  $Q_{sr}$  per electoral vote. In 2004, the lagging candidate (Kerry) should put in extra offensive visits in states like Florida and Ohio, at the cost of weakening the defense of states like California and Illinois. The leading candidate (Bush) should defend his lead in states like Florida and Ohio, at the cost of not challenging Kerry's lead in California and Illinois. This resounds with the result by Snyder (1989) that parties will spend more in safe districts of the advantaged party than in safe districts of the disadvantaged party.

#### E. Additional Issues

Finally, we discuss how the results would be affected by three additional features: the existence of minority candidates; that electoral votes in Maine and Nebraska are allocated in a special fashion; and that campaigns might influence voter turnout rather than vote choice. Starting with the last issue, if the campaigns affected turnout, more resources should still be devoted to states likely to be decisive swing states. Only in decisive swing states can a marginal increase in turnout among own-party supporters change the national election outcome. A difference would be that the marginal voter density would be replaced by the marginal turnout density measuring how easily supporters in a state can be mobilized from the pool of nonvoters.<sup>24</sup>

<sup>24</sup> For a model of turnout in presidential elections, see Ron Shachar and Barry Nalebuff (1999).

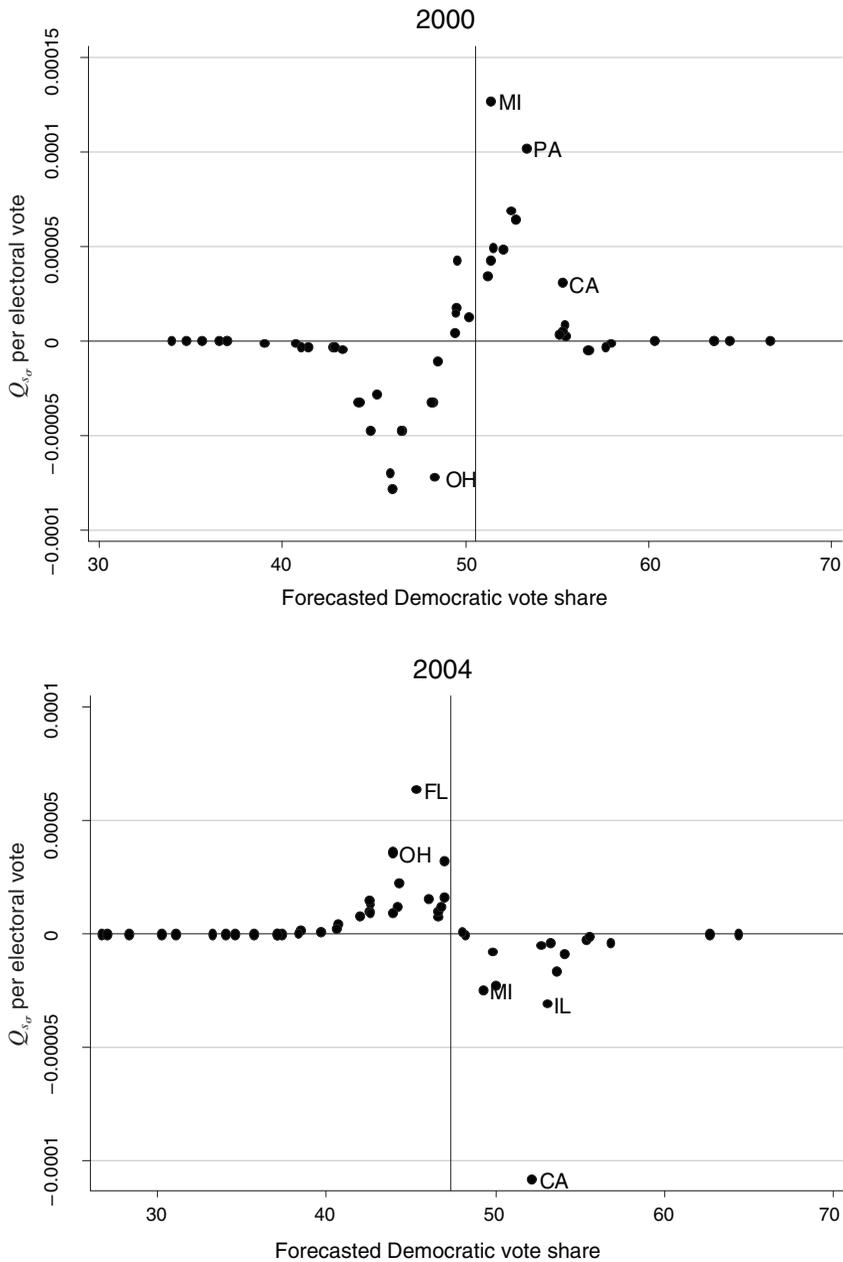


FIGURE 5. INCENTIVES TO INFLUENCE VARIANCE

In terms of this model, minority candidates will matter through their effect on the two-party preference distributions in the states. Suppose that it is known when campaign plans are made that a minority candidate will run for president, and the state and national polls take this into account. Then the effect of the minority candidate is taken into account in computing the probability that each state is a decisive swing state and the equilibrium is unaffected. If there is uncertainty about whether the candidate will run, then the model should be amended to account for this additional uncertainty.

It is more difficult to include the specific procedure of allocating electoral votes in Maine and Nebraska into the model. The model can easily be extended to account for the electoral votes that are allocated in the congressional district elections. However, it is harder to incorporate the two electoral votes that are allocated in proportion to the state election. For the 2000 election, this probably does not have a large impact, since Nebraska was solidly Republican and Maine solidly Democratic, but in 2004, Maine was less clearly Democratic and the allocations would perhaps have been affected.

### V. Electoral Reform

Since the Electoral College creates very sharp incentives to target only a few states, a natural question to ask is whether there are other ways of electing a president that would make candidates care more equally for all voters. Hence, we now analyze candidate visits under a direct national popular vote system (Direct Vote), the reform that has come closest to being accepted.<sup>25</sup> We also discuss the effects of keeping the Electoral College, but having electoral votes allocated proportionally to the popular votes in the states (the Lodge-Gossett Amendment).<sup>26</sup> We focus on the distribution of visits: which states gain or lose from reform, and which electoral system creates a more unequal visit schedule. The model is also used, however, to evaluate and quantify how reform has an impact on attention paid to minorities, and the expected frequency of razor-thin victories and presidents without a majority of the popular vote.

First, we develop the model for Direct Vote. Suppose the president is elected by a direct national vote. The number of Democratic votes in state  $s$  is then equal to the number of voters in the state,  $v_s$ , multiplied by the Democratic vote-share,  $y_s$ . The Democratic candidate wins the election if he receives more than half of the popular votes in the nation:

$$(26) \quad \sum_s v_s y_s \geq \frac{1}{2} \sum_s v_s.$$

Using, again, the Central Limit Theorem of Liapounov, the number of votes won by candidate  $D$  is asymptotically normally distributed with mean and variance,

$$(27) \quad \mu_v = \sum_s v_s \Phi \left( \frac{\Delta u_s - \mu_s - \eta}{\sqrt{\sigma_s^2 + \sigma_{fs}^2}} \right),$$

$$\sigma_v^2 = \sigma_v^2(\Delta u_s, \eta).$$

See Appendix E for the explicit expression for  $\sigma_v^2$ . The probability of a Democratic victory is

$$P^D = 1 - \int \Phi \left( \frac{\frac{1}{2} \sum_s v_s - \mu_v}{\sigma_v} \right) d\eta.$$

As before, both candidates choose the number of days to visit each state, subject to the constraint on the total number of days left in equation (1). The following proposition characterizes the equilibrium allocation.

<sup>25</sup> A Direct Vote reform was proposed by House Representative Emanuel Celler in 1969. It was wildly popular in the House, passing 338–70, but failed to pass in the Senate due to a filibuster.

<sup>26</sup> This amendment passed the Senate by a vote of 64–27, but was rejected in the House.

PROPOSITION 3: A pair of strategies for the parties ( $d^D$ ,  $d^R$ ) that constitute a NE in the game of maximizing the expected probability of winning the election must satisfy  $d^D = d^R = d^*$ , and for all  $s$  and for some  $\lambda > 0$ ,

$$(28) \quad Q_s^{DV} u'(d_s) = \lambda.$$

The interpretation of  $Q_s^{DV}$  is (see Appendix E)

$$(29) \quad Q_s^{DV} \approx (\# \text{ votes cast})_s \times (\text{"voting power"}) \\ \times (\text{marginal voter density, conditional on tied national election})_s.$$

However, "voting power" is now the same for all voters, since any voter is decisive if, and only if, the election is won by no more than one vote. Hence,  $Q_s^{DV}$  varies across states only because of differences in the share of marginal voters and the number of voters.

The allocation under Direct Vote depends crucially on the share of marginal voters, which in turn depends on the estimated variance in the preference distribution,  $\sigma_{f_s}^2$ . Therefore, the restriction  $\sigma_{f_s} = 1$  is removed in the maximum likelihood estimation of equation (16), as well as the assumption that  $\sigma_s$  is the same for all states. (To make the results comparable, the  $Q_s$  relevant for the Electoral College system is reestimated with the same restrictions removed.) The model identifies  $\sigma_{f_s}$  through the response in vote shares to changes that are common to all states, and observable changes in economic growth at the national and state level, incumbency variables, and the home state of the president and vice president. States where vote shares covary strongly with these variables are estimated to have many marginal voters. Rhode Island is estimated to have the largest share of marginal voters and Mississippi the smallest.

#### A. Distribution of Visits

We can now identify states that would have gained or lost candidate attention from reform. Figure 6 shows the 1948–2000 averages of equilibrium number of visits per capita, relative to the national averages, under both systems, assuming that  $u(d_s)$  is of log form.<sup>27</sup> States above 1 on the  $y$ -axis receive more than average visits per capita under the Electoral College system, whereas states to the right of 1 on the  $x$ -axis have more visits per capita than average under the Direct Vote system.

Thus, states below the 45-degree line would have gained from reform, and those above would have lost. Some states, like Maine and New Hampshire, are well off under both systems, while Mississippi is disadvantaged under both. Other states, like Nevada, are among the winners in the present system but among the losers under Direct Vote. The opposite is true for Rhode Island and Kansas.

<sup>27</sup> The graph shows

$$\frac{\bar{Q}_s}{\bar{n}_s}, \\ \frac{1}{n} \sum \bar{Q}_s$$

where the upper bar denotes 1948–2004 averages.

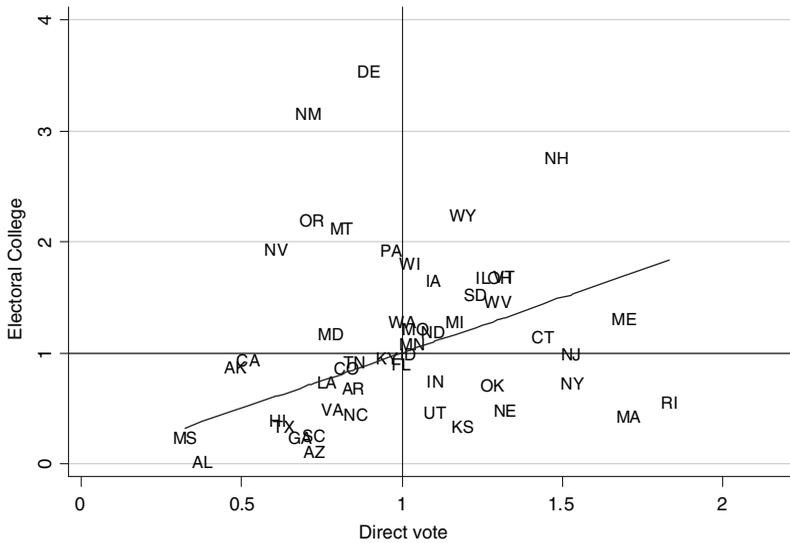


FIGURE 6. EQUILIBRIUM VISITS PER CAPITA AND ADVERTISEMENTS, RELATIVE TO NATIONAL AVERAGE

States gain or lose attention because of their (A) electoral size per capita and (B) influence relative to electoral size. Under the two systems, these components are

$$\begin{aligned}
 EC: \frac{Q_s}{n_s} &= \frac{e_s}{n_s} & \frac{Q_s}{e_s} \\
 DV: \frac{Q_s^{DV}}{n_s} &= \frac{v_s}{n_s} & \frac{Q_s^{DV}}{v_s}
 \end{aligned}$$

Figure 7 plots the 1948–2004 averages of these components. For example, the lower part of Figure 7B, shows why Rhode Island and Massachusetts would gain from reform. Because of their average partisanship, these states are rarely competitive when the national election is close. Consequently, they do not receive much attention under the Electoral College system. Still, these states have quite a few marginal (swing) voters, making them attractive targets under Direct Vote. In the top part of Figure 7A, we see that Nevada and Delaware would get less attention after reform primarily because of their heavy endowment of electoral votes relative to popular votes.

We can now also identify states that would gain attention under the Lodge-Gossett Amendment. This suggested keeping the electoral size per capita (A) constant, while only changing the influence per electoral size (B) from that induced by a state-level winner-takes-all system to a state-level proportional system. In the latter system, marginal voter densities matter, and the winners and losers are thus approximately identified in Figure 7A.<sup>28</sup>

Small states have not, as a group, been advantaged by the Electoral College system. These states have many electoral votes per capita and can attain very high values of  $Q_s$  (see Delaware). On the other hand, since their preferences often lie very far from the national median, many

<sup>28</sup> The correlation between  $Q_s^{Lodge-Gossett}/e_s$  and  $Q_s^{DV}/v_s$  is 0.8.

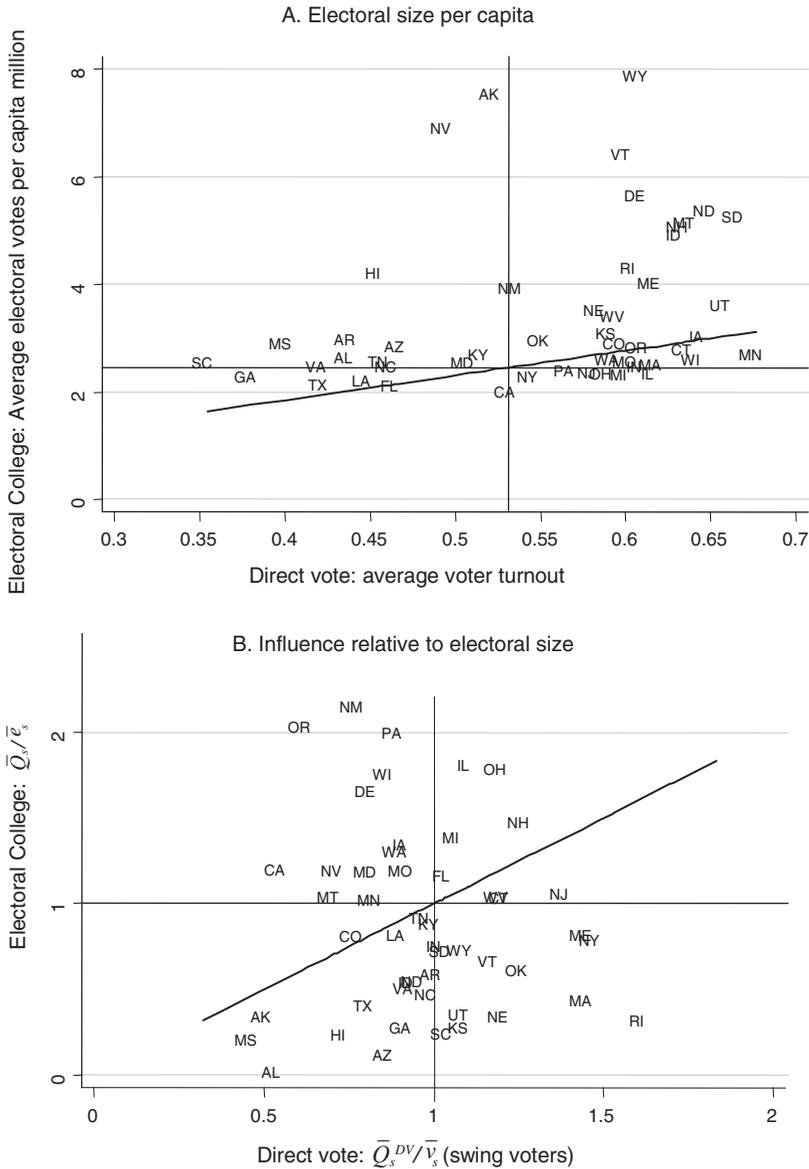


FIGURE 7. VARIABLES AFFECTING INCENTIVES TO VISIT STATES, AVERAGES 1948-2004

small states have extremely low values of  $Q_s$  (see Nebraska).<sup>29</sup> On net, visits per capita are not correlated with size. This is also the case under the Direct Vote system. So small states, as a group, neither gain nor lose from electoral reform.

Which political system spreads visits more equally? The extremely unequal distribution of  $Q_s/e_s$  creates very strong incentives to concentrate visits under the EC. The highest value (in Ohio) is more than twenty times that of the lowest (in Kansas). By comparison, Maine has only

<sup>29</sup> I thank Andrew Gelman for pointing this out to me.

twice as high marginal voter density as California. To make things worse, electoral votes per capita are also more unequally distributed than voter turnout. Consequently, equilibrium visits are much more concentrated under the *EC*.<sup>30</sup> For example, the Gini coefficients of  $Q_s/n_s$  and  $Q_s^{DV}/n_s$  in 2004 were 0.73 and 0.17, respectively. The equilibrium TV advertisements across media markets are also more unequally distributed under the Electoral College system than under Direct Vote. In 2004, the Gini coefficient of equilibrium advertisements ( $Q_m/p_m$ ) across media markets was 0.70 under the Electoral College, compared to 0.13 under Direct Vote.

### B. Other Reform Concerns

A frequent claim is that minorities are concentrated in large, politically competitive states and thus have a higher than average influence on the election.<sup>31</sup> However, this seems inconsistent with the data on advertisements across 75 media markets in 2000 discussed in Section III. In fact, a randomly drawn Hispanic voter had 21 percent fewer ads aired in his media market in 2000 than the average voter. The equivalent number for African Americans was 11 percent fewer ads. Since the model predicts advertisements well, we reach virtually the same results using the model-predicted equilibrium advertisements across these markets in 2000 (20 and 9 percent, respectively). Model-predicted advertisements for all media markets and elections after 1980 indicate that these differences are recent for Hispanics,<sup>32</sup> but long term for African Americans. On average, 1980–2004, there was no difference between Hispanics and the average voter, while a randomly drawn African American voter had 8 percent fewer predicted ads aired in his media market than the average voter. This is perhaps not surprising, given that the greatest concentration of African Americans is found in Washington, DC, and states in the Deep South.

Two often discussed concerns are presidents without a majority of the popular vote and razor-thin victories. Since these are very rare events, it is hard to estimate their probabilities using empirical frequencies. However, given that the elections between 1948 and 2004 are representative of future elections, these probabilities can be estimated by using the model to simulate elections and record event frequencies. The estimated probability of a winning margin of fewer than 1,000 votes is about 40 times higher under the Electoral College system (0.8 percent, compared to 0.02 percent under Direct Vote). The estimated probability of electing a president without a majority of the popular vote is about 4 percent. This implies that we should expect this outcome about once every hundred years. Historically, it has happened around three (perhaps four) times in the last 200 years: 1824 (perhaps), 1876, 1888, and 2000. Arguably, however, the outcomes in 1824 and 1876 had to do with peculiarities in the aggregation of votes.<sup>33</sup>

<sup>30</sup> Relatedly, Persson and Tabellini (2000) and Lizzeri and Persico (2001) conclude that spending will be more narrowly targeted under majoritarian elections, but for very different reasons. Persson and Tabellini (2000) analyze electoral competition in an election with three electoral districts. Their result is driven by the assumption that the same district is always decisive in the Electoral College (majoritarian election), so all resources are spent there. Under proportional elections, resources are spent in proportion to the share of marginal voters in each district. Since the marginal voters are assumed to reside in more than one district, spending is more equally distributed under this system. Lizzeri and Persico (2001) use a very different framework, assuming no uncertainty about the election outcome, given candidate strategies. In each particular election, spending will be targeted to half the voters in half the states under the Electoral College system, and half the voters in the nation under Direct Vote. Therefore, resources are spent in a more concentrated fashion under the Electoral College system. Since candidates use completely mixed strategies, over time all states receive equal expected treatment.

<sup>31</sup> See William C. Kimberling (2005), Kristina Dell, “The Electoral College Explained,” *Time*, November 1, 2004; or Alexander Keyssar in “Pro’s & Cons: The Electoral College,” *The Journal Editorial Report*, October 29, 2004.

<sup>32</sup> I could not find county-level data on the Hispanic population prior to 1980.

<sup>33</sup> In 1824, 6 of the 24 states at the time still chose their electors in the state legislature, so the popular vote outcome is not known. In the chaotic election of 1876, each state delivered to Congress two slates of electors, and a special commission accepted slates favoring of the popular vote loser. See “A Brief History of the Electoral College” (<http://www.fec.gov/elections.html>).

## VI. Conclusion and Discussion

This paper develops a probabilistic-voting model of electoral competition under the Electoral College system of electing a president. The modeling framework is significantly more general than previous work—with  $S$  states, a varying number of electoral votes per state, preference distributions varying across states, uncertainty regarding the election outcome at both state and national level, etc.—which allows a fairly direct application to the election incentives facing a US presidential candidate or incumbent president. In addition, the model can be explicitly solved and directly estimated. This implies that new questions can be asked and a new type of answers given.

Some theoretical results deserve mentioning. First, more resources should be devoted to states that are likely to be decisive swing states, that is, states that are decisive in the electoral college and, at the same time, have tied state elections. Second, the probability of being a decisive swing state equals the number of voters in the state multiplied by the “voting power” in the state, multiplied by the marginal voter density conditional on the state election being tied. Third, the probability of being a decisive swing state is roughly proportional to the number of electoral votes. Fourth, this probability *per electoral vote* is highest for states that have a forecasted state election outcome that lies between a draw and the forecasted national election outcome. Fifth, more precise state-election forecasts make the optimal allocation of resources more concentrated. Sixth, the presidential candidate who is lagging should try to increase the variance in electoral votes. This can be achieved by spending more time in large states where this candidate is behind, and less time in large states where this candidate is ahead.<sup>34</sup>

Most importantly, this paper quantifies these effects. This makes it possible to identify which states have been benefiting from the Electoral College system. It also reveals a very close resemblance between the presidential candidates’ actual strategies and the equilibrium strategies in the last two elections. Finally, it shows the sharp incentives to favor certain states created by the Electoral College system.

This paper also analyzes the effects of a change to direct national popular vote for president. Different factors guide the attention under the two electoral systems: voter turnout and the share of swing voters under Direct Vote; and the probability of being a decisive swing state under the Electoral College. These factors determine what states are likely to gain or lose candidate attention after reform (see Figure 6). Since the probability of being a decisive swing state is much more unequally distributed than the other factors, the Electoral College system induces presidential candidates to pay less equal attention to states than Direct Vote. Finally, neither small states nor minorities have been favored by the Electoral College system.

The model of this paper can be applied both to resources solely used in the campaign, such as visits, and to campaign promises with policy consequences. This has two important implications. First, the theoretical results above also apply to policy incentives (but, of course, we do not yet know how empirically important these are). For example, the Electoral College system provides sharper incentives to target a few states with favorable policies, and Figure 6 shows how incentives to offer favorable policies would change after reform. Second, a general lesson is that while campaign analysis is most interesting in its own right, it is also a useful step to learn about electoral incentives to distort policy. Pure campaign efforts are used only to win elections and

<sup>34</sup> The second, fourth, and fifth points are new, to the best of my knowledge. The first point shows more precisely what people have conjectured or partly demonstrated earlier; see, for example, Snyder (1989). The third point has not previously been shown theoretically, although (because of the second point) it is closely related to the empirical finding of Gelman and Katz (2001) that “voting power” is proportional to size under reasonable assumptions. The sixth point makes more precise the statement of Snyder (1989) that more resources will be spent in safe districts of the advantaged party than in the safe districts of the other party.

are very informative of how politicians perceive electoral incentives. Policy is determined by a multitude of factors, of which election concerns is one. For this reason, a cautious approach is to first test models of political competition on campaigning, and then proceed to test for this signal in policy. This paper takes the first step and leaves the second to future research.

The paper contributes to the literature on comparative political economy that links political institutions to outcomes in politics and economic policy. However, it is slightly methodologically different, since the model is sufficiently rich to allow the complete integration of theory and empirics, tying together theoretical insights and empirical results. Since the modeling framework is quite general, this integration can be carried over to the study of other resource allocation problems in other electoral settings.

### APPENDIX

#### A. Proof of Proposition 1

Since  $P^D(d^{D*}, d^R)$  is continuous and differentiable, a necessary condition for an interior NE is

$$(30) \quad \frac{\partial P^D(d^{D*}, d^{R*})}{\partial d_s^D} = Q_s u'(d_s^{D*}) = \lambda^D,$$

$$(31) \quad \frac{\partial P^D(d^{D*}, d^{R*})}{\partial d_s^R} = Q_s u'(d_s^{R*}) = \lambda^R.$$

Therefore,

$$(32) \quad \frac{u'(d_s^{D*})}{u'(d_s^{R*})} = \frac{\lambda^D}{\lambda^R},$$

for all  $s$ . Suppose that  $d_s^{D*} \neq d_s^{R*}$ . This means that  $d_s^{D*} < d_s^{R*}$  for some  $s$ , implying that  $\lambda^D > \lambda^R$  by equation (32). Because of the budget constraint, it must be the case that  $d_{s'}^{D*} > d_{s'}^{R*}$  for some  $s'$ , which implies  $\lambda^D < \lambda^R$ , a contradiction. Therefore,  $\lambda^D = \lambda^R$ , which implies  $d_s^{D*} = d_s^{R*}$  for all  $s$ .

*Uniqueness.*—Suppose there are two equilibria with equilibrium strategies  $d$  and  $d'$  corresponding to  $\lambda > \lambda'$ . The condition on the Lagrange multipliers implies  $d_s > d'_s$  for all  $s$ , which violates the budget constraint. Therefore, the only possibility is  $\lambda = \lambda'$ , which implies  $d_s = d'_s$  for all  $s$ .

#### B. Data: Variable Definitions and Sources, and Selected Sample

See Tables A1 and A2.

#### C. Analytical Interpretation of $Q_s$

We now show, analytically, why  $Q_s$  approximately equals the probability of being a *decisive swing state*. First, an expression for the approximate probability that a particular state, denoted  $s'$ , is decisive in the electoral college is derived. Disregarding  $s'$ , the electoral vote outcome,  $\sum_{s \neq s'} D_s e_s$ , is approximately normally distributed with mean

$$(33) \quad \mu_{-s'} = \mu - e_{s'} G_s(\cdot)$$

and variance

$$(34) \quad \sigma_{E-s'}^2 = \sigma_E^2 - e_s^2 G_s(\cdot)(1 - G_s(\cdot)).$$

The  $e_{s'}$  electoral votes of state  $s'$  are decisive if

$$\sum_{s \neq s'} D_s e_s \in \left( \frac{\sum_{s \neq s'} e_s}{2}, \frac{\sum_{s \neq s'} e_s}{2} - e_{s'} \right).$$

In this range, a presidential candidate wins if, and only if, he receives the votes of state  $s'$ . The probability of this event is approximately

$$P = \Phi\left(\frac{\frac{\sum_s e_s}{2} - \mu_{-s'}}{\sigma_{E-s'}}\right) - \Phi\left(\frac{\frac{\sum_s e_s}{2} - e_{s'} - \mu_{-s'}}{\sigma_{E-s'}}\right).$$

Inserting for  $\mu_{-s'}$  and  $\sigma_{E-s'}^2$  yields

$$\Phi\left(\frac{\frac{\sum_s v_s}{2} - \mu + e_{s'} G_{s'}(\cdot)}{\sigma_{E-s'}}\right) - \Phi\left(\frac{\frac{\sum_s v_s}{2} - \mu - e_{s'}(1 - G_{s'}(\cdot))}{\sigma_{E-s'}}\right).$$

Making a second-order Taylor-approximation of  $P$  around the point

$$x_0 = \frac{\frac{\sum_s e_s}{2} - \mu}{\sigma_{E-s'}},$$

we get

$$P(\eta) \approx \varphi(x_0) \frac{e_s}{\sigma_{E-s'}} + \frac{e_s^2}{\sigma_{E-s'}^2} \varphi(x_0) x_0 \left( G_s(\cdot) - \frac{1}{2} \right).$$

Given the large number of states,  $\sigma_{E-s'} \approx \sigma_E$ , implying

$$P(\eta) \approx \varphi(x(\eta)) \frac{e_s}{\sigma_E} + \frac{e_s^2}{\sigma_E^2} \varphi(x(\eta)) x(\eta) \left( G_s(\cdot) - \frac{1}{2} \right).$$

Second, we derive the analytical likelihood that the election in state  $s'$  is tied. Given a national shock,  $\eta$ , the state outcome is a tie if  $F_s(-\eta_s - \eta) = 1/2$ , which happens with likelihood  $g_s(-\mu_s - \eta)$ . Conditional on the national shock, the joint likelihood of being decisive in the electoral college and having a tied election is thus roughly  $P(\eta) g_s(-\mu_s - \eta)$ , and the unconditional likelihood equals

$$\int P(\eta) g_s(-\mu_s - \eta) h(\eta) d\eta = Q_s,$$

by the definition of  $Q_s$  in equation (13). Consequently,  $Q_s$  approximates the joint likelihood that a state is decisive in the electoral college and, simultaneously, has a tied state election.

TABLE A1—VARIABLE DEFINITIONS AND SOURCES

Variable name	Definition	Source
Democratic two-party state vote	Democratic percent of state two-party vote	Campbell (1992), Statistical Abstract of the United States, Federal Elections Commission
Democratic two-party national vote	Democratic percent of national two-party vote	Dave Leip's Atlas of Presidential Elections
National trial-heat polls	National Gallup trial-heat polls, percentage of two-party preference for the Democratic candidate	Campbell (1996), Gallup
State trial-heat polls	State polls, percentage of two-party preference for the Democratic candidate	National Journal's Hotline, Electoral Scoreboard
2nd-qtr. GNP growth $x$ incumbent party	Second quarter GNP growth $x$ 1 (-1) if Democratic (Republican) incumbent	Department of Commerce's Survey of Current Business, August election-year issue
1st-qtr. state growth $x$ incumbent party	Growth in a state's total personal income between the prior year's fourth quarter and the first quarter of the election year $x$ 1 (-1) if Democratic (Republican) incumbent. The index is standardized across states within each election year.	Campbell (1992), Commerce Department's Survey of Current Business
Incumbent president	Incumbent president (1=Dem, -1=Rep., 0=none)	Dave Leip's Atlas of Presidential Elections
Elected incumbent seeking reelection	Incumbent president running for reelection (1=Dem, -1=Rep., 0=none)	Dave Leip's Atlas of Presidential Elections
Presidential home-state advantage	Home state of presidential candidate (1=Dem, -1=Rep., 0=none)	Campbell (1992), Dave Leip's Atlas of Presidential Elections
Vice-presidential home-state advantage	Home state of vice-presidential candidate (1=Dem, -1=Rep., 0=none)	Campbell (1992), Dave Leip's Atlas of Presidential Elections
State liberalism (ADA)	The average ADA-scores of each state's members of Congress the year before the election	ADA scores for years 1947–1999, Tim Groseclose: <a href="http://faculty-gsb.stanford.edu/groseclose/homepage.htm">faculty-gsb.stanford.edu/groseclose/homepage.htm</a> , for 2003: <a href="http://www.adaction.org">www.adaction.org</a>
State legislature party division	The Democratic percentage of the two-party vote in the midterm state legislative election	Campbell (1992), Statistical Abstract of the United States
Presidential home region (southern advantage)	See Campbell (1992)	Campbell (1992)
Southern state (1964)	See Campbell (1992)	Campbell (1992)
Deep South state (1964)	See Campbell (1992)	Campbell (1992)
New England state (1960 & 1964)	See Campbell (1992)	Campbell (1992)
Rocky Mountain West state (1976 & 1980)	See Campbell (1992)	Campbell (1992)
North Central state (1972)	See Campbell (1992)	Campbell (1992)
TV advertisements	Number of campaign TV advertisements aired after Sept. 1, 2000, for the 75 largest DMAs	Brennan Center for Justice
TV advertisement costs	Cost of TV advertisements, for the 75 largest DMAs	Brennan Center for Justice

To compare  $Q_s$  with simulated frequencies of being EC decisive and having a state election margin of less than 2 percent,  $Q_s$  must be scaled. The probability of a winning margin of less than  $x$  percent equals

$$\frac{1}{2} - \frac{x}{200} \leq F_s(-\eta_s - \eta) \leq \frac{1}{2} + \frac{x}{200}.$$

The probability that the state-level shock falls in this region is approximately

$$g_s(-\mu_s - \eta) \frac{\sigma_{fs}}{\varphi(0)} \frac{x}{100}.$$

TABLE A2—SAMPLE DESCRIPTION

Year	# States	# Obs.	Missing states
1948	48	47	Alabama (Truman excluded from ballot)
1952	48	47	Alabama*
1956	48	47	Alabama*
1960	50	47	Mississippi (39% of vote to unpledged electors), Alaska*, Hawaii*
1964	50	46	Alabama (Johnson excluded from ballot), Alaska*, Hawaii*, and Mississippi*
1968	50	48	Alabama* and Mississippi*
1972	50	49	Alabama*
1976–2004	50	50	
Sum	744	731	

\* Lagged or twice-lagged votes missing.

The unconditional probability of being pivotal and having a state-election margin of  $x$  percent is therefore approximately

$$Q_s \frac{\sigma_{fs}}{\varphi(0)} \frac{x}{100}.^{35}$$

Since  $\sigma_{fs} = 1$  in Section III,  $[\sigma_{fs}/\varphi(0)][2/100] = \sqrt{2\pi}(2/100) = 0.05013$  is just a constant scaling factor. Table 3 reports percentages; consequently, the scaling factor used there is  $2\sqrt{2\pi}$ .

The scaled values of  $Q_s$  are very similar to the simulated frequencies of the corresponding events. Table 3 reports the scaled  $Q_s$  and simulated frequencies. In 4.5 percent of the 1 million simulated elections, Florida was decisive in the Electoral College and had a state margin of victory of less than 2 percent. The scaled  $Q_s$  for Florida was 4.6 percent. To get the probability that the state margin of victory is within, say, 1,000 votes,  $Q_s$  should be scaled by  $(1,000/v_s) \times (\sigma_{fs}/\varphi(0))$ . Using this formula, the probability of a state being decisive in the Electoral College, and at the same time having an election result with a state margin of victory less than 1,000, was 0.0004 in Florida in the 2000 election. The probability that this would happen in any state was 1 percent. The probability of a victory margin of one vote in Florida is 0.4 per million. The state where one vote is most likely to be decisive is Delaware, where it is decisive 0.9 times in a million elections.

This type of scaling also helps to clarify the relation between  $Q_s$  and “voting power,” i.e., the probability that one vote in state  $s$  decides the election. With continuous vote outcomes, “voting power” can be defined as the probability that the margin of victory is no more than one vote; see Chamberlain and Rothschild (1981). Rewriting  $Q_s$  as follows,

$$(35) \quad Q_s \approx \underbrace{v_s}_{\substack{\# \text{ votes cast} \\ \text{in state } s}} \times \underbrace{Q_s \left( \frac{\sigma_{fs}}{\varphi(0)} \frac{1}{v_s} \right)}_{\substack{\text{“voting} \\ \text{power”}}} \times \underbrace{\frac{\varphi(0)}{\sigma_{fs}}}_{\substack{\text{marginal voter density,} \\ \text{conditional on tied state election}}}$$

shows that  $Q_s$  equals the product of the number of votes cast in the state, the voting power in the state, and the marginal voter density conditional on a tied state election.

<sup>35</sup> Although the marginal voter density plays no role in  $Q_s$ , it enters if  $Q_s$  is rewritten as

$$Q_s \approx \underbrace{Q_s \left( \frac{\sigma_{fs}}{\varphi(0)} \frac{2}{100} \right)}_{\substack{\text{Pr (decisive in electoral college} \\ \text{and state election outcome within 2 percent)}}} \times \underbrace{\frac{\varphi(0)}{\sigma_{fs}}}_{\substack{\text{marginal voter density,} \\ \text{conditional on tied state election}}} \times \frac{100}{2}.$$

D. The Mean and Variance of the  $Q_{s\mu}/e_s$  Distribution

This appendix will discuss the mean and variance in the distribution of  $Q_{s\mu}$  per electoral vote. First, we will show that a simpler functional form closely approximates  $Q_{s\mu}$ . Then the properties of this simpler function will be discussed.

To simplify the analytical form of  $Q_{s\mu}$ , we do a first-order Taylor-expansion of the mean of the expected number of electoral votes  $\mu(\eta)$  around  $\eta = \tilde{\eta}$ , for which

$$(36) \quad \mu(\tilde{\eta}) = \frac{1}{2} \sum_s e_s,$$

that is, the value of the national shock which makes the expected outcome a draw. With this approximation,

$$(37) \quad \mu(\eta) = \sum_s e_s G_s(-\mu_s - \eta) \approx \frac{1}{2} \sum_s e_s - a(\eta - \tilde{\eta}),$$

where

$$(38) \quad a = \sum_s e_s g_s(-\mu_s - \tilde{\eta}).$$

Since the mean of the electoral votes,  $\mu(\eta)$ , is much more sensitive to national shocks than is the variance  $\sigma_E(\eta)$ , the latter is approximated by

$$(39) \quad \sigma_E^2(\eta) \approx \bar{\sigma}_E^2 = \sum_s e_s^2 G_s(-\mu_s - \tilde{\eta})(1 - G_s(-\mu_s - \tilde{\eta})).$$

Using these approximations,

$$(40) \quad \frac{1}{\sigma_E(\eta)} \varphi\left(\frac{\frac{1}{2}\sum_s e_s - \mu}{\sigma_E(\eta)}\right) \approx \frac{1}{\bar{\sigma}_E} \varphi\left(\frac{\eta - \tilde{\eta}}{\bar{\sigma}_E/a}\right).$$

Inserting this into the definition of  $Q_{s\mu}$  in equation (13), we have

$$Q_{s\mu}/e_s \approx \frac{1}{\bar{\sigma}_E} \int \varphi\left(\frac{\eta - \tilde{\eta}}{\bar{\sigma}_E/a}\right) g_s(-\mu_s - \eta) h(\eta) d\eta.$$

Integrating over  $\eta$  and simplifying,

$$(41) \quad Q_{s\mu}/e_s \approx \frac{1}{2\pi\sqrt{a^2\sigma^2\sigma_s^2 + \bar{\sigma}_E^2\sigma_s^2 + \bar{\sigma}_E^2\sigma^2}} \exp\left(-\frac{1}{2}\left(\frac{\tilde{\eta}^2}{\sigma^2 + (\bar{\sigma}_E/a)^2} + \frac{(\mu_s - \mu^*)^2}{\bar{\sigma}^2}\right)\right),$$

where

$$(42) \quad \mu^* = -\frac{\sigma^2}{\sigma^2 + (\bar{\sigma}_E/a)^2} \tilde{\eta},$$

and

$$(43) \quad \bar{\sigma}^2 = \sigma_s^2 + \frac{1}{\left(\frac{1}{\sigma^2} + \frac{1}{(\bar{\sigma}_E/a)^2}\right)}.$$

The approximation above is quite good. The correlation between  $Q_{s\mu}/e_s$  and the right-hand side of equation (41) is 0.9999 for observations from 1948 to 2004.

From equation (41), it is clear that the amplitude of  $Q_{s\mu}$  is larger when  $\tilde{\eta}$  is close to zero. This affects all states in the same way in a particular election, but varies across elections.

The mean  $\mu^*$  of the distribution always lies between a pro-Republican state bias of  $\mu_{st} = 0$ , which corresponds to a 50 percent forecasted Democratic vote share, and  $\mu_{st} = -\tilde{\eta}_t$ , which approximately corresponds to the forecasted national Democratic vote share. The smaller the variance of the national popularity swings,  $\sigma^2$ , the closer is the mean to 50-50. (Note that  $\bar{\sigma}_E$ ,  $a$ , and  $\tilde{\eta}$  do not depend on  $\sigma^2$ .) In the extreme case where  $\sigma^2$  approaches zero,  $\mu_s^*$  approaches 0. In the extreme case that  $\sigma^2$  approaches infinity,  $\mu_s^*$  approaches  $-\tilde{\eta}$ .

The smaller the variance  $\tilde{\sigma}^2$ , the larger incentives to treat states differently. Comparing two states  $s$  and  $t$ , the ratio

$$\frac{Q_{s\mu}/e_s}{Q_{t\mu}/e_t} = \exp\left(-\frac{1}{2}\left(\frac{(\mu_s - \mu^*)^2 - (\mu_t - \mu^*)^2}{\tilde{\sigma}^2}\right)\right)$$

depends on how far the two states are from  $\mu_s^*$ , and on  $\tilde{\sigma}^2$ .<sup>36</sup> By the definition of  $\tilde{\sigma}^2$  above, it is clear that  $\tilde{\sigma}^2$  is increasing in  $\sigma^2$ . It is also increasing in  $\sigma_s^2$ , both directly and because increases in  $\sigma_s^2$  are associated with increases in  $(\bar{\sigma}_E/a)^2$ . The latter is true in the sample and intuitive: the larger the state-level shocks, the more uncertainty about the aggregate outcome. However, due to effects through  $a$  it is hard to prove formally.

### E. Direct Presidential Vote

First, the approximate probability of winning the election is derived under *DV*. Conditional on  $\eta$ , the expected vote share of  $D$  in state  $s$  is

$$\begin{aligned}\mu_{vs}(\Delta u_s, \eta) &= \int F_s(\Delta u_s - \eta - \eta_s) g_s(\eta_s) d\eta_s \\ &= \Phi\left(\frac{\Delta u_s - \eta - \mu_s}{\sqrt{\sigma_s^2 + \sigma_{fs}^2}}\right),\end{aligned}$$

and the expected national vote of  $D$  is

$$\mu_v(\Delta u_s, \eta) = \sum_s v_s \mu_{vs}(\Delta u_s, \eta).$$

Again conditional on  $\eta$ , the variance in  $D$ 's votes in state  $s$  is

$$\sigma_{vs}^2 = v_s^2 \int (F_s(\Delta u_s - \eta - \eta_s) - \mu_{vs}(\Delta u_s, \eta))^2 g_s(\eta_s) d\eta_s,$$

and the variance in the national votes of  $D$  is

$$\sigma_v^2 = \sum_s \sigma_{vs}^2.$$

<sup>36</sup> As in the estimation of Section II, we assume that  $\sigma_s^2 = \sigma_t^2$ .

The approximate probability of  $D$  winning the election is

$$P^D(\Delta u_s) = 1 - \int \Phi\left(\frac{\frac{1}{2}\sum_s v_s - \mu_v}{\sigma_v}\right) h(\eta) d\eta.$$

The equilibrium strategies of Proposition 3 depend crucially on

$$\frac{\partial P^D}{\partial \Delta u_s} = Q_s^{DV} = \frac{\partial P^D}{\partial \mu_v} \frac{\partial \mu_v}{\partial \Delta u_s} + \frac{\partial P^D}{\partial \sigma_v} \frac{\partial \sigma_v}{\partial \Delta u_s} = Q_{s\mu}^{DV} + Q_{s\sigma}^{DV},$$

where

$$(44) \quad \begin{aligned} Q_{s\mu}^{DV} &= \frac{v_s}{\sigma_v \sqrt{\sigma_s^2 + \sigma_{fs}^2}} \int \varphi\left(\frac{\frac{1}{2}\sum_s v_s - \mu_v}{\sigma_v}\right) \varphi\left(\frac{\Delta u_s - \mu_s - \eta}{\sqrt{\sigma_s^2 + \sigma_{fs}^2}}\right) h(\eta) d\eta, \\ Q_{s\sigma}^{DV} &= \int \left(\frac{\partial}{\partial \sigma_v} \Phi\left(\frac{\frac{1}{2}\sum_s v_s - \mu_v}{\sigma_v}\right)\right) \frac{\partial \sigma_v}{\partial \Delta u_s} h(\eta) d\eta. \end{aligned}$$

#### F. Analytical Interpretation of $Q_{s\mu}^{DV}$

Empirically,  $Q_{s\mu}^{DV} \gg Q_{s\sigma}^{DV}$ , and the size of  $Q_{s\sigma}^{DV}$  is negligible compared to  $Q_{s\mu}^{DV}$ . The interpretation of  $Q_{s\mu}^{DV}$  is

$$(45) \quad Q_{s\mu}^{DV} = v_s (\text{pdf of tied election}) E[f_s | \text{election tied}].$$

This follows since, conditional on  $\eta$ , the expected marginal voter density in state  $s$  is

$$\int_{-\infty}^{\infty} f_s(\Delta u_s - \eta - \eta_s) g_s(\eta_s) d\eta_s = \frac{1}{\sqrt{\sigma_s^2 + \sigma_{fs}^2}} \varphi\left(\frac{\Delta u_s - \mu_s - \eta}{\sqrt{\sigma_s^2 + \sigma_{fs}^2}}\right).$$

The probability density function of a tied national election, conditional on  $\eta$ , is

$$\frac{1}{\sigma_v} \varphi\left(\frac{\frac{1}{2}\sum_s v_s - \mu_v}{\sigma_v}\right),$$

and the unconditional

$$(46) \quad \text{pdf of tied election} = \int \frac{1}{\sigma_v} \varphi\left(\frac{\frac{1}{2}\sum_s v_s - \mu_v}{\sigma_v}\right) h(\eta) d\eta.$$

Therefore, the marginal voter density, conditional on a tied election, is

$$(47) \quad \begin{aligned} &E[f_s | \text{election tied}] \\ &= \int \left(\frac{\varphi\left(\frac{\frac{1}{2}\sum_s v_s - \mu_v}{\sigma_v}\right)}{\int \varphi\left(\frac{\frac{1}{2}\sum_s v_s - \mu_v}{\sigma_v}\right) h(\eta) d\eta}\right) \frac{1}{\sqrt{\sigma_s^2 + \sigma_{fs}^2}} \varphi\left(\frac{\Delta u_s - \mu_s - \eta}{\sqrt{\sigma_s^2 + \sigma_{fs}^2}}\right) h(\eta) d\eta. \end{aligned}$$

Inserting equations (46) and (47) into equation (45) yields equation (44).

To see the relationship to “voting power,” note that the probability of a national election margin of  $x$  votes or fewer equals

$$(48) \quad p_x^{DV} = \int \Phi\left(\frac{\frac{1}{2}\sum_s v_s + \frac{x}{2} - \mu_v}{\sigma_v}\right) - \Phi\left(\frac{\frac{1}{2}\sum_s v_s - \frac{x}{2} - \mu_v}{\sigma_v}\right) h(\eta) d\eta \\ \approx x \int \frac{1}{\sigma_v} \varphi\left(\frac{\frac{1}{2}\sum_s v_s - \mu_v}{\sigma_v}\right) h(\eta) d\eta.$$

Voting power is, by definition,  $p_1^{DV}$ . By equations (46) and (48),  $p_1^{DV} \approx$  pdf of tied election; therefore, equation (45) may be written

$$(49) \quad Q_{s\mu}^{DV} \approx v_s \text{ (“voting power”) } E[f_s | \text{election tied}].$$

This is the analog of equation (35) under the Electoral College.

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