In the Solow model we had before:

- the production technology is held constant
  (Assumption 4: $A_t = A$)
- income per capita is constant in the steady state

Neither point is true in the real world:

- 1904-2004: U.S. real GDP per person grew by a factor of 7.6, or 2% per year.
- examples of technological progress abound:
  - From 1950 to 2000, U.S. farm sector productivity nearly tripled
  - The real price of computer power has fallen an average of 30% per year over the past three decades
  - 2001: iPod capacity = 5gb, 1000 songs
  - 2008: iPod capacity = 160gb, 40,000 songs
The Solow Model with Technological Progress

It may seem logical to let $A_t$ grow.

Instead, we will introduce a new variable: labor efficiency $E_t$

$$Y_t = AK_t^\alpha (E_t N_t)^{1-\alpha}$$

Technological progress is labor-augmenting: it increases labor efficiency at the exogenous rate $g$:

$$\frac{\Delta E_t}{E_t} = g > 0$$
The Solow Model with Technological Progress

\[ Y_t = AK_t^{\alpha}(E_t N_t)^{1-\alpha} \]

\( E_t N_t \) is now the number of **effective workers**.

Increases in labor efficiency have the same effect on output as increases in the labor force.

\[ \frac{\Delta E_t N_t}{E_t N_t} \approx \frac{\Delta E_t}{E_t} + \frac{\Delta N_t}{N_t} = g + n \]

Efficiency units of labor grow at a rate \( g + n \).
Definitions

c_t = C_t / E_t N_t, \ i_t = I_t / E_t N_t, \ k_t = K_t / E_t N_t, \ y_t = Y_t / E_t N_t \ now \ denote \ per \ effective \ worker \ variables

\[
Y_t = AK_t^\alpha (E_t N_t)^{1-\alpha}
\]
\[
\iff \frac{Y_t}{E_t N_t} = \frac{AK_t^\alpha (E_t N_t)^{1-\alpha}}{E_t N_t}
\]
\[
\iff y_t = Ak_t^\alpha \equiv f(k_t)
\]

\(y_t = f(k_t)\) is the per effective worker production function

\(k_t\) is the capital-effective labor ratio.
The Capital Accumulation Equation

The evolution of $k_t$ is now given by

$$K_{t+1} = I_t + (1 - \delta)K_t$$

$$\Leftrightarrow \frac{K_{t+1}}{E_t N_t} = \frac{I_t}{E_t N_t} + (1 - \delta) \frac{K_t}{E_t N_t}$$

$$\Leftrightarrow \frac{E_{t+1} N_{t+1}}{E_t N_t} \cdot \frac{K_{t+1}}{E_{t+1} N_{t+1}} = \frac{I_t}{E_t N_t} + (1 - \delta) \frac{K_t}{E_t N_t}$$

$$\Leftrightarrow (1 + n + g)k_{t+1} = i_t + (1 - \delta)k_t$$

$$\frac{N_{t+1}}{N_t} = 1 + n$$ where $n$ is the population growth rate

$$\frac{E_{t+1}}{E_t} = 1 + g$$ where $g$ is the labor efficiency growth rate

$$\frac{E_{t+1} N_{t+1}}{E_t N_t} = (1 + g)(1 + n) \approx 1 + g + n$$ where $g$ is the labor efficiency growth rate
The Solow Model with Technological Progress

We have in per effective worker terms:

\[ y_t = f(k_t) \]  \hspace{2cm} \text{(Production)}

\[ c_t + i_t = y_t \]  \hspace{2cm} \text{(Goods Market Equilibrium)}

\[ (1 + g + n)k_{t+1} = i_t + (1 - \delta)k_t \] \hspace{2cm} \text{(Capital Accumulation)}

\[ c_t = (1 - s)y_t \] \hspace{2cm} \text{(Consumption Function)}

which reduces to

\[ sf(k_t) = (1 + g + n)k_{t+1} - (1 - \delta)k_t \]

With a steady state

\[ \Leftrightarrow sf(k^*) = (g + n + \delta)k^* \]
Break-even investment, \((\delta + n + g)k\)

Investment, \(sf(k)\)

\(k^*\) Capital per effective worker, \(k\)

The steady state
## Steady State Growth Rates

<table>
<thead>
<tr>
<th>Variable</th>
<th>Steady State Growth Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital per effective worker</td>
<td>$k = \frac{K}{EN}$</td>
</tr>
<tr>
<td>Output per effective worker</td>
<td>$y = \frac{Y}{EN}$</td>
</tr>
<tr>
<td>Capital-Labor Ratio</td>
<td>$\frac{K}{N}$</td>
</tr>
<tr>
<td>Labor Productivity</td>
<td>$\frac{Y}{N}$</td>
</tr>
<tr>
<td>Output</td>
<td>$Y$</td>
</tr>
<tr>
<td>Consumption</td>
<td>$C$</td>
</tr>
</tbody>
</table>
Balanced Growth

The steady state in the Solow model with technological progress exhibits **balanced growth**: many variables grow at the same rate.

Predictions:

1. \( Y/N \) and \( K/N \) grow at the same rate \( g \), so \( K/Y \) should be constant.
   This is true in the real world.

2. The real wage \( W/P \) grows at same rate as \( Y/N \), while real rental price \( R/P \) is constant.

\[
\alpha AK^{\alpha-1}(EN)^{1-\alpha} = \frac{R}{P}
\]

\[
(1 - \alpha)AK^{\alpha}(EN)^{-\alpha}E = \frac{W}{P}
\]

This is also true in the real world.
The **golden rule** steady state has the highest possible consumption per person:

\[ c^* = f(k^*) - (g + n + \delta)k^* \]

The golden rule \( k_{gold}^* \) and \( c_{gold}^* \) that maximizes this expression is:

\[ f_k(k^*) = (g + n + \delta) \]

where \( f_k \) is the first derivative of \( f \).
Convergence

$$sf(k^*) = (g + n + \delta)k^*$$

- If countries have the same $f(k)$, $g$, $n$, $s$ then they have the same steady state.
- Prediction: Poor countries where $k < k^*$ should grow faster than rich countries where $k \approx k^*$.
- **Convergence**: The income gap between rich & poor countries should shrink over time, causing living standards to converge.
- In real world, many poor countries do NOT grow faster than rich ones.
In summary, there is no evidence of (unconditional) convergence in the world income distribution over the postwar era (in fact, the evidence suggests some amount of divergence in incomes across nations). But, there is some evidence for conditional convergence, meaning that the income gap between countries that are similar in observable characteristics appears to narrow over time. This last observation is relevant both for understanding among which countries the economic divergence has occurred and for determining what types of models we should consider for understanding the process of economic growth and the differences in economic performance across nations. For example, we will see that many growth models, including the basic Solow and the neoclassical growth models, suggest that there should be "transitional dynamics" as economies below their steady-state (target) level of income per capita grow towards that level. Conditional convergence is consistent with this type of transitional dynamics.

1.6. Correlates of Economic Growth

The discussion of conditional convergence in the previous section emphasized the importance of certain country characteristics that might be related to the process of economic growth. What types of countries grow more rapidly? Ideally, we would like to answer this question...
Conditional Convergence

- Convergence depends on $f(k), g, n, s$ being equal across countries, which is not the case.

- **Conditional Convergence**: Countries converge to their own steady states, which are determined by $s$, $n$, and education ($g$)
  - Income differences after US Civil War disappeared slowly over time.
  - In samples of countries with similar $s$ and $n$, income gaps shrink about 2% per year.
  - In larger samples, after controlling for differences in $s$, $n$ and human capital ($g$), incomes converge by about 2% per year.
The introduction to modern economic growth discusses the importance of understanding which specific characteristics of countries have a causal effect on growth. This involves answering a counterfactual thought experiment that asks what would happen to equilibrium growth if a particular characteristic were changed "exogenously." Answering such questions is challenging because it's difficult to isolate changes in endogenous variables not driven by equilibrium dynamics or other potentially omitted factors.

The text notes that for the purposes of understanding factors correlated with post-war economic growth, investments in physical capital and human capital are two obvious candidates to consider. Figures 1.15 and 1.16 show a strong positive association between the average growth of investment to GDP ratio and economic growth, as well as a positive correlation between average years of schooling and economic growth. These figures suggest that countries that have grown faster are typically those that have invested more in physical capital and those that had greater human capital at the beginning of the postwar era.
Figure 1.1. Estimates of the distribution of countries according to PPP-adjusted GDP per capita in 1960, 1980 and 2000.

\[ \textit{GDP/capita} \] reflects the \textit{absolute} gap between rich and poor countries.
Figure 1.2. Estimates of the distribution of countries according to log GDP per capita (PPP-adjusted) in 1960, 1980 and 2000.

\[ \log(\text{GDP/capita}) \] reflects the \textit{proportional} gap between rich and poor countries.
The Income Distribution across Countries

1. There is a large amount of inequality in income per capita
2. The absolute gap between rich and poor countries has increased considerably between 1960 and 2000
3. The proportional gap has increased noticeably but much less
4. **Stratification phenomenon**: considerable increase in the density of relatively rich countries, while many countries still remain quite poor
5. **Twin Peaks phenomenon**
What Policies to Promote Growth?

- Is free trade good for economic growth?
- Should policy change the saving rate and if yes, how?
- How should we allocate investment between privately owned physical capital, public infrastructure, and “human capital”?
- How do a country’s institutions affect production efficiency and capital accumulation?
- What policies might encourage faster technological progress?
Free Trade and Growth

- Since Adam Smith, economists have argued that free trade can increase production efficiency and living standards.

- Average Annual Growth Rates 1970-1989:

<table>
<thead>
<tr>
<th></th>
<th>open</th>
<th>closed</th>
</tr>
</thead>
<tbody>
<tr>
<td>developed nations</td>
<td>2.3%</td>
<td>0.7%</td>
</tr>
<tr>
<td>developing nations</td>
<td>4.5%</td>
<td>0.7%</td>
</tr>
</tbody>
</table>

- But correlation does not prove causation.

- Examples of economic growth after opening up (Japan 1850s, South Korea 1960s, Vietnam 1990s,...)
Free Trade and Growth

To determine causation, exploit geographic differences among countries:
- Some nations trade less because they are farther from other nations, or landlocked.
- Such geographical differences are correlated with trade but not with other determinants of income.
- Hence, they can be used to isolate the impact of trade on income.

Findings: increasing trade/GDP by 2% causes GDP per capita to rise 1%, other things equal.

Together with many other studies, there is very strong evidence for the effect of openness on growth.
As China, India, and the Eastern bloc have opened up, world markets and opportunities to export have expanded considerably for advanced economies and developing countries alike.

**Figure 5.3. Share of Developing Countries in Trade**

Sources: OECD, STAN Bilateral Trade Database; and IMF staff calculations.
Should Policy Change the Saving Rate?

We can use the Golden Rule to determine whether the U.S. saving rate and capital stock are too high, too low, or about right:

- Because of diminishing returns $f_k = MP_K$ is decreasing in $k$.
- If $f_k(k^*) - \delta > g + n$, U.S. is below the Golden Rule steady state and should increase $s$.
- If $f_k(k^*) - \delta < g + n$, U.S. is above the Golden Rule steady state and should decrease $s$. 
Should Policy Change the Saving Rate?

To estimate $MPK - \delta$, use three facts about the U.S. economy:

1. $K = 2.5Y$: The capital stock is about 2.5 times one year’s GDP.
2. $\delta K = 0.1Y$: About 10% of GDP is used to replace depreciating capital.
3. $MPK K = 0.3Y$: Capital income is about 30% of GDP (see before).

Using 1 and 2, 
$$\delta = \frac{\delta K}{K} = \frac{0.1Y}{2.5Y} = 0.04$$

Using 3 and 1, 
$$MPK = \frac{MPK K}{K} = \frac{0.3Y}{2.5Y} = 0.12$$

Therefore, 
$$f_k(k^*) - \delta = MPK - \delta = 0.08$$
Should Policy Change the Saving Rate?

- We have $f_k(k^*) - \delta = 0.08$
- U.S. real GDP grows an average of 3% per year, so $g + n = 0.03$.
- Therefore $f_k(k^*) - \delta > g + n$
- The U.S. is below the Golden Rule steady state: Increasing the U.S. saving rate would increase consumption per capita in the long run.
- Similar results hold for most other countries.
Savings (green) and Investment (red) to GDP ratio

Shaded areas indicate US recessions - 2015 research.stlouisfed.org
Global investment has risen during the present economic cycle but remains low by historical standards, particularly in the industrial countries. The corresponding rise in saving has been exclusively in emerging market and oil-producing countries, which are building up high current account surpluses.

Sources: OECD Analytical Database; World Bank, World Development Indicators (2006); and IMF staff calculations.

Includes Norway.
How to Increase the US Saving Rate?

1. Reduce the government budget deficit (or increase the budget surplus).

2. Increase incentives for private saving:
   - Reduce capital gains tax, corporate income tax, estate tax as they discourage saving.
   - Replace federal income tax with a consumption tax.
   - Expand tax incentives for IRAs (individual retirement accounts) and other retirement savings accounts.
How to Allocate Investment?

- In the Solow model, there is one type of capital
- In the real world, there are many types, which we can divide into three categories:
  1. Private capital stock
  2. Public infrastructure
  3. Human capital: the knowledge and skills that workers acquire through education
- How should we allocate investment among these types? i.e. which kind of capital yields the highest marginal products?
How to Allocate Investment?

Two views on the role of government:

1. **Laissez-Faire**: Equalize tax treatment of all types of capital in all industries, then let the market allocate investment to the type with the highest marginal product.

2. **Intervention**: Active Industrial policy, Subsidized Education; Government should actively encourage investment in capital of certain types or in certain industries, because they have high marginal productivity or large positive externalities that private investors do not consider.

   e.g. Green industries. Public Schooling
1.7. From Correlates to Fundamental Causes

The correlates of economic growth, such as physical capital, human capital and technology, will be our first topic of study. But these are only proximate causes of economic growth and economic success (even if we convince ourselves that there is an element of causality in the correlations shown above). It would not be entirely satisfactory to explain the process of economic growth and cross-country differences with technology, physical capital and human capital, since there are, presumably, reasons why technology, physical capital and human capital differ across countries. In particular, if these factors are so important in generating large cross-country income differences and causing the takeoff into modern economic growth, why do certain societies fail to improve their technologies, invest more in physical capital, and accumulate more human capital?

Let us return to Figure 1.8 to illustrate this point further. This figure shows that South Korea and Singapore have managed to grow at very rapid rates over the past 50 years, while Nigeria has failed to do so. We can try to explain the successful performances of South Korea and Singapore, and the failure of Nigeria.
What is the Role of Institutions?

Creating the right institutions is important for ensuring that resources are allocated to their best use:

- **Legal institutions**, to protect property rights
- **Capital markets**, to help financial capital flow to the best investment projects
- A **corruption**-free government to promote competition, enforce contracts, etc.
- **Political freedom (?)**: e.g. democracy vs. autocracy
**Endogenous Growth Theories**

- **Solow model**: sustained growth in living standards is due to technological progress. The rate of technological progress is exogenous.

- **Endogenous growth theory**: a set of models in which the growth rate of productivity and living standards is endogenous.

- What policies might encourage **faster technological progress**?
A Basic AK Model

Suppose the aggregate production function is

\[ Y_t = AK_t \]

where \( A \) is the amount of output for each unit of capital (\( A \) is exogenous & constant)

Key difference between this model and Solow: \( MP_K \) is constant here, diminishing in Solow.

Investment: \( I = sY \)

Law of motion for total capital: \( sAK_t = K_{t+1} - (1 - \delta)K_t \)
A Basic AK Model

\[ sAK_t = K_{t+1} - (1 - \delta)K_t \]
\[ \Leftrightarrow \frac{K_{t+1}}{K_t} = (1 + sA - \delta) \]
\[ \Leftrightarrow \frac{\Delta Y_t}{Y_t} = \frac{\Delta K_t}{K_t} = sA - \delta \]

- If \( sA > \delta \), then income will grow forever, and investment is the “engine of growth”
- Here, the permanent growth rate depends on \( s \). In Solow model, it does not.
Does Capital Have Diminishing Returns?

- Depends on definition of capital.
- If capital is narrowly defined (only plant & equipment), then yes.
- Advocates of endogenous growth theory argue that knowledge is a type of capital.
- If so, then constant returns to capital is more plausible, and this model may be a good description of economic growth.
A Two-Sector model

Two sectors:

- **Manufacturing** firms produce goods using the production function
  \[ Y_t = AK_t^\alpha (E_t (1 - u) N_t)^{1 - \alpha} \]

- **Research universities** produce knowledge that increases labor efficiency in manufacturing according the production function
  \[ \Delta E_t = g(u) E_t \]

\( u \) = fraction of labor in research and is exogenous

Law of motion for total capital: \( sY_t = K_{t+1} - (1 - \delta)K_t \)
A Two-Sector model

\[ sY_t = K_{t+1} - (1 - \delta)K_t \]

\[ \iff \frac{sAK_t^\alpha(E_t(1 - u)N_t)^{1-\alpha}}{E_tN_t} = \frac{K_{t+1} - (1 - \delta)K_t}{E_tN_t} \]

\[ \iff sAk_t^\alpha(1 - u)^{1-\alpha} = (1 + g(u) + n)k_{t+1} - (1 - \delta)k_t \]

With a steady state

\[ \iff sf(k^*, u) = (g(u) + n + \delta)k^* \]

where \( f(k, u) = A(1 - u)^{1-\alpha}k^\alpha \)

Output per worker now grows at a rate \( g(u) \)
A Two-Sector model

- $s$: affects the level of income, but not its growth rate (same as in Solow model)
- $u$: affects level and growth rate of income
- Diminishing returns to capital defined narrowly as $K$
- Constant returns to capital broadly defined as $K$ and $E$ (human capital)

Question: Would an increase in $u$ be unambiguously good for the economy?
Case Study: Growth in China

- China is an economic juggernaut
- Population 1.4 billion people

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<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>530</td>
<td>552</td>
<td>448</td>
<td>6,725</td>
<td>1.9%</td>
</tr>
<tr>
<td>Japan</td>
<td>737</td>
<td>1,387</td>
<td>1,921</td>
<td>22,816</td>
<td>2.5</td>
</tr>
<tr>
<td>United States</td>
<td>2,445</td>
<td>5,301</td>
<td>9,561</td>
<td>31,178</td>
<td>1.9</td>
</tr>
</tbody>
</table>

Note: Figures are in U.S. dollars at 1990 prices, adjusted for differences in the purchasing power of the various national currencies.

## Economic Superpowers 2006

<table>
<thead>
<tr>
<th>Rank</th>
<th>GDP in trillion US dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>World</td>
<td>72.7</td>
</tr>
<tr>
<td>1. European Union</td>
<td>17.5</td>
</tr>
<tr>
<td>2. United States</td>
<td>16.8</td>
</tr>
<tr>
<td>3. People’s Republic of China</td>
<td>9.2</td>
</tr>
<tr>
<td>4. Japan</td>
<td>4.9</td>
</tr>
<tr>
<td>5. Germany</td>
<td>3.7</td>
</tr>
</tbody>
</table>
Case Study: Growth in China

Fast output growth attributable to

**K** Capital Accumulation: Investment is huge in China; at the cost of current consumption, so saving rate $s$ is high

**N** China has a huge labor force; comparative advantage in labor-intensive industries (wages are low).

**A and E** Fast productivity growth, in part from changing to a market economy and increased trade:

- China’s exports were $< 10\%$ of GDP in 1980s; now more than 30\%
- China exports many manufactured goods; imports agricultural products and raw materials
- China runs a small trade surplus