3 Growth Theory

- The Sources of Economic Growth
- Growth Dynamics: The Solow Model
- Government Policies to Raise Long-Run Living Standards

Notation: all variables ($Y_t$, $K_t$, $C_t$, ...) are in REAL terms
Why Growth Matters

For developing countries:

- Economic growth raises living standards and reduces poverty
- Infant mortality rates: 20% in poorest 20% of all countries, 0.5% in the richest 20% of all countries
- 25% of the poorest countries have had famines during the past 3 decades
- Poverty is associated with oppression of women and minorities
- Political instability, war, health, corruption, ...
Why Growth Matters

For developed countries:

- Tiny changes in growth rates have huge effects on living standards in the long run.

<table>
<thead>
<tr>
<th>annual growth rate of income per capita</th>
<th>percentage increase in standard of living after…</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>...25 years</td>
</tr>
<tr>
<td>2.0%</td>
<td>64.0%</td>
</tr>
<tr>
<td>2.5%</td>
<td>85.4%</td>
</tr>
</tbody>
</table>
## Income per capita in 1990 Dollars

<table>
<thead>
<tr>
<th>year AD:</th>
<th>1</th>
<th>1000</th>
<th>1500</th>
<th>1820</th>
<th>1950</th>
<th>2003</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>400</td>
<td>400</td>
<td>400</td>
<td>1,257</td>
<td>9,561</td>
<td>29,037</td>
</tr>
<tr>
<td>France</td>
<td>473</td>
<td>425</td>
<td>727</td>
<td>1,135</td>
<td>5,271</td>
<td>21,861</td>
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<tr>
<td>Italy</td>
<td>809</td>
<td>450</td>
<td>1,100</td>
<td>1,117</td>
<td>3,502</td>
<td>19,150</td>
</tr>
<tr>
<td>Japan</td>
<td>400</td>
<td>425</td>
<td>500</td>
<td>669</td>
<td>1,921</td>
<td>21,218</td>
</tr>
<tr>
<td>Mexico</td>
<td>400</td>
<td>400</td>
<td>425</td>
<td>759</td>
<td>2,365</td>
<td>7,137</td>
</tr>
<tr>
<td>China</td>
<td>450</td>
<td>466</td>
<td>600</td>
<td>600</td>
<td>448</td>
<td>4,803</td>
</tr>
<tr>
<td>Iraq</td>
<td>500</td>
<td>650</td>
<td>550</td>
<td>588</td>
<td>1,364</td>
<td>1,023</td>
</tr>
<tr>
<td>Argentina</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>4,987</td>
<td>7,666</td>
</tr>
<tr>
<td>Kenya</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>651</td>
<td>998</td>
</tr>
</tbody>
</table>

Source: Angus Maddison
Differences Even Across Industrialized Countries

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>3,273</td>
<td>5,157</td>
<td>7,412</td>
<td>25,301</td>
<td>1.5%</td>
</tr>
<tr>
<td>Canada</td>
<td>1,695</td>
<td>4,447</td>
<td>7,291</td>
<td>25,267</td>
<td>2.0</td>
</tr>
<tr>
<td>France</td>
<td>1,876</td>
<td>3,485</td>
<td>5,186</td>
<td>22,223</td>
<td>1.8</td>
</tr>
<tr>
<td>Germany</td>
<td>1,839</td>
<td>3,648</td>
<td>3,881</td>
<td>20,801</td>
<td>1.8</td>
</tr>
<tr>
<td>Japan</td>
<td>737</td>
<td>1,387</td>
<td>1,921</td>
<td>22,816</td>
<td>2.5</td>
</tr>
<tr>
<td>Sweden</td>
<td>1,359</td>
<td>3,073</td>
<td>6,769</td>
<td>24,409</td>
<td>2.1</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>3,190</td>
<td>4,921</td>
<td>6,939</td>
<td>23,742</td>
<td>1.5</td>
</tr>
<tr>
<td>United States</td>
<td>2,445</td>
<td>5,301</td>
<td>9,561</td>
<td>31,178</td>
<td>1.9</td>
</tr>
</tbody>
</table>

*Note:* Figures are in U.S. dollars at 1990 prices, adjusted for differences in the purchasing power of the various national currencies.

Growth Accounting

\[ Y_t = A_t K_t^\alpha N_t^{1-\alpha} \]

\[ \Rightarrow \frac{\Delta Y_t}{Y_t} = \frac{\Delta A_t}{A_t} + \alpha \frac{\Delta K_t}{K_t} + (1 - \alpha) \frac{\Delta N_t}{N_t} \]

A change in real output must come from either of three sources:

- \( \frac{\Delta A_t}{A_t} \): **Solow residual**, changes in technology, supply shocks e.g. weather, inventions and innovations, government regulations, oil prices
- \( \frac{\Delta K_t}{K_t} \): **capital accumulation/decumulation**, i.e. investment or depreciation of existing capital
- \( \frac{\Delta N_t}{N_t} \): **changes in the number of hours worked**, e.g. changes in population, length of the workweek, unemployment
### Table 8-3

**Accounting for Economic Growth in the United States**

<table>
<thead>
<tr>
<th>Years</th>
<th>Output Growth $\Delta Y/Y$</th>
<th>Source of Growth</th>
<th>Total Factor Productivity $\Delta A/A$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Capital $\alpha\Delta K/K$</td>
<td>Labor $(1-\alpha)\Delta L/L$</td>
</tr>
<tr>
<td>1948–2002</td>
<td>3.6</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>1948–1972</td>
<td>4.0</td>
<td>1.2</td>
<td>1.0</td>
</tr>
<tr>
<td>1972–1995</td>
<td>3.2</td>
<td>1.3</td>
<td>1.4</td>
</tr>
<tr>
<td>1995–2002</td>
<td>3.7</td>
<td>1.7</td>
<td>0.9</td>
</tr>
</tbody>
</table>

(average percentage increase per year)

*Source: U.S. Department of Labor. Data are for the non-farm business sector.*
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor growth</td>
<td>1.42</td>
<td>1.40</td>
<td>1.13</td>
<td>1.34</td>
<td>0.99</td>
</tr>
<tr>
<td>Capital growth</td>
<td>0.11</td>
<td>0.77</td>
<td>0.69</td>
<td>0.56</td>
<td>1.18</td>
</tr>
<tr>
<td>Total input growth</td>
<td>1.53</td>
<td>2.17</td>
<td>1.82</td>
<td>1.90</td>
<td>2.17</td>
</tr>
<tr>
<td>Productivity growth</td>
<td>1.01</td>
<td>1.53</td>
<td>−0.27</td>
<td>1.02</td>
<td>1.06</td>
</tr>
<tr>
<td><strong>Total output growth</strong></td>
<td><strong>2.54</strong></td>
<td><strong>3.70</strong></td>
<td><strong>1.55</strong></td>
<td><strong>2.92</strong></td>
<td><strong>3.23</strong></td>
</tr>
</tbody>
</table>

Labor Productivity

\[
Y_t = A_t K_t^\alpha N_t^{1-\alpha}
\]

\[
\frac{Y_t}{N_t} = A_t K_t^\alpha N_t^{-\alpha}
\]

\[
\frac{\Delta Y_t}{Y_t} - \frac{\Delta N_t}{N_t} = \frac{\Delta A_t}{A_t} + \alpha \left( \frac{\Delta K_t}{K_t} - \frac{\Delta N_t}{N_t} \right)
\]

- Output per unit of labor, \( \frac{Y_t}{N_t} \) is labor productivity.
- Do not confuse with total factor productivity (TFP) \( A_t \).
- Labor productivity growth equals TFP growth plus \( \alpha \) times the growth rate of capital per unit of labor.
Note the widening gap between labor productivity growth and TFP growth since 1995
Observations about Growth

1. Output growth rates vary over time and across countries.
2. About 2/3 of US growth can be explained by factor accumulation, i.e. capital accumulation and population growth.
3. About 1/3 of US growth is due to technological progress.
4. TFP growth slowdown in the US and all major developed countries after 1973.
5. US surge in labor productivity growth since the late 1990s, but not in Europe.
What caused the post-1973 decline in productivity?

1. *Measurement:* Inadequate accounting for quality improvements (same problem as for the price indices)

2. *The legal and human environment:* Regulations for pollution control and worker safety, declines in educational quality

3. *Oil prices:* Huge increase in oil prices reduced productivity of capital and labor, especially in basic industries. But why no productivity increase after real oil price declines in the 1980s?
What caused the post-1973 decline in productivity?

4. *Worker Quality*
   Baby boom generation lowers the average level of experience which lowers average productivity.

5. *Depletion of Ideas*
   Large backlog of unimplemented ideas after Great Depression and WWII, 1970s is return to normality.

6. *New industrial revolution:*
   Learning process for information technology from 1973 to 1990 meant slower growth
   In the UK and US, productivity also fell at the onset of industrialization around the 1800s.
What caused the post 1995 surge in US $Y/N$ growth?

Labor productivity growth exceeds TFP growth because of faster growth of capital relative to growth of labor

$$\frac{\Delta Y_t}{Y_t} - \frac{\Delta N_t}{N_t} = \frac{\Delta A_t}{A_t} + \alpha \left( \frac{\Delta K_t}{K_t} - \frac{\Delta N_t}{N_t} \right)$$

- ICT growth (information and communications technology) may have been a prime reason.
- But why did ICT not contribute in other countries?
- Why such a lag between investment in ICT (1980s) and growth in productivity (late 1990s)?
The Solow Model

- What is the relationship between the long-run standard of living and the consumption-saving decision, the population growth rate, and rate of technical progress?
- How does economic growth change over time? Will it speed up, slow down, or stabilize?
- Are there economic forces that will allow poorer countries to catch up to richer countries?
Basic Assumptions of the Solow Model

**Assumption 1**: The economy is closed, no government.

\[ G_t = 0, \ S_t = I_t \text{ and } C_t + I_t = Y_t \]

**Assumption 2**: Cobb-Douglas aggregate production:

\[ Y_t = A_t K_t^\alpha N_t^{1-\alpha} \]

**Assumption 3**: Population = labor input \( N_t \) grows at a constant rate \( n \)

**Assumption 4**: No productivity growth (we’ll add it later), \( A_t = A \)
Definitions

c_t = C_t / N_t, i_t = I_t / N_t, k_t = K_t / N_t, y_t = Y_t / N_t denote per capita variables

\[
Y_t = AK_t^\alpha N_t^{1-\alpha}
\]

\[
\Leftrightarrow \frac{Y_t}{N_t} = \frac{AK_t^\alpha N_t^{1-\alpha}}{N_t}
\]

\[
\Leftrightarrow y_t = Ak_t^\alpha \equiv f(k_t)
\]

\[y_t = f(k_t)\] is the per worker production function

\[k_t\] is the capital-labor ratio.
Per-worker production function, $y_t = f(k_t)$

Output per worker, $y_t$

Capital–labor ratio, $k_t$
Basic Assumptions of the Solow Model

Assumption 5: Capital depreciates at a rate $\delta$ such that

$$K_{t+1} = I_t + (1 - \delta)K_t$$

$$\Leftrightarrow \frac{K_{t+1}}{N_t} = \frac{I_t}{N_t} + (1 - \delta)\frac{K_t}{N_t}$$

$$\Leftrightarrow \frac{N_{t+1}}{N_t} \frac{K_{t+1}}{N_{t+1}} = \frac{I_t}{N_t} + (1 - \delta)\frac{K_t}{N_t}$$

$$\Leftrightarrow (1 + n)k_{t+1} = i_t + (1 - \delta)k_t$$

Note: $\frac{N_{t+1}}{N_t} = 1 + n$ where $n$ is the population growth rate
Assumption 6: Consumption function is $C_t = (1 - s) Y_t$ where $0 < s < 1$ is the saving rate.

\[
C_t = (1 - s) Y_t \\
\Leftrightarrow \frac{C_t}{N_t} = (1 - s) \frac{Y_t}{N_t} \\
\Leftrightarrow c_t = (1 - s) y_t
\]

Note: Big simplification relative to the consumption function we derived earlier. (Why?)
The Solow Model

So we have in per capita terms:

\[ y_t = f(k_t) \]  \hspace{1cm} \text{(Production)}
\[ c_t + i_t = y_t \]  \hspace{1cm} \text{(Goods Market Equilibrium)}
\[ (1 + n)k_{t+1} = i_t + (1 - \delta)k_t \]  \hspace{1cm} \text{(Capital Accumulation)}
\[ c_t = (1 - s)y_t \]  \hspace{1cm} \text{(Consumption Function)}

which reduces to

\[ sf(k_t) = (1 + n)k_{t+1} - (1 - \delta)k_t \]
The Steady State

Definition: A steady state $k^*$ is when $k_{t+1} = k_t = k^*$

$$sf(k^*) = (1 + n)k^* - (1 - \delta)k^*$$

$$\iff sf(k^*) = (n + \delta)k^*$$

If $k$ begins at some level other than $k^*$, it will move toward $k^*$:

- For $k < k^*$, saving $>$ the amount of investment needed to keep $k$ constant, so $k \uparrow$
- For $k > k^*$, saving $<$ the amount of investment needed to keep $k$ constant, so $k \downarrow$
Abstracting from productivity growth:

- the economy reaches a steady state, with constant capital-labor ratio, output per worker, and consumption per worker

- the fundamental determinants of long-run living standards are the saving rate $s$ and population growth $n$, productivity $A$:
  - $s \uparrow$ means higher $K/N$, higher $Y/N$, and higher $C/N$
  - $n \uparrow$ means lower $K/N$, lower $Y/N$, and $C/N$
  - $A \uparrow$ means higher $K/N$, higher $Y/N$, and higher $C/N$
Higher Saving Rate $s$

1. Saving increases

2. Capital–labor ratio increases

Steady-state investment per worker, $(n + d)k$

New saving per worker, $s_2 f(k)$

Initial saving per worker, $s_1 f(k)$
Higher Population Growth $n$

1. Population growth rate increases

New steady-state investment per worker, $(n_2 + d)k$

Initial steady-state investment per worker, $(n_1 + d)k$

Saving per worker, $sf(k)$

2. Capital–labor ratio decreases

Capital–labor ratio, $k$
Higher Productivity

1. Productivity increases

2. Capital–labor ratio increases
Evaluating the Solow Model

Predictions of the Solow Model:

1. Countries with higher rates of saving and investment will have higher levels of capital and income per worker in the long run.

2. Countries with higher population growth will have lower levels of capital and income per worker in the long run.
Income per person in 2000 (logarithmic scale)

Population growth (percent per year; average 1960-2000)
The Golden Rule

Question: If higher saving rates increases income per capita, should a policy goal be to raise the saving rate?

- No! If $s = 1$, output per capita is high but consumption is zero.
- The “best” or **golden rule** steady state has the highest possible consumption per person:

$$c^* = (1 - s)f(k^*)$$

$$= f(k^*) - (n + \delta)k^*$$

- The golden rule $k_{\text{gold}}^*$ and $c_{\text{gold}}^*$ are those that maximize this expression:

$$f_k(k^*) = (n + \delta)$$

where $f_k$ is the first derivative of $f$. 

Transition to the Golden Rule Steady State

- The economy does NOT have a tendency to move toward the Golden Rule steady state.
- Achieving the Golden Rule requires that policymakers adjust $s$.
- This adjustment leads to a new steady state with higher consumption.
- But what happens to consumption during the transition to the Golden Rule?
Transition if $k > k^*_{\text{gold}}$
Transition if $k < k^*_{gold}$

The saving rate is increased.
Starting with too much capital $k > k_{gold}$:
In the transition to the Golden Rule, consumption is higher at all points in time.

Starting with too little capital $k < k_{gold}$:
Future generations enjoy higher consumption, but the current one experiences an initial drop in consumption.

If the policymaker cares more about current than future generations, he may decide not to raise $s$.

Biblical Golden Rule: “Do unto others as you would have them do to you”

If we care about future generations, we should aim for $k_{gold}$.