2. Three Key Aggregate Markets

2.1 The Labor Market: Productivity, Output and Employment
2.2 The Goods Market: Consumption, Saving and Investment
2.3 The Asset Market: Money and Inflation
2.2 The Goods Market: Consumption, Saving and Investment

- Last chapter: determination of the **real wage** in labor market equilibrium
- This chapter: determination of the **real interest rate** in goods market equilibrium
  - Consumption and Saving
  - Investment
Recall the *income-expenditure identity*:

\[ Y = C + I + G + NX \]

The total aggregate demand for domestic goods consists of the sum of

- demand for goods for *consumption* \( C \)
- demand for goods for *investment* \( I \)
- net demand for domestic goods \( NX \)
- demand for goods for *government uses* \( G \)
The Trade Balance $NX$

**Assumption 1**: The economy is closed

- No international trade, the trade balance is zero, i.e. $NX = 0$
- No net factor payments, i.e. $NFP = 0$
- Therefore the current account $CA = NX + NFP = 0$
- We will consider the open economy later.
Government Spending $G$

**Assumption 2a:** Demand for goods on behalf of the government is given, $G = \bar{G}$

**Assumption 2b:** Taxes are given, $T = \bar{T}$

- Government spending $G$ and taxation $T$ are determined by the political process.
- We treat them as **exogenous** variables.
- Assume no transfers, $TR = 0$
- Note the government budget deficit: $D^{govt} = \bar{G} + INT - \bar{T}$. 
Consumption and Saving

Under assumptions 1, 2a and 2b:

- **Private disposable income** $Y^d$ is given by

$$Y^d = Y + INT - \bar{T}$$
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- Households allocate $Y^d$ for saving or consumption:
  \[
  
  Y^d = Y + \text{INT} - \bar{T} \\
  Y^d = S + C + \bar{G} + \text{INT} - \bar{T} \\
  Y^d = S^{priv} + C
  
  \]
Consumption and Saving

\[ Y^d = S^{priv} + C \]

- Understanding consumption requires understanding the savings decision.
- Trade-off between *current* consumption and *future* consumption
Consumption and Saving

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**Assumption 3:** Trade-off occurs through utility maximization
Utility Maximization

Household chooses $C_t$ and $C_{t+1}$ to maximize utility

$$U(c_t, c_{t+1}) = u(c_t) + \beta u(c_{t+1})$$

where

- $c_t$ is real current consumption
- $c_{t+1}$ is real future consumption
- $0 < \beta < 1$ is a **discount factor** that captures impatience

subject to

period $t$ budget constraint $P_t c_t + S_t = Y_t^d$

period $t + 1$ budget constraint $P_{t+1} c_{t+1} = Y_{t+1}^d + (1 + i_t) S_t$

where $i_t$ is the nominal interest rate.
Equivalent to choosing $S_t$ to maximize

$$u \left( \frac{Y_t^d - S_t}{P_t} \right) + \beta u \left( \frac{Y_{t+1}^d + (1 + i_t)S_t}{P_{t+1}} \right)$$
Equivalent to choosing $S_t$ to maximize

$$u \left( \frac{Y^d_t - S_t}{P_t} \right) + \beta u \left( \frac{Y^d_{t+1} + (1 + i_t)S_t}{P_{t+1}} \right)$$

Optimality requires

$$-\frac{u_c(c_t)}{P_t} + \beta \frac{u_c(c_{t+1})}{P_{t+1}} (1 + i_t) = 0$$

where $u_c$ is the first derivative and denotes marginal utility.

- Saving one additional dollar more in $t$, means consuming $1/P_t$ less in $t$ which lowers utility by $u_c(c_t)/P_t$

- Saving one additional dollar more in $t$, means consuming $(1 + i_t)/P_{t+1}$ more in $t + 1$ which increases utility by $\beta \frac{u_c(c_{t+1})}{P_{t+1}} (1 + i_t)$.

- marginal cost of saving = marginal benefit of saving
A Specific Example

Suppose that

\[ u(c) = \frac{c^{1 - \frac{1}{\sigma}}}{1 - \frac{1}{\sigma}}, \sigma > 0 \]
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Then

\[ \frac{c_t^{-\frac{1}{\sigma}}}{P_t} = \beta \frac{c_{t+1}^{-\frac{1}{\sigma}}}{P_{t+1}} (1 + i_t) \]

\[ r_t \] is the real interest rate

Note that \( 1 + r_t \approx 1 + i_t - \pi_t \)
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\[
\frac{c_t^{\frac{1}{\sigma}}}{P_t} = \beta \frac{c_{t+1}^{\frac{1}{\sigma}}}{P_{t+1}} (1 + i_t)
\]

\[\Leftrightarrow c_t = c_{t+1} \left( \beta \frac{P_t}{P_{t+1}} (1 + i_t) \right)^{-\sigma} \]
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\[ \Leftrightarrow c_t = c_{t+1} \left( \beta \frac{1 + i_t}{1 + \pi_{t+1}} \right)^{-\sigma} \]

\( r_t \) is the real interest rate

Note that \( 1 + r_t = 1 + i_t - \pi_t + 1 \approx 1 + i_t - \pi_t + 1 \)
A Specific Example

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\[ u(c) = \frac{c^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}, \sigma > 0 \]

Then

\[ \frac{c_t^{\frac{1}{\sigma}}}{P_t} = \beta \frac{c_{t+1}^{\frac{1}{\sigma}}}{P_{t+1}} (1 + i_t) \]

\( \Leftrightarrow \)

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\( \Leftrightarrow \)

\[ c_t = c_{t+1} (\beta (1 + r_t))^{-\sigma} \]

\( r_t \) is the real interest rate

Note that

\[ 1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}} \approx 1 + i_t - \pi_{t+1} \]
Consider again the budget constraints

\[
\begin{align*}
P_t c_t + S_t &= Y_t^d \\
P_{t+1} c_{t+1} &= Y_{t+1}^d + (1 + i_t) S_t
\end{align*}
\]
Consider again the budget constraints

\[ P_t c_t + S_t = Y_t^d \]
\[ P_{t+1} c_{t+1} = Y_{t+1}^d + (1 + i_t) S_t \]

We can eliminate \( S_t \) and write

\[ P_t c_t + \frac{P_{t+1} c_{t+1}}{1 + i_t} = Y_t^d + \frac{Y_{t+1}^d}{1 + i_t} \]
\[ \Leftrightarrow c_t + \frac{c_{t+1}}{1 + r_t} = y_t^d + \frac{y_{t+1}^d}{1 + r_t} \]

where \( y_t^d = Y_t^d / P_t \) and \( y_{t+1}^d = Y_{t+1}^d / P_{t+1} \) denote real disposable incomes.
Final step: use $c_t = c_{t+1} (\beta(1 + r_t))^{-\sigma}$ $\iff c_{t+1} = c_t (\beta(1 + r_t))^\sigma$
and plug into

\[ c_t = y_t^d + \frac{y_{t+1}^d}{1 + r_t} - \frac{c_{t+1}}{1 + r_t} \]
Final step: use $c_t = c_{t+1} \left( \beta (1 + r_t) \right)^{-\sigma} \Leftrightarrow c_{t+1} = c_t \left( \beta (1 + r_t) \right)^\sigma$

and plug into

$$c_t = y_t^d + \frac{y_{t+1}^d}{1 + r_t} - \frac{c_{t+1}}{1 + r_t}$$

$$\Leftrightarrow c_t = y_t^d + \frac{y_{t+1}^d}{1 + r_t} - \frac{c_t \left( \beta (1 + r_t) \right)^\sigma}{1 + r_t}$$
Final step: use $c_t = c_{t+1} (\beta (1 + r_t))^{-\sigma} \iff c_{t+1} = c_t (\beta (1 + r_t))^\sigma$

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$$\iff c_t = y_t^d + \frac{y_{t+1}^d}{1 + r_t} - \frac{c_t (\beta (1 + r_t))^\sigma}{1 + r_t}$$

$$\iff c_t = \left[y_t^d + \frac{y_{t+1}^d}{1 + r_t}\right] \left(1 + (\beta^\sigma (1 + r_t)^{\sigma - 1})\right)^{-1}$$

This last expression allows us to evaluate the determinants of consumption.
\[
c_t = \left[ y_t^d + \frac{y_{t+1}^d}{1 + r_t} \right] (1 + (\beta^\sigma (1 + r_t)^{\sigma - 1}))^{-1}
\]

**Result 1**  Consumption is a fraction \((1 + (\beta^\sigma (1 + r_t)^{\sigma - 1}))^{-1}\) of the net present value (NPV) of lifetime wealth \([y_t^d + \frac{y_{t+1}^d}{1 + r_t}]\). There is consumption smoothing.
\[ c_t = \left[ y_t^d + \frac{y_{t+1}^d}{1 + r_t} \right] (1 + (\beta^\sigma (1 + r_t)^{\sigma-1}))^{-1} \]

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**Result 2** Current consumption \(c_t\) increases with current real disposable income \(y_t^d\)
\[ c_t = \left[ y^d_t + \frac{y^d_{t+1}}{1 + r_t} \right] \left( 1 + \left( \beta^\sigma (1 + r_t)^{\sigma - 1} \right) \right)^{-1} \]

Result 1 Consumption is a fraction \( (1 + (\beta^\sigma (1 + r_t)^{\sigma - 1}))^{-1} \) of the net present value (NPV) of lifetime wealth \( \left[ y^d_t + \frac{y^d_{t+1}}{1 + r_t} \right] \)

There is consumption smoothing.

Result 2 Current consumption \( c_t \) increases with current real disposable income \( y^d_t \)

Result 3 Current consumption \( c_t \) increases with future real disposable income \( y^d_{t+1} \)
\[ c_t = \left[ y_t^d + \frac{y_{t+1}^d}{1 + r_t} \right] \left( 1 + (\beta^\sigma (1 + r_t)^{\sigma-1}) \right)^{-1} \]

**Result 4**  The result of an increase in the real interest rate \( r_t \) on current consumption is ambiguous
\[ c_t = \left[ y_t^d + \frac{y_{t+1}^d}{1 + r_t} \right] \left( 1 + (\beta^\sigma (1 + r_t)^{\sigma - 1}) \right)^{-1} \]

**Result 4** The result of an increase in the real interest rate \( r_t \) on current consumption is ambiguous

- **Substitution effect** –:
  \( r_t \uparrow \) makes saving more attractive and \( c_t \downarrow \). Depends on \( \sigma \), the *elasticity of intertemporal substitution*. 

\("the real expected interest rate"\)}
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**Result 4** The result of an increase in the real interest rate \( r_t \) on current consumption is ambiguous

- **Substitution effect** \(-\): \( r_t \uparrow \) makes saving more attractive and \( c_t \downarrow \). Depends on \( \sigma \), the *elasticity of intertemporal substitution*.
- **Income effect** \(+\): Keeping fixed the NPV of lifetime wealth \( r_t \uparrow \) lowers the price of \( c_{t+1} \) and expands the feasible consumption set, leading to \( c_t \uparrow \)
\[ c_t = \left[ y_t^d + \frac{y_{t+1}^d}{1 + r_t} \right] (1 + (\beta^\sigma (1 + r_t)^{\sigma-1}))^{-1} \]

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- **Substitution effect** -: \( r_t \uparrow \) makes saving more attractive and \( c_t \downarrow \). Depends on \( \sigma \), the *elasticity of intertemporal substitution*.
- **Income effect** +: Keeping fixed the NPV of lifetime wealth \( r_t \uparrow \) lowers the price of \( c_{t+1} \) and expands the feasible consumption set, leading to \( c_t \uparrow \)
- **Wealth effect** -: \( r_t \uparrow \) decreases the NPV of lifetime wealth and \( c_t \downarrow \)
Result 4  The result of an increase in the real interest rate $r_t$ on current consumption is ambiguous

- **Substitution effect** $-$:
  $r_t \uparrow$ makes saving more attractive and $c_t \downarrow$. Depends on $\sigma$, the *elasticity of intertemporal substitution*.

- **Income effect** $+$:
  Keeping fixed the NPV of lifetime wealth $r_t \uparrow$ lowers the price of $c_{t+1}$ and expands the feasible consumption set, leading to $c_t \uparrow$

- **Wealth effect** $-$:
  $r_t \uparrow$ decreases the NPV of lifetime wealth and $c_t \downarrow$

Note, in reality $\pi_{t+1}$ is unknown in period $t$ and $r_t$ is the real *expected* interest rate
\[
c_t = \left[ y_t^d + \frac{y_{t+1}^d}{1 + r_t} \right] \left( 1 + (\beta^\sigma (1 + r_t)^{\sigma-1}) \right)^{-1}
\]

\[
c_t = c_t^a + MPC_t y_t^d
\]

The **marginal propensity to consume** (MPC): If current real disposable income \( y_t^d \) increases, how much does current consumption \( c_t \) increase?

\[
MPC_t = \frac{1}{1 + (\beta^\sigma (1 + r_t)^{\sigma-1})}
\]
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- \(0 < MPC < 1\)
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- \( 0 < MPC < 1 \)
- if \( \sigma > 1 \), substitution effect dominates income effect and \( MPC_t(r_t) \)
\[
ct = \left[ y_t^d + \frac{y_{t+1}^d}{1 + r_t} \right] (1 + (\beta^\sigma(1 + r_t)^{\sigma-1}))^{-1}
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- if \(\sigma < 1\), income effect dominates substitution effect and \(MPC_t(r_t)\)
Question: Do tax rebates stimulate consumer spending?

A tax rebate of $800 raises \( y_d t \) by \( \frac{800}{P_t} \). The rebate must be financed today by increased government borrowing at an interest rate \( 1 + i_t \). But must eventually be paid by higher taxes in the future: so \( y_d t+1 \) decreases by \( (1 + r_t)\frac{800}{P_{t+1}} = (1 + r_t)\frac{800}{P_t} \). The NPV of lifetime wealth is unchanged:

\[
\left[ y_d t + \frac{800}{P_t} + y_d t+1 - (1 + r_t)\frac{800}{P_{t+1}} \right] = y_d t + y_d t+1.
\]

Ricardian Equivalence: Tax rebates have no effect on consumption! Theoretical prediction: most consumers will save their tax rebates and not spend them (as in rebate of 2001).
Question: Do tax rebates stimulate consumer spending?

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- The NPV of lifetime wealth is unchanged:

\[
\left[ y_t^d + \frac{y_{t+1}^d}{1 + r_t} - \frac{(1 + r_t)\$800}{P_t} \right] = y_t^d + \frac{y_{t+1}^d}{1 + r_t}
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Question: Do tax rebates stimulate consumer spending?

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- The NPV of lifetime wealth is unchanged:

\[
\left[ d_t + \frac{y^{d}_t}{P_t} + \frac{y^{d}_{t+1} - (1 + r_t)800/P_t}{1 + r_t} \right] = \frac{y^{d}_t}{1 + r_t} + \frac{y^{d}_{t+1}}{1 + r_t}
\]

- **Ricardian Equivalence**: Tax rebates have no effect on consumption!
- Theoretical prediction: most consumers will save their tax rebates and not spend them (as in rebate of 2001).
Summarizing what we have so far:

\[ Y = C + I + \bar{G} \]

\[ Y = P \left( c^a(r_t) + MPC \times y^d \right) + I + \bar{G} \]

\[ Y = C^a(r_t) + MPC \times Y^d + I + \bar{G} \]

- \( \bar{G} \) exogenous
- \( C \) depends positively on current disposable income \( Y^d \) through \( MPC \)
- We will assume from now on that \( \sigma \approx 1 \) and therefore that \( MPC \) is constant and \( C \) depends negatively on \( r_t \).
- We still need to determine what drives investment \( I \).
Why is investment important?

- Investment fluctuates sharply over the business cycle, so we need to understand investment to understand the business cycle.
- Investment plays a crucial role in economic growth:

\[ K_{t+1} = (1 - \delta)K_t + x_t \]

where \( x_t \) is real investment and \( K_t \) is the real capital stock and remember

\[ \frac{\Delta y_t}{y_t} = \frac{\Delta A_t}{A_t} + \alpha \frac{\Delta K_t}{K_t} + (1 - \alpha) \frac{\Delta N_t}{N_t} \]
In the previous chapter we derived the demand for capital

\[ K^d = (A\alpha)^{\frac{1}{1-\alpha}} \left( \frac{R}{P} \right)^{-\frac{1}{1-\alpha}} N \]

- Demand for capital is such that \( MP_K = \frac{R}{P} \)
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- Demand for capital is such that \( MP_K = \frac{R}{P} \)
- \( \frac{R}{P} \) is real the rental price of capital or the user cost of capital
- Demand for capital depends negatively on the user cost of capital
  \[ \frac{R}{P} = r + \delta \]
- foregone real interest \( r_t \)
- depreciation \( \delta \)
For someone that saves by buying e.g. a government bond:

\[-\frac{u_c(c_t)}{P_t} + \beta \frac{u_c(c_{t+1})}{P_{t+1}} (1 + i_t) = 0\]
For someone that saves by buying e.g. a government bond:

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For someone that saves by buying a unit of capital in $t$ and rents it out to firms in $t + 1$:

\[- \frac{u_c(c_t)}{P_t} + \beta \frac{u_c(c_{t+1})}{P_{t+1}} \left( \frac{R_{t+1} + P_{t+1}(1 - \delta)}{P_t} \right) = 0\]
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Therefore

\[(1 + i_t) = \frac{R_{t+1} + P_{t+1}(1 - \delta)}{P_t}\]

\[\frac{P_t}{P_{t+1}} (1 + i_t) = \frac{R_{t+1}}{P_{t+1}} + (1 - \delta)\]

\[1 + r_t = \frac{R_{t+1}}{P_{t+1}} + (1 - \delta)\]

\[\Leftrightarrow \frac{R_{t+1}}{P_{t+1}} = r_t + \delta\]
If today's real interest rate is high, tomorrow's user cost $R_{t+1}/P_{t+1}$ is high, the desired capital stock for tomorrow is low. \[ R_{t+1}/P_{t+1} = r_t + \delta \]

If today's real interest rate is low, tomorrow's user cost is low, the desired capital stock for tomorrow is high. \[ R_{t+1}/P_{t+1} = r_t + \delta \]

Again note, in reality the future is unknown and $r_t$ is the expected real interest rate.

Bottomline: Demand for investment depends negatively on the (expected) real interest rate!
\[
\frac{R_{t+1}}{P_{t+1}} = r_t + \delta
\]

- If today’s real interest rate is high, tomorrow’s user cost \( \frac{R_{t+1}}{P_{t+1}} \) is high, the desired capital stock for tomorrow is low.  
  \( \Rightarrow \) today’s investment is low
\[
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\]

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  \( \Rightarrow \) today’s investment is high
\[
\frac{R_{t+1}}{P_{t+1}} = r_t + \delta
\]

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- If today’s real interest rate is low, tomorrow’s user cost \( \frac{R_{t+1}}{P_{t+1}} \) is low, the desired capital stock for tomorrow is high.  
  \( \Rightarrow \) today’s investment is high

- Again note, in reality the future is unknown and \( r_t \) is the expected real interest rate.

- Bottomline: Demand for investment depends negatively on the (expected) real interest rate!
Result 1: Current investment \( x_t \) depends negatively on \( r_t \) since it increases the user cost of capital which lowers the desired capital stock \( K_{t+1}^d \).

Result 2: Current investment \( x_t \) depends positively on total factor productivity tomorrow \( A_{t+1} \) because it raises the desired capital stock \( K_{t+1}^d \).

Result 3: Current investment \( x_t \) depends positively on labor input tomorrow \( N_{t+1} \) because it raises the desired capital stock \( K_{t+1}^d \).

\[
K_{t+1}^d = (A_{t+1} \alpha)^{\frac{1}{1-\alpha}} (r_t + \delta)^{-\frac{1}{1-\alpha}} N_{t+1}
\]
\[
K_{t+1}^d = (1 - \delta)K_t + x_t
\]
\[ K_{t+1}^d = (A_{t+1} \alpha) \left( r_t + \delta \right)^{-\frac{1}{1-\alpha}} N_{t+1} \]
\[ K_{t+1}^d = (1 - \delta) K_t + x_t \]

**Result 1** Current investment \( x_t = \frac{I_t}{P_t} \) depends negatively on \( r_t \) since it increases the user cost of capital which lowers the desired capital stock \( K_{t+1}^d \).
\[ K_{t+1}^d = (A_{t+1}^\alpha)^{\frac{1}{1-\alpha}} (r_t + \delta)^{-\frac{1}{1-\alpha}} N_{t+1} \]
\[ K_{t+1}^d = (1 - \delta) K_t + x_t \]

Result 1  Current investment \( x_t = \frac{l_t}{P_t} \) depends negatively on \( r_t \) since it increases the user cost of capital which lowers the desired capital stock \( K_{t+1}^d \).

Result 2  Current investment \( x_t = \frac{l_t}{P_t} \) depends positively on total factor productivity tomorrow \( A_{t+1} \) because it raises the desired capital stock \( K_{t+1}^d \).
\[
K_{t+1}^d = (A_{t+1} \alpha)^{\frac{1}{1-\alpha}} (r_t + \delta)^{-\frac{1}{1-\alpha}} N_{t+1}
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Result 1  Current investment \( x_t = \frac{l_t}{P_t} \) depends negatively on \( r_t \) since it increases the user cost of capital which lowers the desired capital stock \( K_{t+1}^d \).

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Result 3  Current investment \( x_t = \frac{l_t}{P_t} \) depends positively on labor input tomorrow \( N_{t+1} \) because it raises the desired capital stock \( K_{t+1}^d \).
Summarizing what we have so far:

\[ Y = C + I + \bar{G} \]

\[ Y = P \left( c^a(\bar{r}_t) + MPC \times y^d \right) + I(\bar{r}_t) + \bar{G} \]

\[ Y = C^a(\bar{r}_t) + MPC \times Y^d + I(\bar{r}_t) + \bar{G} \]

- \( \bar{G} \) exogenous
- \( C \) depends positively on current disposable income \( Y^d \) through \( MPC \), and negatively on \( r_t \)
- \( I \) depends negatively on \( r_t \).
Goods market equilibrium

Recall that in a closed economy:

\[ S = I = S^{priv} - D^{govt} \]

- We took \( D^{govt} \) as exogenous
- \( S^{priv} = Y^d - C \) private savings increases with \( r \), therefore national saving \( S \) increases with \( r \)
- \( I \) decreases with the real interest rate
- **Goods market equilibrium** occurs at the real interest rate \( r \) for which savings equal investment.
Saving-Investment Diagram

- Real interest rate, \( r \)
- Desired national saving, \( S^d \), and desired investment, \( I^d \)
- Saving curve, \( S \)
- Investment curve, \( I \)

The diagram shows the equilibrium point \( E \) where the saving curve and investment curve intersect at a real interest rate of 6% and desired national saving and investment both equal to 1000.
Shifts in the Saving Curve

e.g. higher current or future disposable income
Shifts in the Investment Curve

- e.g. higher future productivity, employment
The Real Interest Rate