Question 1: Inflation Expectations and Real Money Demand (20 points)

Suppose that the real money demand function is\( L(y, r + \pi^e) = \frac{y^{0.5}}{r + \pi^e} \) where \( y \) is real per capita output, \( r \) is the real interest rate, and \( \pi^e \) is the expected rate of inflation. Real per capita output is constant at its steady state level, \( y^* = 10,000 \). The real interest rate is also constant at its steady state rate \( r^* = 4\% \) per year.

(a) Suppose that the nominal money supply is growing at the rate of 2\% per year and that this growth rate is expected to persist forever. Currently the nominal money supply is \( M_s = 5,000 \). What are the values of the real money demand and the current price level? (5 points)

Solution:

\[
\frac{\Delta M_t^d}{M_t} = \frac{\Delta P_t}{P_t} + \epsilon_y \frac{\Delta y_t}{y_t} + \epsilon_i \frac{\Delta (1 + i_t)}{1 + i_t}
\]

Because \( y_t \) and \( r_t \) are fixed at steady state values, \( \frac{\Delta y_t}{y_t} = 0 \) and \( \frac{\Delta (1 + i_t)}{1 + i_t} = 0 \).

\[
M_t^s = M_t^d \Rightarrow \frac{\Delta P_t}{P_t} = \frac{\Delta M_t^d}{M_t} = \frac{\Delta M_t^s}{M_t}
\]

\[
\pi^e = E\left(\frac{P_{t+1} - P_t}{P_t}\right) = E\left(\frac{M^s_{t+1} - M^s_t}{M^s_t}\right) = 2\%
\]

\[
i = r + \pi^e = 4\% + 2\% = 6\%
\]

Real money demand \( \frac{M_t^d}{L(y, r + \pi^e)} = \frac{10,000^{0.5}}{0.06} = 100 \)

The price level \( P_t = \frac{M_t^d}{L(y^*, r + \pi^e)} = \frac{5,000}{1,666.67} = 3 \)

(b) The central bank now announces that from now on the nominal money supply will grow at the rate of 4\% per year. If everyone believes this announcement, and if markets are in equilibrium, what are the values of real money demand and the current price level? (5 points)
level? Assume that the initial nominal money supply is again \( M^s = 5,000 \). (5 points)

Solution:

\[
\pi^e = \mathbb{E}\left(\frac{P_{t+1} - P_t}{P_t}\right) = \mathbb{E}\left(\frac{M_{t+1} - M_t}{M_t}\right) = 4\%
\]

\[i = r + \pi^e = 4\% + 4\% = 8\%
\]

Real money demand \( \frac{M^d_t}{P_t} = \frac{(10,000)^{0.5}}{0.08} = \frac{100}{0.08} = 1,250 \)

The price level \( P_t = \frac{M^d_t}{L(y, r + \pi^e)} = \frac{M^s_t}{L(y, r + \pi^e)} = \frac{5,000}{1,250} = 4 \)

(c) Explain the effects of such an increase in the rate of money growth on real money demand and the price level. (5 points)

Solution:

The increase in the money supply growth rate increases expected inflation, which decreases real money demand (the expected opportunity cost of holding money has increased). Given that the nominal money supply is fixed, the decrease in real money demand must increase the price level.

(d) If real per capita output jumps above its steady state level, how will nominal money demand respond? What is the elasticity of money demand with respect to income, \( \epsilon_y \), and what value does it take here? (5 points)

Solution:

Given \( \frac{M^d_t}{P_t} = L(y, i) = \frac{y^{0.5}}{1+\pi^e} \), \( M^d_t / P_t \) is clearly increasing in \( y_t \). Intuitively, with higher real income and consumption, more money is needed to facilitate real economic transactions. If nominal money demand remains in equilibrium with money supply, the price level must decrease. If instead the price level remains fixed and the money market is out of equilibrium, nominal money demand will increase. The elasticity of money demand with respect to income, \( \epsilon_y \), is the percent change in money demand caused by a one percent increase in real per capita income, \( y_t \). \( \epsilon_y = 0.5 \). To see this, take logs of \( \frac{M^d_t}{P_t} = \frac{y^{0.5}}{1+\pi^e} \):

\[
\ln(M^d_t) - \ln(P_t) = 0.5 \ln(y_t) - \ln(1 + i)
\]

The coefficient 0.5 will remain on the \( \frac{\Delta y_t}{y_t} \) term after log-linearizing. Holding the nominal interest rate constant,

\[
\frac{\Delta M^d_t}{M_t} \frac{\Delta P_t}{P_t} = 0.5 \frac{\Delta y_t}{y_t} \iff \epsilon_y = \left( \frac{\Delta M^d_t}{M_t} - \frac{\Delta P_t}{P_t} \right) \frac{\Delta y_t}{y_t} = 0.5
\]
Question 2: The Solow Growth Model (25 points)

Consider a closed economy where goods are produced according to
\[ Y_t = AK_t^\alpha (E_t N_t)^{1-\alpha} \]
where \( Y_t \) is output, \( K_t \) is the capital stock, \( E_t \) is labor efficiency and \( N_t \) is labor input. Households save a constant fraction \( s \) of current income \( Y_t \), capital depreciates at a rate \( \delta \), the population grows at a constant rate \( n \) and labor efficiency \( E_t \) grows at a constant rate \( g \).

(a) Derive the “per effective worker” production function. (5 points)

Solution:
\[ y_t = \frac{Y_t}{E_t N_t} = A \left( \frac{K_t}{E_t N_t} \right)^\alpha = Ak_t^\alpha \]

(b) Derive a dynamic equation that describes how \( k_t \), the capital stock per effective worker, evolves over time. (5 points)

Solution:
\[ \frac{K_{t+1}}{E_{t+1} N_{t+1}} = \frac{I_t + (1 - \delta)K_t}{s Y_t + (1 - \delta)K_t} \]
\[ \Leftrightarrow (1 + n + g)k_{t+1} = s Y_t + (1 - \delta)k_t \]
\[ \Leftrightarrow (1 + n + g)k_{t+1} = s Ak_t^\alpha + (1 - \delta)k_t \]

(c) Suppose \( A = 1 \), \( \alpha = 0.5 \), \( s = 0.40 \), \( \delta = 0.05 \), \( n = 0.01 \), \( g = 0.04 \). What is \( k^* \), the steady state value of \( k_t \)? (5 points)

Solution:
In the steady state
\[ sA(k^*)^\alpha = (\delta + n + g)k^* \]
\[ k^* = \left( \frac{\delta + n + g}{sA} \right)^{\frac{1}{\alpha-1}} \]
\[ = \left( \frac{0.1}{0.4} \right)^{-2} = 16 \]

(d) What is the Golden Rule level of the capital-effective labor ratio \( k_{gold}^* \)? What is the growth rate of consumption per capita \( \frac{C}{N} \) when \( k = k_{gold}^* \)? (5 points)
Solution:

$k_{gold}^*$ is such that

$$c^* = A(k^*)^\alpha - (g + n + \delta)k^*$$

is maximized, which requires

$$A\alpha(k_{gold}^*)^{\alpha-1} = g + n + \delta$$

$$k_{gold}^* = \left( \frac{g + n + \delta}{\alpha A} \right)^{\frac{1}{\alpha-1}} = \left( \frac{0.1}{0.5} \right)^{-2} = 25$$

Growth in consumption per capita $\frac{\Delta}{N}$ is $g = 0.04$ or 4%.

(e) Describe how you as a policymaker would act to achieve $k_{gold}^*$.

**Solution:**

The Golden Rule steady state can be achieved by raising the saving rate to $s = \alpha = 0.5$. This could be done by providing tax incentives that encourage saving, e.g., reducing the capital gains or corporate income taxes. (See lecture notes for more options.)

**Question 3: Labor Market (15 points)**

(a) In the classical model of the labor market, what happens to real wages and aggregate labor supply when there is an announcement of a future permanent decrease in labor taxes? (5 points)

**Solution:**

A future permanent reduction in taxes increases the net present value of lifetime income, *ceteris paribus*. This *income effect* will reduce labor supply as less work is needed to reach desired consumption levels, and decreased labor supply will increase the real wage rate. Because the after-tax wage is higher in future periods, there will also be a *substitution effect* towards future work hours decreasing present labor supply.

(b) State some reasons why, in practice, there is always unemployment. (5 points)

**Solution:**

Even when the economy is at potential output, frictional unemployment arises from labor market churn (e.g., search and matching between workers and firms) and structural
unemployment arises from non-cyclical distortions to the market clearing equilibrium wage (e.g., minimum wage laws, trade unions).

(c) Explain why in the US the natural rate of unemployment rose during 1960-1984, and fell during 1985-2006. (5 points)

Solution:
There were greater structural barriers in the 1960s to mid-1980s (e.g., a higher real minimum wage, higher rates of unionization). Frictional unemployment from shifts in labor force participation also influenced the path of the natural rate of unemployment (e.g. workforce getting younger, then aging).

Question 4: Macro Concepts (15 points)

Provide brief explanations for the following

(a) Okun’s law (5 points)

Solution:
A “rule of thumb” relationship between deviations of actual real GDP, $Y$, from potential GDP, $\bar{Y}$, and deviations of the unemployment rate, $u$, from the natural rate of unemployment, $\bar{u}$, that is consistent with GDP at potential:

$$\Delta \frac{Y}{\bar{Y}} = -2(u - \bar{u})$$

The relationship is useful for linking labor market conditions with cyclical fluctuations in aggregate output.

(b) Ricardian equivalence (5 points)

Solution:
The proposition that the timing of taxes doesn’t affect consumption levels because households consume at a constant rate of the net present value of lifetime income, and a temporary tax cut (increase) won’t change lifetime wealth because it will have to be paid for (future taxes will be reduced) by the same discounted amount.

(c) Great Moderation (5 points)

Solution:
A prolonged reduction in the volatility of U.S. business cycles beginning in the mid-1980s. Business cycle volatility then increased noticeably during the Great Recession.