1 GDP, TFP, and MPL

The following data give real GDP, $Y$, capital, $K$, and labor, $N$, for Macroland. Assume that the production function takes the following Cobb-Douglas form: $Y_t = A_t K_t^{1/3} N_t^{2/3}$.

<table>
<thead>
<tr>
<th>Year</th>
<th>$Y$</th>
<th>$K$</th>
<th>$N$</th>
<th>$A$</th>
<th>$\Delta A/A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>2000</td>
<td>3000</td>
<td>250</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>2011</td>
<td>2200</td>
<td>3100</td>
<td>260</td>
<td>3.494</td>
<td>6.00%</td>
</tr>
<tr>
<td>2012</td>
<td>2100</td>
<td>3050</td>
<td>240</td>
<td>3.750</td>
<td>1.23%</td>
</tr>
</tbody>
</table>

a. Calculate total factor productivity growth between 2010 and 2011, and between 2011 and 2012

Solution: $A_t = Y_t / (K_t^{1/3} N_t^{2/3})$

<table>
<thead>
<tr>
<th>Year</th>
<th>$\Delta Y/Y$</th>
<th>$\Delta A/A$</th>
<th>$\Delta K/K$</th>
<th>$\Delta N/N$</th>
<th>$\Delta Y/Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010 – 2011</td>
<td>10.00%</td>
<td>6.00%</td>
<td>(1/3)(3.33%)</td>
<td>(2/3)(4.00%)</td>
<td>9.77%</td>
</tr>
<tr>
<td>2011 – 2012</td>
<td>-4.55%</td>
<td>1.23%</td>
<td>(1/3)(-1.61%)</td>
<td>(2/3)(-7.69%)</td>
<td>-4.43%</td>
</tr>
</tbody>
</table>

b. Decompose contributions to real GDP growth from the capital stock, labor, and TFP

Solution: $\Delta Y/Y \approx \Delta A/A + \alpha \Delta K/K + (1 - \alpha) \Delta N/N$

<table>
<thead>
<tr>
<th>Year</th>
<th>$MP_N = \frac{dY}{dN}$</th>
<th>$\frac{dY}{dN}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>5.33</td>
<td>5.33</td>
</tr>
<tr>
<td>2011</td>
<td>5.64</td>
<td>5.64</td>
</tr>
<tr>
<td>2012</td>
<td>5.83</td>
<td>5.83</td>
</tr>
</tbody>
</table>

Note: $\Delta Y/Y = \frac{Y_{t+1} - Y_t}{Y_t}$

2 Labor market dynamics, taxes, and the minimum wage

Based on ABC Ch. 3 NP #6

Suppose that the production function is $Y = 9K^{0.5}N^{0.5}$. The capital stock is $K = 25$. The labor supply curve is $N_s = 100[(1 - \tau)w]^2$, where $w$ is the real wage rate, $\tau$ is the tax on labor income, and hence $(1 - \tau)w$ is the after-tax real wage rate.

a. Assume the tax on labor income, $\tau$, equals zero. Find the equation of the labor demand curve. Calculate the equilibrium levels of the real wage and employment, the level of full-employment output, and the total after-tax wage income of workers.

Solution: $MP_N = \frac{dY}{dN} = 4.5(0.5 N^{-0.5}) = w \iff N^{0.5} = \frac{4.5K^{0.5}}{w} \iff N = 506.25/w^2$
Equating $N^D = N^S \iff 506.25/w^2 = 100[(1-\tau)w]^2 \iff w^4 = 5.0625 \iff w = 1.5$

$N^S = 100(1.5)^2 = 225$

$Y = 9(25)^{0.5}(225)^{0.5} = 675$

After-tax wage income = $(1-\tau)wN^S = 1.5(225) = 337.5$

b. Repeat part (a) under the assumption that the tax rate on labor income, $\tau$, equals 0.4.

**Solution:** If $\tau = 0.4$, then $N^S = 100[(1-\tau)w]^2 = 36w^2$

$MP_N = \frac{4.5(25)^{0.5}}{N^S} = \frac{22.5}{N^S} = w$

Substituting in for $w$, $N^S = 36(\frac{22.5}{N^S})^2 \iff (N^S)^2 = 18225 \iff N = 135$

$Y = 9(25)^{0.5}(135)^{0.5} = 522.85$

After-tax wage income = $(1-\tau)wN^S = (0.6)(1.94)(135) = 156.86$

So the effect of the tax is to reduce labor supply, bidding up the wage rate, but the decline in output reduces pre-tax wage income, with a bigger drop in after-tax wage income.

c. Suppose that a minimum wage of $w = 2$ is imposed. If the tax rate on labor income, $\tau$, equals zero, what are the resulting values of employment and the real wage? Does the introduction of the minimum wage increase the total income of workers, taken as a group?

**Solution:** Without a tax on labor income, the market clearing wage rate was $1.5 in part (a), so a minimum wage of $2 would be binding ($w = 2$). So $N^D = 506.25/(2^2) = 126.6$

$N^S = 100(w)^2 = 100(2)^2 = 400$

Unemployment = $N^S - N^D = 400 - 126.6 = 273.4$

After-tax wage income = $wN = (2)(126.6) = 253.2$, which is lower than without the minimum wage because employment has fallen considerably.

### 3 Okun’s law

Suppose the Okun’s law coefficient is 2, the full-employment level of output is $17,000 billion, and the natural rate of unemployment is 5.5%.

a. What is the current level of output if the current unemployment rate is 8%? How big is the “output gap” between actual and potential GDP?

**Solution:**

\[
\frac{y - \bar{y}}{y} = -2(u - \bar{u}) \iff \frac{y}{17,000} = -2(0.08 - 0.055) + 1 \iff y = 16,150
\]

Output gap = $\frac{y - \bar{y}}{y} = \frac{16,150 - 17,000}{17,000} = -5.0\%$

b. Suppose the unemployment rate falls to 5%; what are the current levels of output and output gap?

**Solution:**

\[
\frac{y - \bar{y}}{y} = -2(u - \bar{u}) \iff \frac{y}{17,000} = -2(0.05 - 0.055) + 1 \iff y = 17,170
\]

Output gap = $\frac{y - \bar{y}}{y} = \frac{17,170 - 17,000}{17,000} = +1.0\%$

c. Suppose structural changes in the economy raise the natural rate of unemployment to 6.5%, and lowers the full-employment level of output to $16,000 billion. If the current unemployment rate is 8%, what is the current level of output? The output gap?

**Solution:**

\[
\frac{y - \bar{y}}{y} = -2(u - \bar{u}) \iff \frac{y}{16,000} = -2(0.08 - 0.065) + 1 \iff y = 15,520
\]

Output gap = $\frac{y - \bar{y}}{y} = \frac{15,520 - 16,000}{16,000} = -3.0\%$
4 Government deficits and interest rates
2008 Prelim #1

What happens to the real interest rate and investment after an increase in the government budget deficit for a closed economy?

Solution: As a result of the rise in the budget deficit, national saving decreases (public saving falls while private saving remains unchanged). Consequently, the equilibrium amount of saving/investment falls and the equilibrium real interest rate \( r \) increases. The rise in the budget deficit “crowds out” private investment.

Mathematically, \( S = Y - C - G \), so if \( G \uparrow, S \downarrow \). The savings curve shifting inwards (to the left) increases the equilibrium interest rate. \( S = I \), so \( S \downarrow \iff I \downarrow \).

5 Derive optimal savings using the Euler equation

Calculate period \( t \) savings or borrowing for the following two-period consumption-savings problems using the following parameters:

a. \( U(c_t, c_{t+1}) = \log(c_t) + \beta \log(c_{t+1}) \)

b. \( U(c_t, c_{t+1}) = \frac{c_t^{1 - \frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} + \beta \frac{c_{t+1}^{1 - \frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} \)

- \( y_t = 50,000 \)
- \( y_{t+1} = 10,000 \)
- \( \beta = 0.95 \)
- \( \sigma = 1.25 \)
- \( r_t = 4\% \)

If \( Y_t \) increases by $1, how will \( c_t \) change? Which utility form omits the higher marginal propensity to consume?

Solution (a): The set-up of the problem is the same under both utility forms. Starting with log utility:

\[
\max_{c_t, c_{t+1}} \log(c_t) + \beta \log(c_{t+1}) \quad \text{s.t.} \quad P_t c_t + S_t = Y_t \quad \text{(period} \ t \text{ budget constraint)}
\]

\[
P_{t+1} c_{t+1} = Y_{t+1} + S_t (1 + i_t) \quad \text{(period} \ t+1 \text{ budget constraint)}
\]

\( c_t, c_{t+1} \geq 0 \) (non-negativity constraints)

The marginal utility of consumption is infinite at zero consumption, so the non-negativity constraints will never bind and we can drop them. Combining the period \( t \) and \( t+1 \) budget constraints into an inter-temporal budget constraint with lagrange multiplier \( \lambda_t \) yields:

\[
\max_{c_t, c_{t+1}} \log(c_t) + \beta \log(c_{t+1}) \quad \text{s.t.} \quad P_t c_t + \frac{P_{t+1} c_{t+1}}{1 + r_t} = Y_t + \frac{Y_{t+1}}{1 + r_t} \quad \text{(}\lambda_t\text{)}
\]

First-order conditions:

\[
[c_t]: \quad u_c(c_t) - P_t \lambda_t = 0 \iff \lambda_t = \frac{u_c(c_t)}{P_t} \iff \lambda_t = \frac{1}{P_t c_t}
\]

\[
[c_{t+1}]: \quad \beta u_c(c_{t+1}) - \frac{P_{t+1} \lambda_t}{1 + r_t} = 0 \iff \lambda_t = \beta \frac{1 + i_t}{P_{t+1} c_{t+1}}
\]

Combining first-order conditions to derive the Euler equation:

\[
u_c(c_t) = \beta \frac{u_c(c_{t+1}) (1 + i_t) (P_t)}{P_{t+1}} \iff u_c(c_t) = \beta u_c(c_{t+1}) (1 + i_t) \iff c_{t+1} = \beta c_t (1 + r_t)
\]

Divide the budget constraint through by \( P_t \) to express income in real terms (\( y_t = \frac{Y_t}{P_t} \)) and plug in the above expression for \( c_{t+1} \):
\[ c_t = y_t + \frac{y_{t+1}}{1+r_t} - \beta \frac{c_{t+1}}{1+r_t} \quad \iff \quad c_t = \frac{1}{1+\beta} [y_t + \frac{y_{t+1}}{1+r_t}] = \frac{1}{1.95} [50,000 + \frac{10,000}{1.04}] = $30,572 \]

\[ s_t = y_t - c_t = 50,000 - 30,572 = $19,428 \]

The marginal propensity to consume out of an increase in \( y_t = \frac{1}{1.95} = 0.5128 \)

**Solution (b):**

\[
\max_{c_t, c_{t+1}} \frac{c_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \beta \frac{c_{t+1}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \quad \text{s.t.} \quad P_t c_t + \frac{P_{t+1} c_{t+1}}{1+r_t} = Y_t + \frac{Y_{t+1}}{1+r_t} \quad (\lambda_t)
\]

Combining first-order conditions to derive the Euler equation:

\[
c_t^{-1/\sigma} = \beta c_{t+1}^{-1/\sigma} \left( \frac{1+r_t}{P_{t+1}} \right) \quad \iff \quad c_t^{-1/\sigma} = \beta c_{t+1}^{-1/\sigma} \left( \frac{1+r_t}{1+r_{t+1}} \right) \quad \iff \quad c_t = c_{t+1} (\beta (1+r_t))^{-\sigma}
\]

Plug the above expression for \( c_{t+1} \) into the budget constraint:

\[
c_t = y_t + \frac{y_{t+1}}{1+r_t} - \frac{c_t (\beta (1+r_t))^{\sigma}}{1+r_t} \quad \iff \quad c_t = (1 + \beta (1+r_t)^{\sigma-1})^{-1} [y_t + \frac{y_{t+1}}{1+r_t}]
\]

\[
c_t = (1 + 0.95^{1.25}(1.04)^{0.25})^{-1} [50,000 + \frac{10,000}{1.04}] = $30,617
\]

\[ s_t = y_t - c_t = 50,000 - 30,617 = $19,383 \]

The marginal propensity to consume (MPC) out of an increase in \( y_t = (1 + 0.95^{1.25}(1.04)^{0.25})^{-1} = 0.5136 \), so the MPC is higher under the constant relative risk aversion (CRRA) preferences in part (b), with \( \sigma = 1.25 \), than under log utility (which happens to be a special case of CRRA with \( \sigma = 1 \))