**Problem 3.1) Ballistic FET design for high-speed linear amplifiers**

In class we discussed that for a Ballistic FET with a 2D electron gas channel, a parabolic band-
structure $E(k_x,k_y) = \frac{\hbar^2}{2m^*}(k_x^2 + k_y^2)$ leads to a current $I_d \propto (V_{gs} - V_T)^{3/2}$, leading to a gain or transconductance $g_m = \frac{\partial I_d}{\partial V_{gs}} \propto (V_{gs} - V_T)^{1/2}$.

(a) Show why if the gate is driven by a monochromatic input signal $V_{gs} = V_{gs}^{dc} + v_\omega e^{i\omega t}$, the output ac current has many frequencies, not just the frequency of the input signal.

(b) Assume $v_\omega << V_{gs}^{dc}$. Can you design a bandstructure that will remove these higher harmonics?

**Problem 3.2) Using the Landauer formula to explain the broadening of experimental conductance quantization steps**

We derived the expression for current:

$$J = \frac{qg_s g_v}{Ld} \sum_k v_s(k)T(k) \left[ f_L(k) - f_R(k) \right], \quad (1)$$

where $T(k)$ is the transmission probability for mode $|k\rangle$. In the ballistic FET problem for example, we used the approximation $T(k) = 0$ for $E(k) < W_0$ and $T(k) = 1$ for $E(k) > W_0$, where $W_0$ is the conduction band-edge barrier height controlled by the gate. In assignment problem 2.2, you found the current through a rectangular barrier, for which you had derived an exact transmission $T(k)$ in assignment problem 1.3. Equation 1 is the Landauer formula in disguise; in this problem, we gain a deeper insight into the Landauer formula, and use it to explain an experimental feature of quantized conductance: why the conductance quantization steps are not very sharp. This has to do with tunneling, when $0 < T(k) < 1$.

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1 The more general form for the Landauer formula reads $I = \frac{qg_s g_v}{2\pi} \int_{-\infty}^{+\infty} dE \cdot \text{Tr}[\hat{t}^\dagger \hat{t}] \cdot [f_L(E) - f_R(E)]$, where $\text{Tr}[\hat{t}^\dagger \hat{t}]$ is the trace of $\hat{t}^\dagger \hat{t}$, where $\hat{t}$ is the transmission matrix of the amplitudes of individual modes. For a single mode, it is the plain vanilla complex number amplitude $t$ whose square $|t|^2 = |t|^2 = 1 - |r|^2$ is the transmission probability. The transmission matrix $\hat{t}$ retains the interference terms in the amplitudes, which are lost in summing the squares $|t|^2$. 

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The assignments are due on Friday, Oct 9th, 2015.
(a) Consider two contacts with electrochemical potentials $\mu_L$ and $\mu_R$ separated by an arbitrary shaped 1D potential barrier $W(x)$. For a mode of energy $E(k_x)$ that has a group velocity $v(k_x)$, consider the components of the current between the left electrode and the barrier. The carriers there could have come from three sources: (1) directly from the left contact, (2) reflected from the barrier with amplitude $r_{k_x}$, and (c) transmitted through the barrier with amplitude $t_{k_x}$. Show that if the transport is elastic (no energy loss), the current carried by the mode is

$$I_{k_x} = \frac{q g_s g_v}{L} v(k_x) \left[ f_L(k_x) - |r_{k_x}|^2 f_L(k_x) - |t_{k_x}|^2 f_R(k_x) \right],$$  

where $f_L$ and $f_R$ are the Fermi functions of the contacts. Show that Eq. 2 directly leads to Eq. 1.

(b) Show that the current for a multi-channel (multiple subband) conductor is

$$I = G_0 \sum_n \frac{1}{q} \int_{-\infty}^{+\infty} dE \cdot |t_n(E)|^2 \left[ f_L(E) - f_R(E) \right],$$

where $G_0 = \frac{g_s g_v q^2}{\hbar}$ is the conductance quantum. Show that for $T \to 0$ K, if $|t_n(E)|^2 = 1$, then the conductance $G = I/V$ is $G = G_0 M$, where $M$ is the number of channels whose energies lie between $\mu_L < E < \mu_R$. This is the celebrated Landauer formula.

(c) Now consider the measured conductance in Figure 1. This conductance is measured using a split-gate constriction that changes the effective number of modes passing through the constriction. Clearly the conductance steps are not sharply defined, and there is also broadening between steps. Can you explain this feature by identifying the reason, and outlining a method you may go about quantitatively calculating it?
Problem 3.3) WKB: Tunneling, and Flash Memory

Figure 2 shows a 1-dimensional potential for an electron, which is in the state with energy $E_0$ at $t = 0$. Since there is a lower potential for $x > L_w + L_b$, the state $|E_0\rangle$ is a quasi-bound state. The electron is destined to leak out.

(a) Using WKB tunneling probability, and combining semi-classical arguments, find an analytical formula that estimates the time it takes for the electron to leak out. Find a value of this lifetime for $L_b \sim 3$ nm, $L_w \sim 2$ nm, $V_0 \sim 1$ eV, $E_0 \sim 2$ eV, and $E_b \sim 5$ eV. How many years does it take?

(b) This feature is at the heart of flash memory, which you use in computers and cell phones. Find an analytical expression that describes how the lifetime changes if a voltage $V_a$ is applied across the insulator. Estimate the new lifetime for $V_a \sim 2.8$ V. This is the readout of the memory.

Figure 2: Escape and field-emission by tunneling.
Problem 3.4) Green’s Functions, Self-Energy, Transport Through Dots

We re-cast the problem of electron current in terms of the non-equilibrium Green’s functions (NEGF) formalism. The basic strategy was to convert a closed quantum system with Schrodinger equation $i\hbar \frac{d\psi}{dt} = H\psi$ to an open quantum system by the introduction of a self-energy and source term: $i\hbar \frac{d\psi}{dt} = H\psi + \Sigma + s$. We defined the Green’s function as $G(E) = (E\mathbb{I} - H - \Sigma)^{-1}$. By defining the anti-Hermitian parts of the self-energy matrices for reservoir $m$ as $\Gamma_m = i(\Sigma_m - \Sigma_m^\dagger)$, and the source-influx as $\Sigma_{in} = \Gamma_1 f_1 + \Gamma_2 f_2$, we identified the correlation function as $G_n(E) = G\Sigma_{in} G^\dagger$, and the spectral function as $A(E) = i(G - G^\dagger)$. The density of states of the system was found to be $D(E) = \text{Trace}[A(E)]/2\pi$. The transmission was found to be $T_{12}(E) = \text{Trace}[\Gamma_1 G \Gamma_2 G^\dagger]$, the current spectrum $\tilde{I}_1(E) = \frac{e}{h} T_{12}(E)[f_1(E) - f_2(E)]$, and the net current was $I_1 = \int_{-\infty}^{+\infty} dE \tilde{I}_1(E)$. Neglect spin throughout this problem.

Consider the problem of transport through a quantum dot sitting in a wide-bandgap insulator between two metal electrodes. A single energy eigenvalue $E_0$ of the isolated dot falls between the metal Fermi levels of interest. Assume that the coupling to the left and right electrodes measured by the self-energy term is identical, both being $\Sigma = \epsilon_0 + i\eta$.

(a) Calculate the density of states of this system, and show its dependence on $\eta$. Prove that if we started with one energy state, the integral of the broadened DOS is still unity. Explain.

(b) Calculate the current through the dot as a function of the voltage at any temperature.

(c) Make a plot of the density of states and the I-V curves at $T = 300$ K and $T = 4$ K for $E_0 = 1$ eV, $\epsilon_0 = 0.01$ eV, and $\eta = 0.1$ eV.

(d) Show that the conductance can not exceed the quantum of conductance $G_0 = \frac{e^2}{h} \leq \frac{e^2}{h}$.

Problem 3.5) Conductivity by Non-Equilibrium Green’s Functions (NEGF)

In class we discussed a 2-site 1D chain to illustrate the NEGF method for evaluating quantum transport. In Problem 3.4, you did the 1-site problem, for which analytical results are available. Here, you will set up a 4-site transport problem. Assume the uncoupled on-site energies to be $E_0 = 4$ eV, the hopping energy $t = 1$ eV, and the ‘contacts’ to be identical to the inter-site hopping terms. Plot i) the system DOS, ii) the transmission $T(E)$, and iii) the I-V curves at $T = 300$K and $T = 4$K.