ECE 5390/MSE 5472, Fall Semester 2015
Quantum Transport in Electron Devices & Novel Materials

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Assignment 2

Policy on assignments: Please turn them in by 5pm of the due date. The due date for this assignment is **Wednesday, Sept 23rd, 2015**.

General notes: Present your solutions *neatly*. Do not turn in rough unreadable worksheets - learn to **take pride in your presentation**. Show the relevant steps, so that partial points can be awarded. $\boxed{\text{BOX}}$ your final answers. Draw figures wherever necessary. Please print out this question sheet and staple to the top of your homework. Write your name and email address on the cover. Some problems may lead to publishable results - be on the lookout!

Problem 2.1) Currents from wavefunctions

(a) Electrons sit in the n_z state of a heterostructure 2D quantum well of length L_z and infinite depth in the $z-$direction and are free to move in an area $L_x L_y$ in the $x-y$ directions. The energy bandstructure is $E(k_x, k_y) = \frac{\hbar^2(k_x^2+k_y^2)}{2m^\star}$. Show that the probability current density for state $|\mathbf{k}\rangle = (k_x, k_y, k_{n_z})$ is the following:

$$\mathbf{j}(k_x, k_y, k_{n_z}) = \frac{1}{L_x L_y} \cdot [\frac{\hbar}{m^\star}(k_x \hat{x} + k_y \hat{y})] \cdot \frac{2}{L_z} \sin^2(k_{n_z} z). \tag{1}$$

(b) Provide an expression for k_{n_z} and explain the result. Integrate the $z-$component to show that the 2D probability current is in the form $\mathbf{j}_{2d}(\mathbf{k}) = \frac{1}{L^d}\mathbf{v}_g(\mathbf{k})$, where $\mathbf{v}_g(\mathbf{k}) = \frac{1}{\hbar}\nabla_k E(\mathbf{k})$ is the group velocity. This is a more general result that applies also for particles that may appear 'massless'.

(c) Now fill the quantum well (k_x, k_y) states so that the 2D carrier sheet density is n_s. This defines a Fermi level E_F: find how n_s is related to E_F at any temperature. Also, what happens to E_F if $\hbar \to 0$? Why?

(d) Find the current per unit width flowing in the $+x$ direction as a function of temperature and n_s. Assume a spin degeneracy g_s and valley degeneracy g_v for each $\mathbf{k}$-state. This is of course equal to the current flowing in the $-x$ direction. Find the magnitude of the current density for $m^\star \sim 0.2m_0$ and $n_s \sim 10^{13}/\text{cm}^2$, $g_s = 2$, and $g_v = 1$ (these values are typical for GaN HEMTs; g_v is larger for Silicon MOSFETs).

(e) In some 2D semiconductors such as graphene, the energy bandstructure is not parabolic, but conical: $E(k_x, k_y) = \hbar v_F \sqrt{k_x^2 + k_y^2}$, where v_F is a characteristic 'Fermi' velocity. Find the $+x$ directed current per unit width for graphene as a function of temperature. Find the magnitude of the current per unit width for $v_F \sim 10^8$ cm/s, $n_s \sim 10^{13}/\text{cm}^2$, and $g_s = 2$, and $g_v = 2$, the values for 2D graphene.

Problem 2.2) Tunneling vs thermionic currents across a 1D barrier

In problem 1.3 of assignment 1, you found the transmission (or tunneling) probability of an electron across a 1D rectangular barrier. Assume that on the two sides of the barrier we have identical 1D metals, and the barrier is a 1D insulator. This sort of situation occurs very often: the structure is called a M-I-M structure for obvious reasons.

(a) Using the quantum expression for current, find an expression for the total 1D current I_{tot} flowing in the 1D M-I-M structure at any temperature T when a voltage V is applied across the metal electrodes. Assume plane waves in the metal with an effective mass of $m^{\star}$. Neglect any change in the potential barrier shape, but explain why it *must* change!

(b) Plot the total current I_{tot} in the M-I-M structure in the log-scale vs the applied voltage V in linear scale at temperatures $T = 4, 77, 300$ K for $m^{\star} = m_0$, $V_0 = 1.0$ eV, and barrier thickness $a = 10$ nm, and for $a = 1$ nm. Re-plot in linear scale and explain why this device is also called a M-I-M *diode*.

(c) The part of the current I_{th} flowing *over the barrier* is called the 'thermionic' current. Derive an expression for this part, and plot it for the same conditions as part (b). From the temperature dependence explain the name 'thermionic'.

(d) Now derive an expression for, and plot the *tunneling* portion of the current I_{tun} vs voltage for the same condition as the thermionic part for various temperatures. Examine the temperature dependence of this portion of the current.

(e) Now show the decomposition of the plots in part (b) into the thermionic and tunneling components: $I_{tot} = I_{th} + I_{tun}$ of the current. Clearly indicate which component dominates under what conditions. Are they consistent with what may be intuitively expected?

Problem 2.3) Current saturation in a Carbon Nanotube

If high energy electrons collide with the lattice and emit optical phonons at a very fast rate, they come to equilibrium with the lattice rather than the source and drain electrodes. Assume we have a metallic carbon nanotube that has a 1D energy dispersion $E(k) = \hbar v_F |k|$ with a spin degeneracy of $g_s = 2$ and a valley degeneracy $g_v = 2$. Show that if the optical phonon energy is $\hbar\omega_{op}$ and the above ultrafast optical phonon emission occurs, then the saturation current in the nanotube is given by $I_{sat} = \frac{q g_s g_v \omega_{op}}{2\pi}$. Find the magnitude of this current for $\hbar\omega_{op} \sim 160$ meV, and compare with experimental data (give references).

Problem 2.4) Ballistic FETs

We derived the characteristics of a ballistic field-effect transistor in class. Assume the gate barrier has a thickness $t_b = 4$nm, and a dielectric constant $\epsilon_b = 20\epsilon_0$.

(a) Plot the 77K and 300K $I_d - V_{ds}$ and $I_d - V_{gs}$ characteristics of ballistic FETs made of a semiconductor with dispersion $E(k) = \hbar^2 k^2 / 2m^{\star}$ with $m^{\star} = 0.05$ and $m^{\star} = 0.2$. Use a spin degeneracy of $g_s = 2$ and a valley-degeneracy of $g_v = 1$. Compare and comment: the smaller effective mass represents a narrow-gap semiconductor (such as GaAs) and the larger effective mass represents a wide-bandgap semiconductor (such as GaN).

(b) Find an expression for the effective carrier injection velocity v_{inj} by writing the current per unit width as $I_d = qn_s v_{inj}$ where $n_s \sim C_g(V_{gs} - V_T)$ in the on-state of the ballistic FET. Make plots for the parameters in part (a). Note that not all the n_s carriers are *actually* moving at uniform velocity of v_{inj}. Make a 'spectral' plot of the number of carriers vs the velocity in the direction of the source/drain contacts, that runs from -ve to +ve velocities, for 77K and for 300K for the parameters for part (a).

(c) Find expressions for the gain (transconductance per unit width, $g_m = \frac{\partial I_d}{\partial V_{gs}}$) for the ballistic FET as a function of the gate voltage V_{gs} and small $V_{ds} << kT/q$, and for V_{ds} in current saturation. Make plots for the parameters of part (a) and comment.

(d) A popular method to extract the field-effect mobility in FETs in the 'resistor' or linear region of operation where the electric field driving transport is $F \sim V_{ds}/L$ is the following: For a channel length L use $qn_s \sim C_b(V_{gs} - V_T)$ with the drift current per unit width $I_d = qn_s\mu\frac{V_{ds}}{L}$ to write $I_d = C_b(V_{gs} - V_T)\mu\frac{V_{ds}}{L}$, and take the slope of the measured $I_d - V_{gs}$ curve to extract μ. Because C_b, V_{ds} and L are precisely known, this gives the unknown μ. Find an expression for the effective 'mobility' that will be measured when this technique is applied to a *ballistic* FET, and why the results must not be trusted.